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## Essentials of Microeconomics: Exercises

Krister Ahlersten


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# Microeconomics 

## Exercises

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## 1. Consumer Theory

### 1.1 Preferences

## Exercise 1.1.1

A basic assumption about consumers in microeconomics is that they have preferences over different baskets of goods. Explain the concepts "preference", "preference order", and "basket of goods".

## Exercise 1.1.2

a) If there are only two goods, it is possible to illustrate a consumer's preferences over them with an indifference map. Draw an indifference map with three indifference curves.
b) There are a few standard assumptions about what an indifference map can and cannot look like. Which are these assumptions, and what reasoning lies behind them?

## Exercise 1.1.3

a) What is the marginal rate of substitution, MRS? State the definition and explain, in words, what it means.
b) MRS will have an influence on the shape of an indifference curve. What influence?

## Exercise 1.1.4

a) Often, we assume that consumers have diminishing MRS. Explain what that means and how it is reflected in indifference curves.
b) Can you draw an indifference curve that does not have diminishing MRS, but that is still allowed?

## Exercise 1.1.5

a) In Figure E.1.1, we have drawn an indifference curve for a certain consumer. Calculate an estimate of her marginal rate of substitution, MRS, in point $A$.
b) Can we say anything about whether point $B$ is better or worse for the consumer, as compared to point $A$ ?
c) What about point $C$ ?


## Exercise 1.1.6

Explain the relation between marginal willingness to pay and marginal rate of substitution, MRS.

## Exercise 1.1.7

a) Explain what substitute goods and complementary goods are.
b) Draw a diagram for two goods, with the quantity of good 1 on the X -axis. What will the indifference curves for substitute goods look like? What will they look like for complementary goods?

### 1.2 The Budget Line

## Exercise 1.2.1

a) Explain in words what the budget line is.
b) Suppose we have two goods. The price of good 1 is 10 and the price of good 2 is 15 . The income is 30 . Construct a diagram, with the quantities on the X -and Y -axes, and draw a budget line in the diagram.
c) How do the prices and the income affect the shape of the graph? What happens if the price of one good rises? What happens if income increases?

## Exercise 1.2.2

a) State the definition of the marginal rate of transformation, MRT. Explain what it means in words.
b) Calculate MRT in Exercise 1.2.1.

## Exercise 1.2.3

a) Suppose there are two goods in a market, and that you buy q1 of the first and q2 of the second. Give a mathematical expression for the total cost.
b) Now, use the answer to a) to show that the marginal rate of transformation, MRT, is equal to the slope of the budget line.

### 1.3 Utility Maximization

## Exercise 1.3.1

a) Explain briefly, what utility maximization is.
b) What is a utility function?
c) What is the criterion that a consumer maximizes her utility? Give the answer in the form of a mathematical expression.

## Exercise 1.3.2

a) Suppose a consumer has two goods from which to choose. Draw a graph, with quantities on the X - and Y -axes, that illustrates how she can choose, given prices and income.
b) Also, illustrate a few indifference curves in the graph.
c) Show how the consumer maximizes her utility and where in the graph this occurs.
d) Can you give an example of a situation in which the consumer will find more than one point where she maximizes her utility? Think about what the indifference curves must look like to make this possible.

## Exercise 1.3.3

Look at Figure E.1.1 again. Suppose the consumer maximizes her utility at A, and that the price of good 2 is 100 . What is the price of good 1 ? How large is the consumer's income?

## 2. Demand

### 2.1 Price Changes

## Exercise 2.1.1

a) Suppose there are two goods a consumer can choose between, and that the prices are equal. First, construct a diagram, with quantities on the X - and Y -axes, where you show a utility maximizing choice for the consumer.
b) Then, show what happens if you vary the price of good 1. Construct one budget line corresponding to the case when the price is cut by half, and another one when it is doubled. Will the consumer maximize her utility in the same point as before? Show how to derive the price-consumption curve using this technique.
c) Use the price-consumption curve to derive the consumer's demand curve for good 1.
d) Suppose that you also have another consumer's demand curve. Show in a new diagram how you can derive the market's demand curve, assuming the market only consists of these two consumers. You may assume that the consumers' demand curves are straight lines.

### 2.2 Income Changes

## Exercise 2.2.1

Start, similarly to the previous exercise, with a consumer who has two goods between which she can choose. However, instead of varying the price, you now vary the income. Derive the incomeconsumption curve. Use the cases when the income is either doubled or cut by half. Then, use the income-consumption curve to derive the Engel curve.

## Exercise 2.2.2

a) Suppose there are two goods, that the prices are given, and that there is a consumer with a certain income. Show in a diagram how it is possible to split the effect of a price fall on good 1 into the income- and substitution effects. Assume that the good is a normal good.
b) If the good had been an inferior good, what would have been different in the graph?
c) If the good had been a Giffen good, what would have been different?

## Exercise 2.2.3

Can a Giffen good be a normal good? Why or why not? Use a market with only two goods in your reasoning.

### 2.3 Elasticities

## Exercise 2.3.1

a) State the definitions of price elasticity (of demand), income elasticity, and cross-price elasticity. What do these definitions mean in words?
b) In the graph in Figure E.2.1, D1 is the demand for a certain good at different prices. Calculate the price elasticity of the good at point A and point B . Do you get the same answer in both points? Why or why not?
c) If the slope of D1 would change, so that demand becomes a horizontal line through point A, what would the price elasticity in point A be?
d) If income increases by $10 \%$, D1 shifts to D2. Calculate an approximate value for the income elasticity at point A .
e) Suppose the price of the good is 5 , and that is increases by $5 \%$. As a consequence, the demand of another good decreases by $20 \%$. Calculate the cross-price elasticity for the other good. Is the other good a substitute good or a complementary good to the first one?

Figure E.2.1


## 3. Production

### 3.1 Definitions

## Exercise 3.1.1

a) Sometimes it is said that producer theory is similar to consumer theory. In what ways are they similar?
b) Describe in words what a production function is. Which variables are typically inputs?
c) What is the difference between the short and the long run?
d) What does "returns to scale," mean?

## Exercise 3.1.2

a) State the definition of marginal product, MP, both as a mathematical definition and with your own words.
b) What is the "law of diminishing marginal returns"? How has it been derived?


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## Exercise 3.1.3

a) State the definition of the marginal rate of technical substitution, MRTS. What does that mean, in your own words?
b) Show how to derive a relation between the marginal products of labor and capital, MPL and MPK, and MRTS.

### 3.2 The Production Function

## Exercise 3.2.1

In the short run, the relation between number of hours worked and quantity produced looks like in the table.

| $L$ | $q$ |
| :---: | :---: |
| 0 | 0 |
| 20 | 30 |
| 40 | 100 |
| 60 | 170 |
| 80 | 210 |
| 100 | 200 |

a) Draw a graph of what the production curve looks like.
b) Explain the concepts of "average product of labor," $\mathrm{AP}_{\mathrm{L}}$, and "marginal product of labor," $\mathrm{MP}_{\mathrm{L}}$, and what they correspond to in the graph.
c) Draw another graph below the production curve, illustrating the shapes of $\mathrm{AP}_{\mathrm{L}}$ and $\mathrm{MP}_{\mathrm{L}}$. Explain how to find the most characteristic points for $\mathrm{AP}_{\mathrm{L}}$ and $\mathrm{MP}_{\mathrm{L}}$ on the production curve and indicate the relations in the graphs.

## 4. Costs

### 4.1 Costs in the Short Run

## Exercise 4.1.1

Suppose the production of a certain quantity of a good has a certain cost. Can you think of a situation in which producing more of the good costs less?

## Exercise 4.1.2

A firm has the following costs for the short-run production of different quantities of a good:

| $q$ | $C$ |
| :---: | :---: |
| 1 | 30 |
| 20 | 40 |
| 40 | 60 |
| 60 | 80 |
| 80 | 130 |
| 100 | 220 |

a) Construct a diagram of the cost function, where you have the quantity on the X -axis and the cost on the Y-axis.
b) How do you find the fixed cost, FC, from the information in the graph? Draw a line indicating the fixed cost at different quantities produced.
c) How do you find the variable cost, VC? Draw it.
d) Draw a new graph below the first one. Draw the marginal cost curve, MC, and the curves for average total cost, ATC, and average variable cost, AVC.
e) Which are the most characteristic points in the total cost curve? Indicate them at the appropriate points in the lower graph. Which are the relations between the characteristic points in the upper and lower graphs?

### 4.2 Costs in the Long Run

## Exercise 4.2.1

a) In the long run, both labor, L, and capital, K, are variable costs. Show in a graph, where you have the quantity of L on the X -axis, and the quantity of K on the Y -axis, how one can indicate combinations of L and K that cost the same to produce. What is this type of lines called?
b) Then show how one can indicate combinations of L and K that produce the same quantity of the good. What is this type of lines called?
c) The firm always wants to minimize its cost of production. Choose a certain quantity in your graph, and show how the firm would minimize its cost of producing that quantity.
d) What is the mathematical criterion for a cost-minimizing choice of L and K ? What does that correspond to in the graph?
e) Show, in your graph, how to derive the long-run expansion path.
f) Show how to derive the short-run expansion path.
g) Use the information in your graph to derive the long-run cost curve. First, choose levels for the cost and the production in the graph you have constructed. Then, draw a new graph, with the quantity produced, q , on the X -axis, and the cost, C , on the Y -axis.


## Exercise 4.2.2

In Figure E.4.1, we see the long-run average cost for the production of a good, LRAC.

Figure E.4.1

a) In the short run, capital is a fixed cost. Draw, for a few different values of K, what the short-run average cost, SRAC, looks like in relation to the long-run average cost.
b) Sometimes, one talks of (dis-) economies of scale. What in the graph indicates whether we have economies or diseconomies of scale?

## 5. Perfect Competition

### 5.1 Definitions and Assumptions

## Exercise 5.1.1

a) What does "perfect competition" mean? State a few of the underlying assumptions.
b) Explain in words why the demand curve a firm faces in a perfectly competitive market is horizontal.
c) For an individual firm in a perfectly competitive market, the marginal revenue, MR, is equal to the price, p. Why is that?

### 5.2 The Firm's Short-Run Profit Maximization

## Exercise 5.2.1

We will now study the choice of which quantity to produce for an individual firm in the short run. Draw a graph with produced quantity on the X -axis and cost/revenue (i.e. amount of the currency of your choice) on the Y-axis.
a) You are given data over total cost, TC, at different quantities produced. Draw the corresponding TC curve.

| q | TC |
| :---: | :---: |
| 0 | 0 |
| 20 | 60 |
| 40 | 80 |
| 60 | 100 |
| 80 | 130 |
| 100 | 240 |

b) For a firm in a perfectly competitive market, the total revenue curve, TR, is unusually easy to draw. What will it look like? Draw TR in your figure. Remember that if you sell nothing, your revenue is zero. The price of the good is 2.20 .
c) Below the graph, construct another graph with the same scale on the X -axis.

First, draw the curve for average variable cost, AVC. Be careful to get the minimum point in the right place. How can you know at which quantity AVC reaches its lowest point?
Then, draw the marginal cost curve, MC. At least one point is easy to find. Which one? Where will the MC curve be above the AVC curve and where will it be below it? Lastly, draw the marginal revenue curve, MR.
d) Show how to find the point where the firm maximizes its profit. Where is that in the graph?
e) The profit can be found in two different ways. Show both of them. Approximately, how large is the profit.
f) How can one find the firm's short-run supply curve from the graph? Indicate it in the graph.
g) Can you find the firm's long-run supply curve in the graph?

### 5.3 The Firm's Long-Run Profit Maximization

## Exercise 5.3.1

a) Describe in a few sentences how to derive the market's short-run supply curve from the individual firms' short-run MC curves.
b) Describe how to find the markets' long-run supply curve.

## Exercise 5.3.2

On the left-hand side of Figure E.7.2, you see the total market supply and demand. Together, they determine the market price, $\mathrm{p}^{*}$, and total quantity, $\mathrm{Q}^{*}$. On the right-hand side, you see a representative individual firm's marginal cost, MC , and average variable and average total cost, AVC and ATC.

The firm faces the price determined by the market, and therefore $M R=p^{*}$.

a) Will this firm make a profit, a loss, or break even in the short run? Why? How much will it produce?
b) Describe the forces that will affect this situation in the long run. How will a long-run equilibrium arise? What will happen to $\mathrm{p}^{*}$ ? What will happen to the number of firms in the market? How will it affect this firm's and other firms' profits or losses?


## 6. Monopoly

## Exercise 6.1.1

Why do monopolies arise? Give a few examples of underlying structures that can generate a monopoly in a market.

### 6.2 Monopoly Profit Maximization and Efficiency Problems

## Exercise 6.2.1

A certain monopoly firm has a marginal cost that depends on the quantity produced. The marginal cost is $\mathrm{MC}=2 * \mathrm{Q}$. You are also given a few values regarding the firm's average total cost, ATC, at different quantities:

| Q | ATC |
| :---: | :---: |
| 2 | 20 |
| 5 | 12.5 |
| 7 | 12 |
| 10 | 13 |
| 12 | 15 |
| 15 | 18 |
| 20 | 23 |
| 25 | 27 |

As a direct consequence of the shape of the demand curve, the marginal revenue curve becomes $\mathrm{MR}=30 \quad 2^{*} \mathrm{Q}$.
a) Construct a graph with quantity on the X -axis and your currency of choice on the Y -axis. Draw the MC-, MR-, ATC- and demand curves in the graph.
b) Why is the MR curve steeper than the demand curve?
c) How large quantities will the firm produce if it maximizes its profit?
d) Which price will they charge?
e) Calculate the profit.
f) Indicate the producer- and consumer surpluses in the graph.
g) Indicate the deadweight loss in the graph. Can you calculate how large it is? (Calculate how large the area you have indicated is.)
h) If the firm had operated in a perfectly competitive market instead, what would the equilibrium price have been? How would producer- and consumer surplus have been different?
i) Is the monopoly Pareto efficient? Why or why not?

### 6.3 Price Discrimination

## Exercise 6.3.1

A monopoly firm can take advantage of its market power by using price discrimination. Briefly describe price discrimination of the 1 st, 2 nd, and 3rd degrees. Also, state what conditions have to be fulfilled in order to use the different types of price discrimination.


## 7. Game Theory

### 7.1 Basic Concepts

## Exercise 7.1.1

For a game (in the game theoretic sense), we need to specify the players. What else needs to be specified?

What is the difference between a normal-form game and an extensive-form game?
Define in words what a dominant strategy is.
What is a payoff-matrix?

### 7.2 Games on Normal Form

## Exercise 7.2.1

Two individuals, A and B , who like each other, have arranged a date. They will meet either at a pop concert or at a techno party. However, they have not decided on which of the two.

A prefers techno whereas B prefers pop. However, they both prefer being at the same event as the other to going alone to the pop concert or to the techno party.

Suppose they cannot communicate, and therefore must decide separately. Then the game can be represented as in Figure E.7.1. The worst outcome is that they end up alone at their least preferred event. The best outcome for A is that they both go to the techno party, but that is only the second best outcome for B. The best outcome for B (and the second best for A) is that they both go to the pop concert.

a) What is a Nash equilibrium? Give a definition in words.
b) Find all Nash equilibria in the game.
c) To avoid this type of problems in the future, A and B decide on the following rule: If a game such as the one in Figure E.7.1 arises, then we go to the one that A prefers." Does that rule constitute an improvement for B ?

### 7.3 Games on Extensive Form

## Exercise 7.3.1

One day you lose your wallet. In it, you had 500 and some valuables that others cannot use, such as a few old photos. It will cost you another 500 to get new copies of the photos and replace the other valuables. Consequently, the wallet is worth 1,000 to you.

Fortunately, someone finds your wallet. She opens it and sees that it contains 500 . She thinks that if she keeps the money and throws the wallet away, she will get 500 . However, if she returns it to you she might get a reward. After all, it is worth 1,000 to you. Suppose you give her either 600 in reward or nothing.

We can represent this game as in Figure E.7.2.

a) What is the name of the method used to find the subgame perfect equilibrium?
b) Which is the subgame perfect equilibrium in Figure E.7.2?
c) Is the equilibrium efficient or not? Why or why not?
d) Can you think of a way to change the structure of the game, such that a better equilibrium will arise?

## 8. Oligopoly

## Exercise 8.1.1

a) Explain the difference between monopoly, duopoly, and oligopoly.
b) What does a "kinked demand curve" mean?
c) What is a reaction function?

### 8.2 The Cournot Model

## Exercise 8.2.1

A popular model to analyze duopolies with is the Cournot model.
a) Which are the assumptions behind the Cournot model?
b) In Figure E.8.1 we have drawn the reaction functions for two firms, and labeled them r1 and r 2 . Which is the Nash equilibrium and why?


### 8.3 The Bertrand Model

## Exercise 8.3.1

In the Bertrand model, we have two firms that set prices (instead of quantities), without knowing the price that the other firm has set.

This can be thought of as a closed bid auction. Two firms get to make an offer on how much they will demand in compensation for a certain assignment. The one that has made the lowest bid wins the contract, and in the case that they have made the same bid they get to split it in two.

Assume that the two firms are identical: They have the same cost of production, etc. Which price would the Bertrand model predict, i.e. which price is a Nash equilibrium?


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## 9. Monopolistic Competition

## Exercise 9.1.1

We have a firm that produces shoes. There are many competitors in the market but through a series of aggressive PR-campaigns, we have built a rumor around our brand, the X-shoe, so that it is perceived as somewhat special. Unfortunately, several of our competitors have done the same. Because of this, we have some power of price setting in the market. If we increase our price, some of our customers will change brands, but not all of them. Our most diehard fans will stay. If we, on the other hand, lower the price we will attract some customers from our competitors, but not all. There are no barriers to entry for new firms. If they want to, they may even copy our PR-strategy.
a) Show in a graph how this situation can be described. The marginal cost depends on the quantity produced: $\mathrm{MC}=2 * \mathrm{Q}$. The (inverse) demand curve is $\mathrm{p}=30 \mathrm{Q}$. The marginal revenue curve of the firm is MR $=30 \quad 2 * \mathrm{Q}$. Furthermore, the firm's average total cost, ATC, at different quantities are:

| Q | ATC |
| :---: | :---: |
| 2 | 20 |
| 5 | 12.5 |
| 7 | 12 |
| 10 | 13 |
| 12 | 15 |
| 15 | 18 |
| 20 | 23 |
| 25 | 27 |

b) Which price and which quantity will this firm choose if it wants to maximize profit? How large will the profit be?
c) Compare your answer to the answer to Exercise 6.2.1. Is there any difference? If there is not, why is this conceived of as another market form that monopoly?
d) If no barriers to entry exist, the long-run situation will be different. How will this change the graph you have drawn?
e) Is the situation in the short run, and in the long run, efficient? Why or why not?

## 10. Labor

### 10.1 The Supply of Labor

## Exercise 10.1.1

A certain individual can choose between work and leisure. She prefers leisure, but if she works, she receives a wage that makes it possible for her to buy things she also wants. In Figure E.10.1, we have illustrated a part of her problem of choosing. Her wage is increased from 20 to 32 and her budget line therefore rotates from BL1 to BL2. We have also drawn two of her indifference curves regarding leisure, LS, and wage, w.
a) Obviously, the individual's utility is increased because of the increase in wage. She also chooses to work more. (She reduces leisure.) To what extent does this depend on the substitution effect and the income effect, respectively?

b) In part a), the individual chose to work more when the wage was increased. It is, however, possible to get the opposite effect, i.e. that she would choose to work less when the wage is increased. Draw an indifference curve in the figure that would have caused the individual to do that.
Explain that effect in words. Why does it arise?
c) The effect you studied in part b), does it typically arise for high or for low wages? Alternatively, is it independent of wage?

### 10.2 The Demand for Labor

## Exercise 10.2.1

When a firm is to determine how much labor it needs, it is often interested in how revenue is affected by, say, one more hour of labor. This is called "the marginal revenue product of labor," MRPL, and can be defined as

$$
M R P_{L}=\frac{\Delta T R}{\Delta L}
$$

## Exercise 10.2.2

Suppose we have perfect competition in both the labor market and in the output market. Show that this leads to the equilibrium wage being equal to the marginal revenue product of labor, i.e. that

$$
w=M R P_{L}
$$

## Exercise 10.2.3

a) How will the equilibrium criterion in Exercise 10.2.2 change if the firm is a monopolist in the output market?
b) Will the firm, in that case, demand more, less, or an equal amount of labor?

## Exercise 10.2.4

a) Suppose the output market is, again, a perfectly competitive market and that there are many workers. However, let the firm be a monopsonist in the labor market. As compared to Exercise 10.2.2, will the firm demand more, less, or an equal amount of labor? In your answer, you can assume that the supply of labor increases with higher wages.
b) Will the equilibrium wage be affected? In that case, how?

## 11. General Equilibrium

### 11.1 Definitions

## Exercise 11.1.1

a) What is a Pareto improvement?
b) What is Pareto efficient?
c) What is a zero-sum game?
d) There are three criteria for Pareto optimal welfare. State all three.
e) State the two theorems of welfare economics.

### 11.2 Efficient Production

## Exercise 11.2.1

In Figure E.11.1, we have drawn a so called Edgeworth-box. It shows isoquants and quantities for the production of two goods, apples and bananas, given different combinations of labor and capital.
a) Point $a$ does not represent an efficient production of apples and bananas. Why not?
b) Indicate all points in the graph that are Pareto improvements compared to point $a$.


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Figure E.11.1
Capital
c) Indicate all points in the graph that are both Pareto improvements as compared to a, and are Pareto efficient. An approximate answer is sufficient.
d) What is the criterion for Pareto efficient production? What does that correspond to in the figure?
e) What is the "production contract curve"? Draw an approximate production contract curve in the graph.

f) In Figure E.11.2, we have drawn the two axes for a production transformation curve (production possibilities curve). Use the information from Figure E.11.1 to construct the full curve.
g) Suppose we produce in point $b$ in Figure E.11.2. That point is not efficient. What is the alternative cost of changing to an efficient production?

## 12. Choice under Uncertainty

## Exercise 12.1.1

If you throw a die, you will get a number between 1 and 6 with equal probability. What is the expected value?

## Exercise 12.1.2

In Figure E.12.1, we have drawn the amount of utility a certain individual gets from different levels of wealth.

a) Does this person have diminishing, increasing, or constant marginal utility of wealth?
b) Is she risk-averse, risk-neutral, or a risk-lover? Why?

Suppose she has 500,000 and is invited to participating in a lottery that with a probability of $50 \%$ increases her wealth to $1,000,000$ and with equal probability causes her to lose everything.
c) What is the expected value of the lottery?
d) What level of utility does she achieve if she does not participate in the lottery? Indicate that point in the graph.
e) What is her expected utility if she does participate in the lottery? Indicate that point as well.
f) Indicate in Figure E.12.1, what represents the risk premium.

## 13. Other Market Failures

### 13.1 Basic Concepts

## Exercise 13.1.1

a) What is an externality?
b) What is the difference between positive and negative externalities? Do both of these constitute economic problems?
c) What is a public good? State two criteria that must be fulfilled for a good to be a public good.
d) What does "free riding" mean? State an example when free riding can be a problem.

### 13.2 Externalities

Exercise 13.2.1

A firm that produces pulp also emits smelly pollution. The more pulp it produces, the more pollution it emits. The pollution primarily affects the people who live in the area.


Suppose the pulp is sold in a perfectly competitive market and that the firm has linear marginal cost, MC, which increases with production.

Suppose also that the marginal cost of pollution, ME (the marginal cost of the externality), increases proportionally to the quantity produced, and is approximately $1 / 3$ as large as the firm's marginal cost.
a) Draw a diagram with quantity of pulp on the X -axis and cost/revenue on the Y -axis. Indicate the profit maximizing choice of quantity given the assumptions.
b) How should the social cost be represented in the graph? Draw it.
c) Show in the graph how to find the socially optimal quantity. Will that quantity be higher or lower than in the answer to a)?
d) Suggest a solution how to motivate the firm to produce the socially optimal quantity.

### 13.3 Public Goods

## Exercise 13.3.1

Figure E.13.1


Two individuals, A and B , have decided to arrange a small park between their respective houses. However, they have very different opinions how big this park should be. In Figure E.13.1, we see their different marginal willingness to pay. We have also drawn the marginal cost, MC, of producing different quantities of park.

Show how A and B can decide on the optimal quantity of park.

## Suggested Solutions

## 1. Consumer Theory

### 1.1 Preferences

## Exercise 1.1.1

A basket of goods is a certain mix of different goods and/or services. For example, 2 ice creams and 1 liter of milk is a basket of goods. Everything you consume at a certain point in time is another, probably more complex, basket of goods.

Preferences are what one prefers to other things, i.e. an expression of one's taste.
A preference order is a sort of (imaginary) list over how a certain consumer values all possible baskets of goods. For any two items on the list, there are three possibilities: The consumer prefers the first to the second, the second to the first, or she is indifferent between the two.

Assumptions about preference orders are typically:

- Complete. All possible baskets are possible to find in the preference order. In other words, the consumer always knows what she prefers.
- Transitive. If she prefers A to B, and B to C, then she prefers A to C.
- Non-satiation. More of a good is always better.
- Convexity. If there are two baskets between which a consumer is indifferent, then she will prefer (or at least be indifferent) a mix on the two. For instance, she will prefer the average basket to both of the original baskets.


## Exercise 1.1.2

Assumptions about indifference curves:

- A consumer always prefers points that are to the northeast of a given point and, vice versa, prefers a given point to all points southwest of it. This depends on the assumption of non-satiation. This implies that an indifference curve cannot slope upwards.
- For the same reason, indifference curves that lie northeast of a given indifference curve must correspond to a higher level of utility.
- The slope of an indifference curve diminishes as quantity increases (i.e. to the right in a graph). This depends on the diminishing marginal utility of consumption. See, however, the answer to Exercise 1.1.4 b.
- Two indifference curves cannot intersect. This is easy to see if one thinks of indifference curves as elevation contours on a map.


## Exercise 1.1.3

a) Definition of MRS:

$$
\text { MRS }=\frac{\Delta q_{2}}{\Delta q_{1}}
$$

The marginal rate of substitution is similar to a price that the consumer is willing to pay for a good in terms of another good. To get one additional unit of good 1 , how many units of good 2 is she willing to give up?
b) MRS is the slope of an indifference curve. If one has a given value of MRS in a certain point, the indifference curve going through that point must have that slope.


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## Exercise 1.1.4

a) Diminishing MRS means that the more one has of a certain good, the less interested one is to get even more of it. Consequently, one is willing to give up a lot of it (since one does not value it) to get more of another good.
Diminishing MRS will mean that the indifference curves will slope less to the right in a graph.
b) dPerfect substitutes (that have indifference curves that are straight lines) do not have diminishing MRS. They have constant MRS. Complementary goods do not have diminishing MRS for additional units of only one of the goods. They have a constant MRS equal to zero.

## Exercise 1.1.5

a) MRS is the slope of the indifference curve at a certain point. Below, we have drawn the approximate slope of the curve through point A . To get a numerical value, we read off the values at the X - and Y -axes and insert them into the definition:

$$
M R S=\frac{\Delta q_{2}}{\Delta q_{1}}=\frac{2}{3}
$$

At point A , the consumer is consequently willing to trade 3 units of good 1 against 2 units of good 2.
b) Since A and B are on the same indifference curve, the consumer must be indifferent between them.

c) At point $C$, the consumer gets more of good 1 but less of good 2 . If there had not been any indifference curve in the figure, we would not have been able to answer the question. However, since $C$ is to the northeast of the indifference curve, it must be better than both $A$ and $B$.

## Exercise 1.1.6

The marginal willingness to pay is how much a consumer is willing to pay for an additional unit of the good. This can be expressed in money or in how much of another good she is willing to give up.

The latter is the same as the marginal rate of substitution.

## Exercise 1.1.7

a) Substitute goods are goods where one of them can be used instead of the other, for instance green and blue pens. Many goods are imperfect substitutes: For instance, a habitual coffee drinker could drink tea instead of coffee, but would still prefer coffee if it is available. If the consumer is completely indifferent between the goods, we say that they are perfect substitutes.
Complementary goods are goods that typically go together. A standard example is left and right shoes. One might also argue that cars and petrol are complementary goods. It is possible to define substitute- and complementary goods in terms of cross-price elasticity. Suppose the price of good 1 rises. The demand on that good would then typically decrease. If demand for good 2 also decreases, good 2 is a complementary good. In the opposite case, it is a substitute good.
b) See Figure S.1.2 for indifference curves for perfect substitutes and perfect complementary goods.

Figure S.1.2


### 1.2 The Budget Line

## Exercise 1.2.1

a) The budget line corresponds to all baskets that cost the same as the consumer's income. The budget line therefore defines her possible consumption choices.
b) Since we have 30 and the price of good 1 is 10 , we can maximally consume 3 units of good 1 . The price of good 2 is 15 , so we can maximally consume 2 units of good 2 . We indicate these points on the axes (see Figure S.1.3) and draw a straight line between them. The area on and under the line corresponds to all baskets we can consume.

Figure S.1.3

c) The relative price will define the slope of the budget line (which is $\mathrm{p} 1 / \mathrm{p} 2$ ). The income will affect the distance of the budget line from the origin.
If the price of good 1 falls, we can consume more of it. Say that the price falls from 10 to 6. Then we can maximally buy 5 units of it. Consequently, the budget line rotates to the broken line in the figure.

If income increases, we can consume more of both goods. Say that the income increases to 40 , but that the prices remain the same. Then we can maximally consume 4 units of good 1 or 2.7 units of good 2 . Consequently, the whole line shifts outwards to the dotted line in the figure.

## Exercise 1.2.2

a) The marginal rate of transformation, MRT, can be calculated as

$$
M R T=-\frac{p_{1}}{p_{2}}
$$

MRT is consequently the relative price between good 1 and good 2 : the value of good 1 in terms of good 2 .
b) Inserting the prices from last exercise, we get

$$
M R T=-\frac{10}{15}=-\frac{2}{3}
$$

If one wants to trade good 1 for good 2, one has to give up 3 units of good 1 to get 2 units of good 2.

Exercise 1.2.3
a) If you buy the quantity $q 1$ of good 1 to a price of p 1 , and the quantity $q 2$ of good 2 to a price of p 2 , the total cost will be

$$
p_{1} * q_{1}+p_{2} * q_{2}
$$

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b) When the income is m , the budget line is all baskets that cost exactly m . In other words, all baskets that satisfy the condition that

$$
p_{1} * q_{1}+p_{2} * q_{2}=m
$$

Solving for q2, we get the function for the budget line:

$$
q_{2}=\frac{m-p_{1} * q_{1}}{p_{2}}=\frac{m}{p_{2}}-\frac{p_{1}}{p_{2}} * q_{1}
$$

We can interpret this as "the quantity of good 2 we can maximally consume if we have already chosen the quantity q 1 of good 1 , given that the prices are p 1 and p 2 , and our income is m ".

The function we have derived for the budget line is that of a straight line. The first term ( $\mathrm{m} / \mathrm{p} 2$ ) is the intercept with the Y-axis, and the term in front of q1 (i.e. p1/p2) is the slope. Furthermore, it is identical to the expression for MRT. Consequently, MRT corresponds to the slope of the budget line.

### 1.3 Utility Maximization

## Exercise 1.3.1

a) Utility maximization means that a consumer chooses in such a way that she gets as much utility as possible. She does not choose utility directly. Utility is, instead, an indirect result of consuming a certain mix of goods. Usually, there are limitations to how she can choose. Among the baskets that she can choose, she chooses the one that gives her the highest level of utility.
b) A utility function is a mathematical expression that gives a value in numbers for, say, different combinations of goods or a certain wealth. For instance:

$$
\begin{aligned}
& \text { For two goods: } U\left(q_{1}, q_{2}\right)=q_{1} * q_{2} \\
& \text { For wealth } \mathrm{W}: U(W)=\sqrt{W}
\end{aligned}
$$

Which function to use depends on the individual we are studying.
c) The criterion for utility maximization is that the marginal rate of substitution, MRS, equals the marginal rate of transformation, MRT:

$$
M R S=M R T
$$

This implies that, at the point of maximization, the slope of the indifference curve is equal to the slope of the budget line.

## Exercise 1.3.2

a) In Figure S.1.4, the area under the budget line corresponds to all baskets a consumer can choose, given her income and the prices.
b) The curves I1, I2, and I3 in Figure S.1.4 are three of the consumer's indifference curves for the two goods.
c) The consumer wants to end up on an indifference curve as far to the northeast as possible. She also has to afford it. The only indifference curve she can both afford and that is as far as possible to the northeast is I2. The only point on I2 she can afford is point A. The basked corresponding to point A is therefore the only utility maximizing choice she can make in this case.

d) It is possible to construct situations in which a consumer can find several points that all maximize her utility. Take the case of perfect substitutes. In Figure S.1.5, we have four indifference curves for perfect substitutes. Suppose that the prices of the goods are the same (which is reasonable, since they are perfect substitutes). That means that the budget line, the broken line, will be a straight line with the same slope as the indifference curves. The consumer can then choose any point on the budget line. They all maximize her utility.

Figure S.1.5


Note that, if one of the goods is more expensive than the other one is, the budget line will no longer have the same slope as the indifference curves. Say that good 2 is cheaper than good 1. Then the budget line will be steeper, for instance as the dotted line in the figure. The consumer would then choose to consume only the cheaper good, i.e. good 2. A solution such as this, when one ends up at one of the axes, is called a corner solution. In such a case, the criterion that MRS $=$ MRT is no longer valid.

## Exercise 1.3.3

If she maximizes utility, then MRS $=$ MRT at point A. In Exercise 1.1.5, we calculated MRS to be $2 / 3$. The slope of the budget line must therefore be $2 / 3$. This implies that $\mathrm{p} 1 / \mathrm{p} 2=2 / 3$ and that $\mathrm{p} 1=100 * 2 / 3=67$.

She can maximally buy 2 units of good 2, and the price is 100 . Therefore, her in-come must be $2 * 100=200$.

## 2. Demand

### 2.1 Price Changes

## Exercise 2.1.1

a) The answer to the first part is given in Figure S.1.4.
b) When the price of one of the two goods changes, the budget line will rotate inwards or outwards depending on whether the price rises or falls. In Figure S.2.1, we show what it will look like.

In the beginning, we have the budget line BL1. Since the prices are equal, the slope will be $\mathrm{p} 1 / \mathrm{p} 2=1$.

If the price of good 1 is doubled, the consumers can only maximally buy half as much as before. BL2 will consequently intersect the X-axis at half the distance from the origin. If the price is instead cut by half, she can maximally buy twice as much as before. BL3 will therefore intersect the X -axis at twice the distance from the origin, as compared to BL1.

## Try this...

The sequence $2,4,6,8,10,12,14,16, \ldots$ is
the sequence of even whole numbers. The looth place in this sequence is the number...?

We now have three different budget lines, corresponding to three different prices of good 1. How the consumer will choose depends on her preferences, i.e. on her indifference curves. What these look like is difficult to know. In the figure, we have assumed that the good is a normal good. We then draw the in-difference curves such that higher prices imply lower demand, and vice versa. Thereby we get the indifference curves I1, I2, and I3, that all give one point on each budget line where the consumer maximizes utility. Those points are labeled A, B, and C.

Had we repeated this procedure for all possible prices of good 1 , we had ended up with a curve. Now, we only have the three points A, B, and C, so we just sketch what the curve could look like between the points. The resulting curve is the price-consumption curve.
c) Next step is to derive the individual demand curve. Each budget line that we have drawn corresponds to a certain price of good 1 , and the chosen quantities can be read off from the X -axis. We can then construct a new graph using that information.

Often, the new graph is drawn straight below the first one. Then the X -axes will be identical and the derivation becomes more obvious. We can then draw vertical lines from the first graph to highlight the connections regarding the quantities.

The information we have about the prices is that $\mathrm{p}_{12}$ is twice as high as p 11 and that $\mathrm{p}_{13}$ is half as high as $\mathrm{p}_{11}$. Consequently, $\mathrm{p}_{12}$ must be twice as high up as p 11 on the Y -axis in the new graph, and $\mathrm{p}_{13}$ must be half as high up as $\mathrm{p}_{11}$.

We then draw horizontal lines from the prices until they intersect the corresponding quantities. In the figure, these points are labeled $a, b$, and $c$.


To derive the entire demand curve for this consumer, we would have to repeat the whole procedure for each point along the price-consumption curve. Instead, we sketch the shape between the points. This results is the individual demand curve.
d) To derive the market demand curve, we need all individuals' individual demand curves. In Figure S.2.2, we have simplified these to two straight lines, D1 and D2, representing the demand of two individuals. The market demand is the horizontal sum of the two.

If the individual demand curves are straight lines, then the market demand curve will be a succession of straight lines that bends whenever a new consumer's demand enters. If we have only two consumers, there will be only one bend.

For prices between 3 and 4 in Figure S.2.3, only consumer 1 demands the good, so market demand will be equal to her demand. At a price of 3 , consumer 2 starts to demand the good as well. Therefore, we will get a bend in the market demand curve at that price. At a price of 0 (or near 0 ), consumer 1 demands 20 units and consumer 2 demands 25
units. The market's demand is therefore 45 units. Therefore, the market demand curve will correspond to the thick full line in the figure.

Figure S. 2.2


### 2.2 Income Changes

## Exercise 2.2.1

Since the starting point is the same as in Exercise 2.1.1, BL ${ }_{1}$ in Figure S.2.3 looks the same as in Figure S.2.1. However, now we will vary the income instead of the price. As the prices are constant, the slope of the budget line will be the same in all cases and only shift inwards or outwards for different levels of income.

If the income is doubled, the consumer can maximally buy twice as much of either good 1 or good 2 as she could before. $\mathrm{BL}_{2}$ will consequently intersect the X - and Y -axes at twice the distances from the origin as compared to $\mathrm{BL}_{1}$. If income is cut by half, she can only buy half of what she could before, and $\mathrm{BL}_{3}$ must intersect the axes at half the distances as compared to $\mathrm{BL}_{1}$.

Will the consumer buy more or less of good 1 if income increases? That is not obvious. That depends on whether we have a normal good or not. She will buy more of the good if it is a normal good, and less of it if it is an inferior good.

If both good 1 and good 2 are normal goods, she will buy more of both of them when income increases. The indifference curves will then look like $I_{1}, I_{2}$, and I3 in Figure S.2.3. If good 1 were an inferior good, they would have looked more like $I_{3}, X_{3}$, and $Y_{3}$. Similarly if good 2 were an inferior good. As you can probably see from the diagram, good 1 and good 2 cannot simultaneously be inferior goods: If the consumer's income increases, she must buy more of at least one good. That good is then a normal good.

Let us assume that both goods are normal goods. The consumer will maximize her utility in points $A, B$, and $C$ for the corresponding income levels. If we sketch what the points in between should look like, we get the income-consumption curve.

We have used three different incomes and can read off the corresponding quantities of good 1 from the graph. Then we have everything we need to derive the Engel curve.

Often, the Engel curve is drawn straight below the other diagram, to stress the fact that the Xaxes are the same and that the quantities of good 1 correspond directly to each other. Draw a vertical line from points $A, B$, and $C$ down to the new diagram. On the Y-axis, we indicate the incomes. They are $\mathrm{m}_{1}, \mathrm{~m}_{2}$ (at twice the distance from the origin compared to $\mathrm{m}_{1}$ ), and $\mathrm{m}_{3}$ (at half the distance as compared to $\mathrm{m}_{1}$ ). As we do not have any numerical values for them, we do not include that information.

Thereafter, we draw horizontal lines from the incomes until they intersect the corresponding quantities. We then get the points of intersection $a, b$, and $c$. If we connect these points, we get the Engel curve.

Figure S.2.3


## Exercise 2.2.2

a) We have a consumer who maximizes her utility at a certain point on the budget line. In that point, the budget line must be a tangent to an indifference curve. We therefore have an indifference curve such as I1 and a budget line such as BL1 in Figure S.2.4. The point of utility maximization is point A .

If the price of good 1 is cut by, say, half, the consumer can maximally buy twice the amount she could before. The budget line therefore rotates outwards to BL2, and intersects the X -axis at twice the distance.

Suppose that the consumer buys more of good 1 if the price falls. The new point of utility maximization, where another indifference curve just about touches the budget line BL2, must then be to the right of point A. For instance, it could be at point B. The total effect of the price fall is that the consumer shifts from consuming the quantity q11 of good 1 to the quantity q 12 . q 12 must then be larger than q 11 .


We will now split the total effect into the substitution- and income effects. We start with the former. The substitution effect is the change in demand on good 1 that only depends on the change in relative prices and not on the increase in the level of utility. To find the size of that effect, we construct an imaginary budget line, BL $*$. Two things are important with BL*. First, the relative prices must be the same as after the price decrease, implying that BL* must have the same slope as $\mathrm{BL}_{2}$. Second, the level of utility at the point of utility maximization on BL* must be the same as the one the consumer originally had (since the substitution effect does not measure the increase in utility). BL* must consequently be a tangent to the same indifference curve to which the original budget line, $\mathrm{BL}_{1}$, is a tangent: $\mathrm{I}_{1}$.

However, since BL* and $\mathrm{BL}_{1}$ have different slopes, the point of tangency will not be the same for both of them. For $\mathrm{BL}_{1}$ it is point $A$; for $\mathrm{BL} *$ it is point $C$. From the graph, we see directly that the substitution effect must be positive for price decreases and negative for price increases: If $\mathrm{BL} *$ has a slope that is smaller than that of $\mathrm{BL}_{1}$, the point of tangency must be to the right of the previous point. (Note however that the substitution effect can be zero.)

The substitution effect is the increase in consumption of good 1 that corresponds to the distance between point $A$ and point $C$ along the X -axis. The consumer has the same level of utility but consumes more of good 1 since it is cheaper than before in relative terms. Instead, she consumes less of good 2. Note that this is not something you can observe. BL* and point $C$ are theoretical constructs.

The income effect is the part of the total effect that remains after the substitution effect has been subtracted. It corresponds to the part of the change in demand on good 1 that depends on the fact that the consumer increases her utility; in this case, that she moves from the indifference curve $\mathrm{I}_{1}$ to $\mathrm{I}_{2}$. If $\mathrm{BL} *$ would have been a real budget line, an increase in income that had shifted $\mathrm{BL} *$ to $\mathrm{BL}_{2}$ would have given the same increase in the demand of good 1. Therefore the name "the income effect."
b) If the good were an inferior good, there would be a slight difference. An inferior good is a good one buys less of when income increases, since one shifts over to other goods of higher quality. Regarding the substitution- and income effects, the inferior good must have a negative income effect. In Figure S.2.4 that would mean point $B$ would have been somewhere to the left of $C$ instead. To construct such a graph, draw the indifference curve such that $\mathrm{BL}_{2}$ is a tangent to it to the left of $C$, for instance as $\mathrm{I}_{3}$ in the graph.

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c) A Giffen good is a special type of inferior good. The odd thing about it is that the income effect is so large that it dominates over the substitution effect. To construct such a graph, draw the indifference curve such that $\mathrm{BL}_{2}$ is a tangent to it to the left of point $A$. See $\mathrm{I}_{4}$ in the graph.

## Exercise 2.2.3

A normal good is a good one buys more of if income increases. A Giffen good, on the other hand, is a good one buys less of when the price of it decreases. Is it possible for a good to have both of these characteristics simultaneously?

The effect of a price change can be divided into the income- and substitution effects. Suppose the price on good 1 falls. The substitution effect depends on the fact that one of the goods has become cheaper relative to the other. Then one can trade more units of the cheaper one against a certain amount of good 2 . That effect cannot imply that one demands more of good 2 . It must imply that one demands more of good 1. (In extreme special cases, it can imply that demand is unchanged.) The substitution effect must therefore imply an increase in the demand of good 1 if the price of that good falls.

By definition, one buys more of a normal good if income increases. Therefore, the income effect must also be positive for a normal good. Therefore, for a normal good, both the substitution effect and the income effect must increase demand for good 1.

However, a Giffen good is a good one buys less of when the price falls. That is clearly inconsistent with it being a normal good.

### 2.3 Elasticities

## Exercise 2.3.1

a) The definitions are:

$$
\begin{aligned}
& e_{p}=\frac{\Delta Q / Q}{\Delta p / p} \\
& e_{m}=\frac{\Delta Q / Q}{\Delta m / m} \\
& e_{12}=\frac{\Delta Q_{1} / Q_{1}}{\Delta p_{2} / p_{2}}
\end{aligned}
$$

Here, $e_{p}$ is the price elasticity (of demand), $e_{m}$ is the income elasticity, and $e_{12}$ is the cross-price elasticity between good 1 and good 2. Q is the demand for the good and $\Delta \mathrm{Q}$ is the change in demand. $p, \Delta p, m$, and $\Delta m$ are the corresponding for price and income.

These definitions are all about the percentage change in the demand of a good as an effect of a percentage change in another variable. Either a change in the price of the same good (price elasticity), in the price of another good (cross-price elasticity), or in income (income elasticity). The higher the value is the more sensitive demand is to changes in the other variable.
b) If is often practical to rewrite the expression for (for instance price elasticity) in the following way:

$$
e_{p}=\frac{\Delta Q / Q}{\Delta p / p}=\frac{\Delta Q}{\Delta p} \cdot \frac{p}{Q}
$$

If demand is a straight line, the value of $\Delta Q / \Delta p$ is constant. For $D_{1}$ in the exercise, we can easily calculate it as $-30 / 15=-2$. To then calculate the price elaticities in points $A$ and $B$, we insert the values of p and Q at those points:

$$
\begin{aligned}
& e_{p}(A)=-2 \cdot \frac{10}{10}=-2 \\
& e_{p}(B)=-2 \cdot \frac{5}{20}=-0,5
\end{aligned}
$$

As you can see, we do not get the same answer in both cases. Even though the slope is the same, the proportion $\mathrm{p} / \mathrm{Q}$ changes depending on where on the line we measure it.
c) If $\mathrm{D}_{1}$ would change so that it became a horizontal line through $A$ instead, the proportion $\mathrm{p} / \mathrm{Q}$ would be the same as before, i.e. $10 / 10=1$, but the slope would be 0 and $\Delta \mathrm{Q} / \Delta \mathrm{p}$ is not possible to calculate (as $1 / 0$ is not defined). It can be shown that the closer to a horizontal the line we get, the higher the value of $\Delta \mathrm{Q} / \Delta \mathrm{p}$ will be. Consequently, the (negative) value of the price elasticity will also be higher.
d) To calculate the income elasticity in point $A$, we insert values into the formula. We know that the change in income is $+10 \%$, i.e. that $\Delta \mathrm{m} / \mathrm{m}=0.10$. In point $A, \mathrm{Q}=10$ and increases to 20 when the income increase shifts the demand curve. Consequently $\Delta Q=20-10=10$. Inserting these values we get that

$$
e_{m}=\frac{10 / 10}{0,10}=\frac{1}{0,10}=10
$$

The value is an approximation. We have very large changes in the variables, i.e. $\Delta \mathrm{Q}$ and $\Delta \mathrm{p}$ are large relative Q and p . The value becomes more exact the smaller the changes are.
e) To calculate the cross-price elasticity, we need $\Delta \mathrm{Q}_{2} / \mathrm{Q}_{2}$ and $\Delta \mathrm{p}_{1} / \mathrm{p}_{1}$, where $\mathrm{Q}_{2}$ is the quantity of the other good. The price increase is $5 \%$, so $\Delta p_{1} / p_{1}=0.05$, and the demand on the other good fell by $20 \%$, so $\Delta \mathrm{Q}_{2} / \mathrm{Q}_{2}=-0.20$. The cross-price elasticity is consequently

$$
e_{21}=\frac{-0,20}{0,05}=-4
$$

Since the cross-price elasticity is negative, the good is a complementary good.

## 3. Production

### 3.1 Definitions

## Exercise 3.1.1

a) Consumer theory is similar to that part of producer theory that deals with long-run production.

- In consumer theory, consumers are assumed to be utility-maximizers. This corresponds to the assumption in producer theory that producers are profitmaximizers.
- In consumer theory, consumers are limited because of prices and income. Correspondingly, producers are limited by the prices of the input factors (wages and rental rates for capital) and costs.
- In consumer theory, the consumer has indifference curves that indicate her utility of certain combinations of goods. Correspondingly, producers have isoquants that indicate the maximum production of certain combinations of inputs.
b) A production function is a mathematical expression that describes how large the production will be if one uses different combinations of the input factors. Usually, labor, L , and capital, K , are variables in the production function:

$$
q=f(L, K)
$$

Sometimes, a variable for technology is also used in the production function.
c) There are often input factors that are not possible to vary immediately. For example, a factory could be of a certain size and not possible to sell or increase quickly. The short run is then defined as the period during which it is impossible to change those input factors. Often, one assumes that capital is not possible to vary in the short run but that labor is fully variable.

Long run is then defined as the period when all input factors are variable.
d) Returns to scale are about the relation between the size of a firm and its production. If you have a firm of a certain size and then increase all its inputs by the same factor, will then the total output also increase by the same factor.

If the answer is yes, we have constant returns to scale. If the answer is that the production increases more, we have increasing returns to scale. In the third case, we have decreasing returns to scale.

## Exercise 3.1.2

a) We often distinguish between the marginal product of labor and of capital. They are defined as

$$
\begin{aligned}
& M P_{L}=\frac{\Delta q}{\Delta L} \\
& M P_{K}=\frac{\Delta^{q}}{\Delta K}
\end{aligned}
$$

$\mathrm{MP}_{\mathrm{L}}$ answers the question "if we increase the input with one unit of labor, by how much does the production increase?" $\mathrm{MP}_{\mathrm{K}}$ is the corresponding measure if one increases the input with one unit of capital.
b) The law of diminishing marginal returns states that if one increases, say, labor input by more and more units, then eventually one reaches a point where additional units increases production by fewer and fewer units. For example, because of congestion in the factory.

This law is not a result from economic theory but partly an empirical result and partly based on speculation.

## Exercise 3.1.3

a) The marginal rate of technical substitution is

$$
M R T S=\frac{\Delta K}{\Delta L}
$$

If labor input, L , is decreased by one unit, by how much must one increase capital, K , to hold production constant? The answer is given by MRTS.
b) To show the relation with the marginal products, we start by the following reasoning: Suppose we increase the input of labor, $\Delta \mathrm{L}$, by one unit. Depending on how productive the labor is, this will increase production. The total increase will then be $\mathrm{MP}_{\mathrm{L}} * \Delta \mathrm{~L}=\Delta \mathrm{q}$.

Suppose we then decrease the input of capital, $\Delta \mathrm{K}$, by an amount that makes the associated decrease in production just as large as the previous increase. We then get that $\mathrm{MP}_{\mathrm{K}} * \Delta \mathrm{~K}=-\Delta \mathrm{q}$. Summing these two together must give a total change of zero:

$$
\begin{aligned}
& \left\{\begin{array}{l}
M P_{L} * \Delta L=\Delta q \\
M P_{K} * \Delta K=-\Delta q
\end{array}\right. \\
& M P_{L} * \Delta L+M P_{K} * \Delta K=0
\end{aligned}
$$

Now, rearrange the last expression: Move $\mathrm{MP}_{\mathrm{K}}{ }^{*} \Delta \mathrm{~K}$ over to the right-hand side, and then divide both sides by $\mathrm{MP}_{\mathrm{K}}$ and by $-\Delta \mathrm{L}$. You will then have an expression for MRTS:

$$
-\frac{M P_{L}}{M P_{K}}=\frac{\Delta K}{\Delta L}=M R T S
$$

This is the relation we asked for.

### 3.2 The Production Function

## Exercise 3.2.1

a) The production curve will look like in the upper part of Figure S.3.1.
b) The average product of labor is the total quantity of units produced divided by number of working hours. The marginal product of labor is how many extra units are produced if we add one more unit of labor:

$$
\begin{gathered}
A P_{L}=\frac{q}{L} \\
M P_{L}=\frac{\Delta q}{\Delta L}
\end{gathered}
$$

$\mathrm{MP}_{\mathrm{L}}$ corresponds to the slope of the production function in the graph. $\mathrm{AP}_{\mathrm{L}}$ corresponds to the slope from the origin to a certain point on the production curve.
c) The shapes of $\mathrm{MP}_{\mathrm{L}}$ and $\mathrm{AP}_{\mathrm{L}}$ are given in the lower graph.


The most characteristic points on the production curve are points $A, B$, and $C$. The corresponding points below are labeled $a, b$, and $c$.

In point $A$, the slope of the curve is the same as the slope of a line from the origin to point $A$. Consequently, $\mathrm{MP}_{\mathrm{L}}$ and $\mathrm{AP}_{\mathrm{L}}$ have the same value there. In the corresponding point $a$, they intersect each other. Also note that $\mathrm{AP}_{\mathrm{L}}$ slopes downwards on both sides of point $a$.

In point $B$, the slope of the production curve is zero. Consequently, $\mathrm{MP}_{\mathrm{L}}$ intersects the X axis at point $b$.

In point $C$ the production curve reaches its highest slope. Both to the left and to the right of $C$ the curve is less steep. Consequently, $\mathrm{MP}_{\mathrm{L}}$ reaches its maximum at $c$.

## 4. Costs

### 4.1 Costs in the Short Run

## Exercise 4.1.1

The answer depends of which assumptions we make.

- If we assume that the firm always chooses an efficient production, then it is impossible that more units would cost less. That follows directly from the definition of efficient. If it was cheaper to produce 100 units of the good than 90 units, one could easily decrease the cost of producing 90 units by instead producing 100 and then throw away 10 units.
- If the firm has not chosen an efficient production, it is sometimes possible to produce more of the good at a lower cost. However, that assumes that the firm did not maximize its profit before the change.
- The firm can often lower its cost in the long run, as compared to the short run.


## Exercise 4.1.2

a) The short-run total cost curve, TC, looks like in the upper part of Figure S.4.1.
b) The fixed cost is that part of the cost that is not possible to change in the short run, for instance the cost of a factory building. We can therefore find the fixed cost as the intercept on the Y-axis. Consequently, the fixed cost is approximately 30. As it does not vary with production, it is a horizontal line.
c) The variable cost is that part of the total cost that is possible to vary in the short run, i.e. the part that changes with changes quantity. In the diagram that corresponds to the total cost minus the fixed cost. In other words, we take the total cost, TC, and move it downwards so that it starts at the origin.
d) The most characteristic points in the graph are those where a line from the origin to TC and VC , respectively, has the lowest slope. (Remember that this is a way to find the average costs.) At those points, the average costs (ATC and AVC) are at their minimums. These are indicated as points $A$ and $B$ in the graph.
In the lower graph, they correspond to points $a$ and $b$. Since the slope through $B$ is lower than through $A, b$ must lie below $a$ in the lower graph. We then draw AVC through point $b$ and ATC through point $a$. Both curves must slope upwards on both sides of their respective minimum point.
The marginal cost curve, MC, must intersect the ATC and AVC curves at points $a$ and $b$. For instance, as in the lower part of Figure S.4.1.


### 4.2 Costs in the Long Run

## Exercise 4.2.1

a) Combinations of $L$ and $K$ that cost the same are given by the isocost lines in the upper part of Figure S.4.2. In the figure, they are labeled $C_{1}, C_{2}$, and $C_{3}$, with corresponding (invented) levels of cost.
b) Combinations of L and K that produce the same number of units are given by the isoquants in the upper part of Figure S.4.2. They are labeled $\mathrm{q}_{1}, \mathrm{q}_{2}$, and $\mathrm{q}_{3}$, and they have been assigned numerical values.
c) For a given production level, the firm minimizes the cost where the isoquant just about touches an isocost line. Since the isocost lines have the same slopes, as long as wage and rental rates do not change, it is possible to find the points of cost-minimization by letting a ruler with the same slope glide along the graph until it hits a certain isoquant. The point where it does so first, is the point of cost minimization for the corresponding quantity. In the figure, we have identified three such points. $A, B$, and $C$.
d) The criterion is that the slope of the isocost line is the same as the slope of the isoquant. The slope of the isocost line is $-\mathrm{w} / \mathrm{r}$, where w is the wage and r is the rental rate. The slope of the isoquant is $\Delta \mathrm{K} / \Delta \mathrm{L}$ and corresponds to the marginal rate of technical substitution, MRTS.
The criterion can then be written

$$
-\frac{w}{r}=M R T S\left[=\frac{\Delta K}{\Delta L}\right]
$$

e) The long-run expansion path corresponds to all combinations of $L$ and $K$ that minimize cost for a certain quantity.
It can be constructed by finding all points of tangency, i.e. all points where the criterion $-\mathrm{w} / \mathrm{r}=$ MRTS is fulfilled. In the figure, we have sketched it through the points we have already found.
f) In the short run, the firm can only vary its labor costs. Production can then only be increased by increasing $L$ in the figure. The short-run expansion path must then be a straight horizontal line at that level of $K$ one has already chosen. In the figure, we have chosen a level of K that corresponds to point $A$.

g) The information we need comes from the points of cost minimization, $A, B$, and $C$. We need both the cost of production and the quantity produced at those points. These values are (A: 6/100; B: 8/300; C: 10/500).

We indicate the corresponding points in a new graph, $a, b$, and $c$, and connect them to a line. In the long run, the cost of producing nothing must be zero so the curve must start at the origin. The cost curve will then look like in the lower graph of Figure S.4.2.
Note that, over the whole interval we have decreasing average cost. In other words, we have economies of scale.

## Exercise 4.2.2

a) See Figure S.4.3. No short-run average cost, SRAC, can be lower than the long-run average cost, LRAC. For each level of the fixed cost, SRAC must be above or on LRAC.

Each point on LRAC must also have an SRAC that just barely touches it. This depends on the fact that each point on LRAC corresponds to a certain optimal choice of the level
of capital. An SRAC corresponding to that same level of capital must then go through that point as well. For other quantities of capital, the short run cost is often higher than the long run cost, and SRAC will then be above LRAC at all other points.

The SRAC's will then be "contained" within LRAC, as in Figure S.4.3.

It is also possible to reach these conclusions in the opposite way: If we take all possible short-run cost curves and draw them, LRAC will correspond to the lower edge of the whole collection.

Figure S.4.3

b) Economies of scale means that the average cost of production becomes lower and lower the more units one produces. That corresponds to the case when LRAC slopes downwards. Diseconomies of scale refer to the opposite case: The average cost increases and LRAC slopes upwards.

The intervals for (dis-) economies of scale are indicated in the figure.

## 5. Perfect Competition

### 5.1 Definitions and Assumptions

## Exercise 5.1.1

a) Frequently used assumptions include

- All agents are price takers, i.e. they take prices as given. This will be true if there are many buyers and sellers.
- Homogenous products.
- There are no barriers to entry or exit. All input factors, labor and capital, are completely variable.
- All agents have perfect information about all existing alternatives in the market.
- No cartels, i.e. no agents can cooperate about prices.
b) This depends on the fact that all agents are price takers. If the firm cannot affect the price, then the price is independent on the quantity produced. Consequently, the price becomes a horizontal line.
c) Since the firm cannot affect the price, each unit will be sold at the same price. Thereby, the marginal revenue is exactly what the consumer pays for the good. The MR curve therefore becomes identical to the price curve, i.e. a horizontal line.



### 5.2 The Firm's Short-Run Profit Maximization

## Exercise 5.2.1

a) The TC curve will look like in Figure S.5.1.
b) The revenue only depends on the price and the quantity sold. The TR curve then becomes a straight line with a slope equal to the price. Here, it will be a straight line with a slope of 2.20 .

c) The minimum point of the AVC curve is found in the following way: In the upper graph, let a ruler have one point at the origin and one point on the curve. What is the smallest slope it can have? The slope corresponds to the average cost, and the quantity where the slope is the smallest corresponds to the minimum value of the average cost. Note that there is no fixed cost in this case, and that the variable cost therefore is the same as the total cost. In the figure, the minimum point is labeled $B$ in the upper part, corresponding to point $b$ in the lower part. The AVC curve must slope downwards on both sides of $b$.

In the point where TR intersects TC, MC is equal to AVC, which is reflected in the lower graph.

The point easiest to find for MC is point $b$. To the right of $b$, MC must be above the AVC curve and to the left it must be below it.

The marginal revenue, MR , is the same as the price in this case, since we have perfect competition. The MR curve is, consequently, a horizontal line at 2.20.
d) The profit maximizing quantity is the one where $\mathrm{MR}=\mathrm{MC}$, i.e. at point $a$ in the graph. The quantity is approximately 78.
e) The profit can be read from the upper graph as the difference between TR and TC at the maximizing quantity: $\mathrm{TR}=175, \mathrm{TC}=120$ and $\mathrm{TR}-\mathrm{TC}=175-120=55$.

It can also be read off from the lower graph. The average revenue is 2.20 and $\mathrm{AVC}=1.50$. On average, the firm, consequently, makes $2.20-1.50=0.70$ per unit. The quantity is 78 , so in total it makes $0.70 * 78=55$.
f) The supply curve is the quantity the firm offers at different market prices. If the market price shifts, the firm will choose the quantity where the price (=MR) intersects the MC curve. However, this is only true as long as it is able to recover its variable cost. We must therefore be on or above the AVC curve. The short-run supply curve is, consequently, the part of the MC curve that lies above the AVC curve. That part is indicated with a thicker full line in the figure.
g) Since there is no fixed cost for the firm in this case, the average total cost, ATC, is the same as the average variable cost, AVC. The long-run supply curve must therefore be identical to the short-run supply curve.
Had there been a fixed cost, the long-run supply curve would have been the same as the part of the MC curve that lies above ATC.

### 5.3 The Firm's Long-Run Profit Maximization

## Exercise 5.3.1

a) The short-run supply curve of an individual firm is the part of the MC curve that lies above AVC curve. The short-run supply curve of the market is the horizontal sum of the short-run supply curves of all individual firms.
b) In the long run, the number of firms can vary. The market's long-run supply curve is therefore not the sum of the long-run supply curves of individual firms.

The market's long-run supply curve is instead obtained from the relation between the production cost and the scale of the production. A downward-sloping supply curve reflects economies of scale, whereas an upward-sloping one reflects diseconomies of scale. In the intermediate case, the supply curve is horizontal.

## Exercise 5.3.2

a) The firm makes a short-run loss. That depends on the fact that the price it is able to demand per unit, $\mathrm{p}^{*}$, is below the average total cost, ATC. The firm will still produce the quantity $\mathrm{q}^{*}$ (where $\mathrm{MR}=\mathrm{MC}$ ) in the short run, since it is then able to recover some of its fixed cost. This is due to $\mathrm{p}^{*}$ being above the average variable cost, AVC.

b) In the long run, the firm cannot accept making a loss. Either it decides to shut down, or the market will change. In a perfectly competitive market, where there are no barriers to leave (or to entry), some of the firms will leave the market.

As there are fewer firms in the market in the long run, the market can offer fewer units at each given price. That means that the supply curve will shift to the left. As long as the equilibrium price is below ATC, firms will continue to get pushed out of the market.

When enough many firms have left the market, $S$ has shifted to $S^{\prime}$ in Figure S.5.2, and consequently the price has been pushed up from $p^{*}$ to $p^{\prime}$. At that price, $p^{\prime}=M R$ is a tangent to ATC. The average revenue is then just as large as ATC, and the profit is therefore zero. Since none of the firms makes a loss, neither in the short run or in the long run, no more firms will be pushed out of the market. A long-run equilibrium has therefore been reached.

## 6. Monopoly

### 6.1 Monopolies

## Exercise 6.1.1

Monopolies arise because there is some sort of barrier to entry in the market. These can be either natural or constructed.

- Economies of scale can give rise to natural monopolies. If a large part of the total cost of production is fixed, it might be possible for only one firm to recover its costs. Examples include railroads and telecommunications networks.
- Cost advantages in the production could also be called a natural monopoly. The advantage could be due to an invention or a patent.
- Strategic limitations. The monopolist could create barriers to shut out competitors.
- Political limitations.
- Patents and exclusive rights for a certain production.


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### 6.2 Monopoly Profit Maximization and Efficiency Problems

## Exercise 6.2.1

a) The MC-, MR-, ATC- and demand curve will look like in Figure S.6.1.
b) For the first unit sold, the price is the same as the marginal revenue. In this case, it is 30 . For the second unit, one has to lower the price on both units. In this case, this means we can sell two units at a price of 29 each. Total revenue is then $2 * 29=58$. For the first unit, total revenue was 30 . For the second, MR is then $58-30=28$. However, the price was 29 . The marginal revenue becomes smaller since we had to lower the price of the first unit as well. The same argument is then repeated for each additional unit. Therefore, the MR curve will be steeper than the demand curve. If the demand curve is a straight line, the MR curve is also a straight line. It will have the same intercept on the Y-axis as the demand curve, but it will have a slope that is twice as large. In this case, the demand curve intersects the X -axis at 30 , so the MR curve must intersect it at 15 .

c) To maximize her profit, the monopolist chooses to produce at the quantity that makes $M R=M C$. In this case, this occurs at a quantity of 7.5. At that quantity, $\mathrm{MR}=\mathrm{MC}=15$.
d) She will charge the highest possible price at the chosen quantity. At a quantity of 7.5 , the demand curve is at 22.50. Consequently, she will charge a price of 22.50 .
e) Profit is the difference between revenue and cost. 7.5 units are sold at a price of 22.50 . The revenue is then $7.5 * 22.50=168.75$.
The average cost we read off from the ATC curve. At a quantity of 7.5 , it is 12.50 . The total cost is then $7.5^{*} 12.50=93.75$.
The profit, $\pi$, is consequently $168.75-93.75=75$. This corresponds to the grey rectangle in Figure S.6.1.
f) We draw a new graph with the MC-, MR-, and demand curves (see Figure S.6.2). The monopoly price, $\mathrm{p}_{\mathrm{M}}$, is the quantity where $\mathrm{MR}=\mathrm{MC}$, i.e. $\mathrm{Q}_{\mathrm{M}}$.
The consumer surplus for a certain unit of the good is the difference between what a consumer is willing to pay for it and what she actually does pay for it. The price she pays is $\mathrm{p}_{\mathrm{M}}$, and the price she is willing to pay for different quantities can be read off from the demand curve, D. The consumer surplus is consequently the area A in Figure S.6.2. The producer surplus for a certain unit is the difference between what the producer is paid for it and what it costs her to produce it. The producer is paid $\mathrm{p}_{\mathrm{M}}$ and her cost of producing it can be read off from the MC curve (= the supply curve). The producer surplus is consequently the areas $\mathrm{B}+\mathrm{D}$.

g) The deadweight loss is the value of the production that is lost because the monopoly is not efficient. In an efficient economy, more units would be produced as long as the consumers are prepared to pay at least the cost of producing the additional units. The optimal quantity is then where the demand curve intersects the MC curve, i.e. $\mathrm{Q}_{\mathrm{C}}$ with a corresponding price of $\mathrm{p}_{\mathrm{C}}$.
The value of the additional production, the deadweight loss, is then the area $\mathrm{C}+\mathrm{E}$. The corresponding triangle is indicated in Figure S.6.1. The area of a triangle is $\mathrm{B}^{*} \mathrm{H} / 2$, where B is the base and H is the height. We can choose B as the line between 15 and 22.50 on the Y -axis. Its length is $22.50-15=7.50$. Then we choose H as the line between 7.5 and 10 on the X -axis. Its length is $10-7.5=2.5$.

The area of the triangle is consequently $7.50 * 2.5 / 2=9.38$, which corresponds to the value of the deadweight loss.
h) First, look at Figure S.6.1. The equilibrium price in a perfectly competitive market would have been where the demand curve intersect the MC curve, i.e. $\mathrm{p}_{\mathrm{C}}$ would have been 20 and the quantity consumed would have been $\mathrm{Q}_{\mathrm{C}}=10$.
Then, look at Figure S.6.2. The consumer surplus would have increased to $\mathrm{A}+\mathrm{B}+\mathrm{C}$. The producer surplus would have changed to $\mathrm{D}+\mathrm{E}$.
i) If the production is changed to $\mathrm{Q}_{\mathrm{C}}$ and the price to $\mathrm{p}_{\mathrm{C}}$, and the consumers afterwards transfer a value corresponding to the area B back to the producers then both producers and consumers improve their situations. The consumers increase their surplus from A to $\mathrm{A}+\mathrm{C}$, and the producers increase theirs from $\mathrm{B}+\mathrm{D}$ to $\mathrm{B}+\mathrm{D}+\mathrm{E}$.
As both sides can improve their situation simultaneously, the monopoly cannot be Pareto efficient.


### 6.3 Price Discrimination

## Exercise 6.3.1

- $1^{\text {st }}$ degree price discrimination is when a seller charges each buyer what she is maximally willing to pay for the good. The seller must then have knowledge about each buyer's willingness to pay, which is usually impossible. The buyers must also be kept from reselling the goods to other buyers.
- $2^{\text {nd }}$ degree price discrimination is when a seller constructs different packages of goods, e.g. different sizes. The idea is that different packages appeal to different types of buyers, who, in turn, have different willingness to pay. By forcing the buyers to choose between the packages, they will sort themselves into appropriate groups. The buyers must in this case be kept from splitting the packages and reselling the parts.
- $3^{\text {rd }}$ degree price discrimination is when a seller splits the market into different submarkets, where buyers from different submarkets typically have different willingness to pay. Examples are different prices for students and retired people, or different markets for technology products in Europe and the US. It must be possible to tell the buyers from different submarkets apart, and the buyers in the low-price market must be kept from reselling the products.


## 7. Game Theory

### 7.1 Basic Concepts

## Exercise 7.1.1

a) Usually, we have to specify

- Players.
- Actions. What the players can do at different stages of the game.
- Information. What each player knows at each stage of the game.
- Strategies.
- Payoffs.

Oftentimes, other things need to be specified as well.
b) In a game on normal form, there is no time dimension. In a game on extensive form, the players choose in a specific order.
c) A dominant strategy for a player is a strategy that is always at least as good as every alternative strategy and sometimes better. If a player has a dominant strategy, there is no reason ever to use another one.
d) A payoff matrix is a graphical representation of certain two-player normal-form games. It is constructed by drawing a net of squares, where the columns belong to one player and corresponds to the possible strategies of that player, and the rows belong to the other. Each square represents the outcome of a certain strategy pair, and contains the corresponding payoffs. The game in Figure S.7.1 is a payoff matrix.

### 7.2 Games on Normal Form

## Exercise 7.2.1

a) A Nash equilibrium is a set of strategies. Each player should have one, and only one, strategy. For each player it should be the case that there is no way that she can receive a higher payoff by unilaterally changing her strategy.
b) Individual A chooses in the vertical direction and individual B in the horizontal. In Figure S.7.1, we have drawn arrows to squares that a player would prefer to other squares (in the direction that the player can choose).
We see that both the square $\{$ Techno, Techno $\}$ and the square $\{$ Pop, Pop $\}$ are efficient outcomes and that both are Nash equilibria. For instance, given that both have chosen Techno, there is no way that any of them can receive a higher payoff by unilaterally changing her strategy to Pop.
The problem that they cannot coordinate without communicating is, however, still there. For both of them, there is a risk that the other has chosen the other event.
c) Let us first state that A cannot lose by this rule, since it will always give her the preferred outcome.
For B it is more complicated. \{Techno, Techno\} is not a bad outcome for her, but \{Pop, Pop $\}$ is better. However, the rule eliminates the risk of the outcomes \{Techno, Pop\} and \{Pop, Techno\}. One could say that B sacrifices her preferred outcome to avoid the bad ones, whereas A does not sacrifice anything. Both of them will increase their (expected) utility this way, but the increase will not be fairly distributed.

| Figure S.7.1 |  |  |  |
| :---: | :---: | :---: | :---: |
|  |  | B |  |
|  |  | Techno | Pop |
| A | Techno | 10, 9 | 2, 2 |
|  | Pop | 0, 0 | 9, 10 |



### 7.3 Games on Extensive Form

## Exercise 7.3.1

a) It is called backward induction.
b) We eliminate backwards. The last part of the game is when Owner chooses whether to reward or not to reward Finder. That part of the game looks like in Figure S.7.2.

Figure S.7.2


If she chooses to reward Finder with 600, she will make 400 for herself. However, if she does not reward Finder, she makes 1,000 . Consequently, she does not reward Finder.

We then eliminate that part of the game and replace it with the outcome Owner will choose. The game then looks as in Figure S.7.3.


Finder chooses between keeping the wallet and returning it. If she returns it, she will get nothing whereas if she keeps it, she will get 500 . Consequently, she keeps it. The subgame perfect equilibrium is then $\{$ Keep wallet $\}$.
c) If they had played the strategy-pair \{Return wallet, Reward\} Finder would have received 600 and Owner 400. The subgame perfect equilibrium is consequently not efficient since there is another outcome that gives them both more.
d) This problem can be circumvented by introducing an intermediary. Finder hands the wallet in to the police who give her 600 . Owner then get to collect her wallet at the police against a fee of 600 .


## 8. Oligopoly

## Exercise 8.1.1

a) A monopoly is a market where there is only one seller, a duopoly is a market with two sellers, and an oligopoly is a market with only a few sellers.
b) The expression "kinked demand curve" refers to a model for an oligopoly market. The idea is that it is much easier to lose customers to competitors than to win them over from them.
A price decrease leads to the competitors also lowering their prices not to lose customers, and a price increase leads to no one else increasing theirs to win over customers instead. This leads to a case where a firm's individual demand reacting strongly to price increases but weakly to price decreases. In other words, the demand curve is broken at the present price.
c) A reaction function is a mathematical function that determines the optimal reaction to someone else's action.
For instance, suppose another firm has chosen which quantity to produce. Then there is an optimal quantity for our company to produce, where we maximize our profit. Our firm's reaction function then tells us which quantity to produce, given the other firm's choice.

### 8.2 The Cournot Model

## Exercise 8.2.1

a) Assumptions in the Cournot model are

- There are two firms.
- They set quantities.
- They do so with no information about what the other firm has chosen.
b) To find all Nash equilibria, we reason like this:
- Take the point of intersection between the two reaction functions, in Figure S.8.1 labeled $A$. That point must be a Nash equilibrium since:
- Firm 1 cannot get a better outcome by unilaterally changing its strategy. Given that firm 2 has chosen the quantity $\mathrm{q}_{2}{ }^{*}, \mathrm{q}_{1}{ }^{*}$ is the optimal choice according to the reaction function.
- The same is true for firm 2.
- Then, take any other choice of quantity. For instance, $\mathrm{q}_{2}$ ' for firm 2. No such point can be a Nash equilibrium since:
- Firm 1's optimal response can be read off from the reaction function. In this case at point $B$, i.e. the quantity $\mathrm{q}_{1}{ }^{\prime}$. Note that, if firm 1 would have chosen any other quantity it would have been able to improve its choice by moving to the quantity $\mathrm{q}_{1}$ '. So none of those points can be part of a Nash equilibrium.
- Since firm 2's reaction function does not intersect firm 1's in point $B$, firm 2's choice of quantity cannot be optimal given firm 1's choice. Consequently, firm 2 can improve on its situation by unilaterally changing its strategy. Therefore, point $B$ cannot be a Nash equilibrium.
- The same argument can be used for every point in the graph, except for point $A$, which consequently is the only Nash equilibrium.

Figure S.8.1


### 8.3 The Bertrand Model

## Exercise 8.3.1

The easiest way of finding the Nash equilibrium is by putting ourselves in the place of one of the firms, and then reason like this:
The firms have the same costs of production and therefore the same marginal cost, MC. In a perfectly competitive market, the equilibrium price would have been MC and none of the firms would have made any profit. Let us compare three different bids that are below, above, or equal to MC.

- $\mathrm{p}<\mathrm{MC}$. If we choose to sell our product at a price below the cost, we will make a loss. Therefore, we would be able to improve on the situation by shutting down. A bid below MC cannot be a Nash equilibrium regardless of what the other firm chooses.
- $\mathrm{p}>\mathrm{MC}$. Here, we get three different cases, depending on what the other firm has chosen:
- Firm 2 has chosen a price above ours. We win the whole contract and they get nothing. This cannot be a Nash equilibrium since the other firm could improve on its situation by changing its bid to slightly below ours, and then win the whole contract.
- Firm 2 has chosen a price below ours. They win the whole contract and we get nothing. This cannot be a Nash equilibrium either since we could improve on our situation by submitting an offer slightly below theirs.
- Firm 2 has chosen the same price as we have. We split the contract and the profit. However, both they and we can improve on our situation by submitting another bid just below the one that the opponent has chosen and get the whole profit, so this cannot be a Nash equilibrium either.
- $\quad \mathrm{p}=\mathrm{MC}$. Again, we get three different cases depending on our opponent's choice:
- Firm 2 has chosen a price above ours. We get the whole contract but make no profit. We could improve on our situation be submitting a higher bid that is still below theirs. Then we still get the contract but increase our profit.
- Firm 2 has set a price below ours. They win the contract but make a loss. They could improve by increasing the price and make the loss smaller.
- Firm 2 has chosen the same price as we, $p=$ MC. We split the contract but make no profit. However, no one can improve on her situation by unilaterally changing her strategy. If we lower our price, we win the whole contract but make a loss. If we increase the price, we lose the contract and still make no profit. This is, consequently, the only Nash equilibrium in this game. The surprising result is that the outcome is the same as in a perfectly competitive market, even though we have only two firms.


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## 9. Monopolistic Competition

## Exercise 9.1.1

a) The graph will be identical to the one in Figure S.6.1 (from Exercise 6.1.2).

b) Just as in Exercise 6.1.2, $\mathrm{p}=22.50, \mathrm{q}=7.5$ and the profit will be $\pi=7.5^{*}(22.50-12.50)$ $=75$.
c) The answer is the same as in the monopoly case. In the short run, the firm is a monopolist on the good. The difference is:

- It is only a short-run monopoly. In the long run, it is not.
- Often, the demand curve for a firm in a market characterized by monopolistic competition is very elastic (close to a horizontal line). This means that the demand is very sensitive to price changes. This, in turn, depends on the fact that there are close substitutes to the good. In our case, there are other shoes of comparable quality.
d) When more and more firms enter the market by copying our shoes and PR-strategy, we will be able to sell fewer shoes at any given price. That means our demand curve shifts inwards.
This process will continue as long as there are excess profits in the market. The final position for the firm's demand curve is when the profit is zero. That occurs when the demand curve has shifted far enough to touch the ATC curve. The price will then be the same as the average total cost, ATC. Therefore the profit is $\pi=q^{*}(p-A T C)=0$.

In Figure S.9.1, we show both the short-run equilibrium, with $D_{1}$ and $\mathrm{MR}_{1}$, and the final long-run equilibrium, with $\mathrm{D}_{2}$ and $\mathrm{MR}_{2}$.
e) The situation is not efficient either in the short or in the long run. In both cases, the firm produces at a quantity where the consumers value additional units higher than the cost of production. The fact that those additional units are not produced is a sign of inefficiency. Regarding inefficiency in the long run, one should weight the value to society of the additional units lost against the value of having additional close substitute goods from which to choose.

Figure S.9.1


## 10. Labor

### 10.1 The Supply of Labor

## Exercise 10.1.1

a) She will shift from consuming 12 hours of leisure to consuming 9 hours of leisure (see Figure S.10.1). The total effect is therefore $9-12=-3$ hours (meaning that she works 3 more hours).


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Figure S.10.1


To find the substitution effect, we shift $\mathrm{BL}_{2}$ downwards until it becomes a tangent to the original indifference curve, to $\mathrm{BL} *$. Remember that the substitution effect is the change in consumption that only depends on the change in relative prices (the slope of the budget line). We should therefore shift $\mathrm{BL}_{2}$ and not $\mathrm{BL}_{1}$ in order to find it. The new, theoretical, point of optimization occurs straight below the one on $\mathrm{BL}_{2}$, i.e. at 9 hour of leisure. The substitution effect is, consequently, 9-12=-3 hours.
The income effect is the part of the total effect that is not explained by the substitution effect. However, in this case the total effect is completely explained by the substitution effect. The income effect is therefore zero.
The increase in utility for this individual, consequently, is due to an increased consumption of wage, or rather what the wage can buy. (It is, again, useful to think of money as "all other goods".)
Note that the substitution effect must be negative in this case, whereas the income effect can be either positive or negative. If leisure is a normal good, the income effect will be positive, if it is an inferior good, the income effect will be negative. Typically, leisure is a normal good.
b) If the income effect is large enough to counterweight the substitution effect, it is possible to get the opposite total result.
In Figure S. 10.1 we have drawn an indifference curve, I', that would have had that effect after a shift from $\mathrm{BL}_{1}$ to $\mathrm{BL}_{2}$.
For the income effect to counterweight the substitution effect, it is necessary that the individual is already rather satisfied in her demand for other goods, so that she primarily wants to consume more leisure.
c) This outcome occurs typically for already high wages.

### 10.2 The Demand for Labor

## Exercise 10.2.1

Divide and multiply $\Delta \mathrm{TR} / \Delta \mathrm{L}$ with $\Delta \mathrm{q}$, and then separate the expression in two parts. The first term is MR and the second is $\mathrm{MP}_{\mathrm{L}}$ :

$$
M R P_{L}=\frac{\Delta T R}{\Delta L}=\frac{\Delta T R}{\Delta q} \cdot \frac{\Delta q}{\Delta L}=M R \cdot M P_{L}
$$

## Exercise 10.2.2

Neither the individual workers nor the individual employers can affect the wage, since we have perfect competition in the labor market. The wage will consequently be w. The marginal cost of labor is therefore $\mathrm{MC}_{\mathrm{L}}=\mathrm{w}$.

What the firm benefits from hiring more workers is the value of the units produced. The marginal product of labor (i.e. how many additional units that are produced for every additional unit of labor) is $\mathrm{MP}_{\mathrm{L}}$, and the value of that is $\mathrm{MRP}_{\mathrm{L}}$.

The firm hires until they no longer benefit from doing so, i.e. as long as $\mathrm{w}<\mathrm{MRP}_{\mathrm{L}}$. As soon as $\mathrm{w}=\mathrm{MRP}_{\mathrm{L}}$, they stop hiring and this is consequently the criterion for equilibrium.

## Exercise 10.2.3

a) The difference is that the monopolist faces a downward sloping supply curve. This, in turn, means that the marginal revenue will lower than the price: $\mathrm{MR}<\mathrm{p}$. If the marginal product of labor is $\mathrm{MP}_{\mathrm{L}}$, then the marginal revenue product of labor is no longer $\mathrm{p}^{*} \mathrm{MP}_{\mathrm{L}}$, but $\mathrm{MRP}_{\mathrm{L}}=\mathrm{MR}^{*} \mathrm{MP}_{\mathrm{L}}$.
The labor market is still perfectly competitive, so no agent can affect the wage, w. The marginal cost of labor is then still w , and the marginal revenue product is $\mathrm{MRP}_{\mathrm{L}}$. The criterion is then the same as before, $\mathrm{w}=\mathrm{MRP}_{\mathrm{L}}$. However, $\mathrm{MRP}_{\mathrm{L}}$ has changed.
b) The point of equilibrium in Exercise 10.2.2 is $w=\mathrm{MRP}_{\mathrm{L}}$, where $\mathrm{MRP}_{\mathrm{L}}=\mathrm{p} * \mathrm{MP}_{\mathrm{L}}$. In this exercise, the criterion is the same but $\mathrm{MRP}_{\mathrm{L}}=\mathrm{MR} * \mathrm{MP}_{\mathrm{L}}$. Furthermore, $\mathrm{MR}<\mathrm{p}$. To separate these two cases, let us use the superscript $C$ for the case when the output market is competitive and $M$ when the firm is a monopolist. Then we can write the equilibrium criterion for the first case as $\mathrm{w}^{\mathrm{C}}=\mathrm{MRP}_{\mathrm{L}}{ }^{\mathrm{C}}=\mathrm{p}^{\mathrm{C} *} \mathrm{MP}_{\mathrm{L}}{ }^{\mathrm{C}}$, and for the second case as $\mathrm{w}^{\mathrm{M}}=\mathrm{MRP}_{\mathrm{L}}{ }^{\mathrm{M}}=\mathrm{MR}^{\mathrm{M}} * \mathrm{MP}_{\mathrm{L}}{ }^{\mathrm{M}}$.
Since the wage is set in a perfectly competitive market, w must be the same in both cases, $\mathrm{w}^{\mathrm{C}}=\mathrm{w}^{\mathrm{M}}$. This implies that $\mathrm{p}^{\mathrm{C}} * \mathrm{MP}_{\mathrm{L}}{ }^{\mathrm{C}}=\mathrm{MR}^{\mathrm{M} *} \mathrm{MP}_{\mathrm{L}}{ }^{\mathrm{M}}$.
However, since $\mathrm{MR}^{\mathrm{M}}<\mathrm{p}^{\mathrm{C}}$ we must have that $\mathrm{MP}_{\mathrm{L}}{ }^{\mathrm{M}}>\mathrm{MP}_{\mathrm{L}}{ }^{\mathrm{C}}$, else the equality cannot hold.

Furthermore, the $\mathrm{MP}_{\mathrm{L}}$ curve should be a downward sloping function of L , according to the law of diminishing marginal returns (i.e. additional workers add less marginal product). Therefore, the monopolist must choose fewer workers as compared to a perfectly competitive market in order to make $\mathrm{MP}_{\mathrm{L}}{ }^{\mathrm{M}}>\mathrm{MP}_{\mathrm{L}}{ }^{\mathrm{C}}$.

## Exercise 10.2.4

Monopsony is a direct parallel case to monopoly. Instead of an MR curve that is steeper than the demand curve, as for the monopolist, the MC curve will be steeper than the supply curve.

In Figure S.10.2, we have drawn the situation for the monopsonist. The supply curve, $w(L)$, is a direct function of the wage, and the firm's marginal cost of labor, $\mathrm{MC}_{\mathrm{L}}$, will be twice as steep (as long as they are straight lines). In a perfectly competitive market, the firm would have hired until $\mathrm{w}=\mathrm{MRP}_{\mathrm{L}}$. However, the marginal cost of hiring is no longer equal to the wage, as the firm must increase the wage for all those already hired as well. The firm will, instead, hire as long as the increase in cost is lower than the increase in revenue, i.e. until $\mathrm{MC}_{\mathrm{L}}=\mathrm{MRP}_{\mathrm{L}}$.
a) As we can see in the figure, this means that the monopsonist hires fewer workers, and that
b) Their wage will be lower than it would in a perfectly competitive market.

Figure S.10.2


## 11. General Equilibrium

### 11.1 Definitions

## Exercise 11.1.1

a) A Pareto improvement is a reallocation such that

- No one gets less utility, and
- At least one individual, possibly many, gets more utility.
b) A Pareto efficient allocation is an allocation where no Pareto improvements are possible.
c) A zero-sum game is a strategic situation in which the sum of losses and gains is always zero. In other words, what the winners gain is what the losers lose.
d) The criteria are:
- $\quad \mathrm{MRS}_{\mathrm{A}}=\mathrm{MRS}_{\mathrm{B}}$; Efficient consumption. Both individual A and individual B have the same marginal valuation of the goods.
- MRTS $_{1}=$ MRTS $_{2}$; Efficient production. It is not possible to produce more of one good without producing less of the other.
- $\quad \mathrm{MRS}=\mathrm{MRT} ;$ Efficient product mix. No consumer can receive more utility by changing the mix of products, without anyone else receiving less utility.


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e) The theorems of welfare economics:

- $I^{\text {st }}$ theorem of welfare: If all trade occurs in perfectly competitive markets, the allocation that arises in equilibrium is efficient.
- $2^{\text {nd }}$ theorem of welfare: Each point along the contract curve is a competitive equilibrium for some initial allocation of goods.


### 11.2 Efficient Production

## Exercise 11.2.1

a) In point $a$, we produce 20 apples and 60 bananas, but it is possible to produce more of both without producing less of any. Therefore, $a$ cannot be an efficient production.
b) All points in the grey area in Figure S .11 .1 constitute Pareto improvements as compared to $a$.

c) We want to have all Pareto efficient points that are also Pareto improvements as compared to point $a$. That corresponds to all points along the production contract curve that are also in the grey area, i.e. the part that is indicated with a thick full line.
d) The criterion is that MRTS $_{\text {Apple }}=$ MRTS $_{\text {Banana }}$.

MRTS, the marginal rate of technical substitution, is the slope of the isoquants. For an efficient production, a necessary condition is that an isoquant for one of the goods just about touches an isoquant for the other good. In such a point, the curves have the same slope. The criterion is then fulfilled at all points where the slopes are the same.
e) The production contract curve is all points where the criterion from d) is fulfilled. It will be a line starting in the southwest corner, ending in the northeast corner, and running through all points where two isoquants just about touch each other. In Figure S.11.1, we have sketched such a line.
f) In Figure S.11.1, we have three points where two curves just about touch each other. They correspond to production levels of 20, 60, and 80 apples and 80,60 , and 20
bananas. Thereby, we get three points on the transformation curve (see Figure S.11.2). We then connect these three points with an always-downward sloping line.
g) The alternative cost is what one has to give up in order to produce more of something else. Changing from point $b$ to any other point in the grey area in Figure S.11.2, we do not have to give up anything at all. Consequently, the alternative cost is zero. If we want to move to an efficient production that does not lie within the grey area, however, we must produce either fewer apples or fewer bananas. That reduction in production is then the alternative cost.



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## 12. Choice under Uncertainty

## Exercise 12.1.1

The expected value is the sum of all probabilities times the value of each outcome:

$$
\frac{1}{6} \cdot 1+\frac{1}{6} \cdot 2+\frac{1}{6} \cdot 3+\frac{1}{6} \cdot 4+\frac{1}{6} \cdot 5+\frac{1}{6} \cdot 6=3,5
$$

## Exercise 12.1.2

a) The slope of the utility curve becomes lower the wealthier she becomes. That implies that she receives less and less utility from additional amounts of wealth. Therefore, she has diminishing marginal utility.

Figure S.12.1

b) She is risk-averse. That follows immediately from the fact that she has diminishing marginal utility.
c) The expected value is

$$
\frac{1}{2} \cdot(-500,000)+\frac{1}{2} \cdot(+500,000)=0
$$

d) She receives the utility of having 500,000, which we can read off at point $a$ in Figure S.12.1: the utility is 7 .
e) If she loses, she has 0 , which gives her a utility level of 0 . If she wins, she has $1,000,000$, which gives her a utility level of 10 . Each case occurs with a probability of $50 \%$. The expected utility is consequently

$$
\frac{1}{2} \cdot 0+\frac{1}{2} \cdot 10=5
$$

That corresponds to point $b$ in the figure.
f) The risk premium is the maximum amount she would be willing to pay not to take part in the lottery.
What certain level of wealth gives her the same level of utility as participating in the lottery gives her? We can read that off at the intersection between a utility level of 5 (the expected utility of participating in the lottery) and the utility function: point $c$. That corresponds to a certain wealth of 250,000 . Consequently, she is prepared to go from an (uncertain) expected level of wealth of 500,000 , to a certain level of wealth of 250,000 . She is then prepared to pay $500,000-250,000=250,000$.
The corresponding distance is indicated in the figure.


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## 13. Other Market Failures

### 13.1 Basic Concepts

## Exercise 13.1.1

a) An externality is a situation in which the consumption or the production of goods and/or services has positive or negative effects on the utility of other people, and that is not reflected in the price.
b) A positive externality increases others' utility whereas a negative externality decreases it. Both types constitute economic problems. Too few goods with positive externalities are produced, too many with negative externalities.
c) A public good is a good that is both

- Nonrival. Many individuals can consume the same unit of the good.
- Nonexclusive. No one can be kept from consuming the good.
d) Free riding means that someone consumes a good without paying a fair share of the price. One way to do that is to lie about one's valuation of a public good. If the others pay their true valuations, one will be able to consume more than one pays for. This can become a problem if all, or enough many, try to free ride, since then the good might not be produced at all.


### 13.2 Externalities

## Exercise 13.2.1

a) Since the pulp is sold in a perfectly competitive market, the firm's demand curve is horizontal and $\mathrm{MR}=\mathrm{p}$. The MC curve should be linear and increasing. The situation could look like in Figure S.13.1. The firm chooses to produce at the point where the MC curve intersects the MR curve, i.e. at the quantity $\mathrm{q}_{\mathrm{c}}$.
b) The neighbors' marginal cost is $1 / 3$ of the firm's marginal cost. The ME curve (the cost of the marginal external effect) therefore has $1 / 3$ of the slope of the MC curve.
c) To find the social optimum, we need the total marginal cost of production. That consists of the vertical sum of the MC curve and the ME curve. (For each quantity, the total cost is the sum of the corresponding MC and ME. Hence, we sum vertically.) We have drawn this as the marginal social cost curve, MSC, in the figure. The intersection between MSC and MR corresponds to the quantity where the cost to society starts to be higher than the willingness to pay for additional units of the good. $\mathrm{q}_{\mathrm{s}}$ is consequently the optimal quantity, all things considered.

d) If one knows the value of ME, one could introduce a per-unit tax. In this case, a tax of $1 / 3$ of MC would make the firm produce at the quantity $\mathrm{q}_{\mathrm{s}}$.
An alternative is to introduce a maximum allowed quantity of production, corresponding to $\mathrm{q}_{\mathrm{s}}$. The firm will then produce as much as it is allowed.

### 13.3 Public Goods

## Exercise 13.3.1

First, we need to construct a curve for the aggregate willingness to pay, i.e. how much A and B together are willing to pay for different quantities of the park. To that end, we sum the individual demand curves vertically. In the direction of the Y-axis, we can put B's maximum valuation on top of A's, and thereby get the aggregate maximum willingness to pay.


At the far right, at enough high quantities, only A demands additional units. Over that interval, the aggregate demand is equal to A's demand. If we tie the two parts together, we get the curve labeled $D_{A}+D_{B}$ in Figure S.13.2.

The point where the aggregate willingness to pay intersects the MC curve is the optimal choice. In the figure, it is labeled $q^{*}$.

Since they have different marginal willingness to pay, they should not have to pay the same amount. A should pay the amount labeled $A$ on the Y -axis, whereas B should pay the amount labeled $B$.

