# International Trade Theory

## A critical review

Murray C. Kemp

Routledge Studies in International Business and the World Economy

# International Trade Theory

Economic theory grew out of the European industrial revolution and the associated globalization of the world economy in the seventeenth and eighteenth centuries. Central to the new theory were questions concerning international trade and investment: What determines the pattern of trade and investment? What are the implications of its trade and investment for a country's well-being? These questions are as relevant and alive today, in the midst of a new wave of globalization, as in the time of Montesquieu and Adam Smith.

This book, the third in a three-volume set, brings together several chapters on the current state of the theory of international trade. Critical in tone, the chapters show that several long-established propositions, concerning free trade for example, are seriously defective. On the other hand, Kemp's chapters are also constructive. Thus the international equalization of factor prices, once thought to be possible only under perfectly competitive conditions with a unique market equilibrium, is shown to be possible even when producers exercise market power and the equilibrium is not unique. Similarly, the book offers a much more realistic analysis of international transfers by allowing for the possibility that donors and recipients care about the well-being of each other. Finally, the book explores the implications of the fact that many countries can no longer survive autarchy, that is, without trade.

Written by one of Australia's foremost economists and covering subject areas such as the theory of international trade, international finance and investment and aid, this book is highly interesting and topical. Its accessible style makes it an important book for anyone with a desire to understand the causes and implications of international trade.

**Murray C. Kemp** is Emeritus Professor at the University of New South Wales in Sydney and has previously served as President of the International Economics and Finance Society.

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*Mazen Labban*

**43 Petroleum Taxation**
Sharing the oil wealth: a study of petroleum taxation yesterday, today and tomorrow
*Carole Nakhle*

**44 International Trade Theory**
A critical review
*Murray C. Kemp*

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**Murray C. Kemp**

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# Introduction

The present volume brings together several chapters on the current state of the theory of international trade. Most of the chapters have been published, but many are not readily available electronically. The chapters are critical in tone. However, many of them are also constructive. On the one hand, it is suggested that several of the oldest and best-established propositions in trade theory are in need of substantial qualification. Here I have in mind the theory of comparative advantage, both in the form given the theory by Torrens and Ricardo and in the form given it by Gottfried Haberler. I also recall the theory of international transfers, both requited (indemnities) and unrequited (foreign aid). For many years the theory hobbled along under the implausible assumption that each country derives satisfaction only from its own consumption. Now, at last, a more convincing theory can be constructed.

On the other hand, it is suggested that some well-established propositions are much broader in scope than has been thought. Here I have in mind the theory of factor price equalization, once thought to be valid only under constant returns to scale, perfectly competitive markets and a unique equilibrium, but now seen to be possible under increasing returns to scale, when producers exercise market power and when multiple equilibria prevail. I also recall the traditional gains-from-trade proposition, greatly extended in recent years to embrace missing markets and (bequest-free) overlapping generations but now found wanting in other contexts, dynamic as well as static.

The present volume is companion to two earlier volumes also published by Routledge (see Kemp 1995, 2001). In the earlier volumes, the focus is on the normative theory of international trade. Taken together, the three volumes provide a fairly complete survey of the modern general-equilibrium theory of international trade.

The main focus in Parts I and II is on descriptive theory; however, in Part III, I do briefly resume the earlier discussion (in Kemp 1995, 2001) of normative theory.

The descriptive theory of international trade has two easily distinguishable major components. The first or classical component goes back to the pioneering studies of Adam Smith and David Ricardo, the second or neoclassical component to the twentieth century work of Eli Heckscher, Bertil

Ohlin, Abba Lerner and Paul Samuelson. The classical component, commonly known as the Ricardian theory, emphasizes the role of international differences in technology in determining patterns of international trade; the neo-classical component, commonly known as the Heckscher-Ohlin or Lerner-Samuelson theory, emphasizes the role of international differences in primary factor endowments in determining trade patterns.

Each branch of the theory is highly simplified, stripped of all elements that might compete with differences in technology or endowments as determinants of trade patterns. By isolating a single powerful influence on trade, each branch has proven to be a useful teaching device. However, textbook presentations of the material are usually incomplete and sometimes quite misleading: They rely on several assumptions that are vital to the conclusions drawn but never made explicit, and they rely on explicit but implausible assumptions that are not vital to the conclusions and therefore misleadingly suggest that the theory is of limited applicability. In the present volume, it is my primary objective to bring hidden assumptions to the surface and to assess the implications of weakening any implausible assumptions or withdrawing them altogether.

I now offer a brief, chapter-by-chapter indication of the contents of each of the three parts of the book. Part I contains just four chapters. However, each of them unearths vital but hitherto unrecognized assumptions underpinning the Torrens-Ricardo and Haberler Principles of Comparative Advantage. The first chapter focuses on the Torrens-Ricardo Principle. It brings to light the hidden assumptions that, both in autarky and under free trade, each country can produce all commodities and that, in autarkic equilibrium, each country consumes all commodities. Without those assumptions, all or part of the Torrens-Ricardo Principle must be abandoned. In the second chapter it is shown that Haberler's Principle rests on the same assumptions and, without them, also assumes a greatly weakened form. In the third chapter, on the other hand, it is shown that, unless the preferences of all countries are sufficiently similar, parts of the Torrens-Ricardo Principle must be abandoned – even if, in the autarkic equilibrium, all commodities are consumed in each country. Finally, in the fourth chapter, it is shown how randomness in preferences, production sets or factor endowments can destroy the predictive power of long-run comparative advantage over patterns of production and international trade.

In Part II, I turn to the neo-classical theory of trade developed by Lerner and Samuelson, focusing again on the hidden assumptions of the theory. Thus Chapters 5 and 6 draw attention to the universal but implausible assumption that each country has an autarkic equilibrium. It is shown that if not all countries possess an autarkic equilibrium, then well-known and generally accepted propositions, such as that of Mill and Edgeworth concerning the possibility of impoverishing technical improvements and the companion proposition that a country inevitably benefits from an increase in the demand for its exports by the rest of the world, must be heavily qualified or abandoned.

Indeed, it emerges that a technical improvement in one trading country might be disastrous for all countries in ensuring the *non-existence* of free-trade equilibrium. It is noted also that, if not all countries possess an autarkic equilibrium, the Torrens-Ricardo Principle of Comparative Advantage must be reformulated in a conditional form.

Chapters 7 and 8 introduce and put to work a dynamic version of the static Lerner-Samuelson model. In the dynamic model, the reallocation of resources in response to any disturbance is costly and therefore gradual; moreover, given a positive rate of time preference, the reallocation stops short of that indicated by the static model. Thus, the familiar comparative statics of the Lerner-Samuelson model must be abandoned even as long-run approximations; only when the rate of time preference is zero can they be accorded long-run validity.

Chapters 9 to 11 question the relevance to practical policy-making of the prevailing theory of international lump sum transfers. In that theory it is assumed that each country is completely indifferent to the other's well-being. Specifically, it is assumed that the well-being of each country depends solely on that country's consumption pattern; and, on the basis of that assumption and of the further assumption of local Walrasian stability, it is shown that, in the absence of a third country, the recipient country always benefits and the donor country always suffers from a transfer. However, to assume that each country is indifferent to the other's well-being is to render the theory irrelevant to any discussion of post-war reparations between countries that have recently been intent on the annihilation of each other and to any discussion of friendly and unconditional aid from one country to another.

It is shown in Chapters 9 to 11 that if the conventional assumption is replaced by something more appropriate, then the conventional conclusion must be abandoned. In Chapters 9 and 10 it is shown that if the well-being of each formerly warring country is negatively influenced by the well-being of the other then the defeated donor might benefit at the expense of the victorious recipient and that this is so even in the absence of a third or bystander country and even when the world economy is stable in the sense of Walras. The finding is counter-intuitive. One might have expected that, under the revised assumptions, the donor would incur the *additional* burden of improving the lot of a nation it scorns and that the recipient would reap the *additional* satisfaction of putting down a people it cordially dislikes. In Chapter 11, on the other hand, it is shown that if each country derives satisfaction from a sufficiently small improvement in the well-being of a sufficiently poor trading partner then *either* no country extends aid to the other *or* one country extends aid and both countries benefit from the aid.

Chapter 12 reconsiders the implications of foreign aid from an alternative point of view. Foreign aid is, for the most part, government to government. Moreover, much of inter-governmental aid is *tied* by the requirement that at least a specified portion of the aid be spent by the recipient on the exports of the donor. That is, the aid is not freely offered; the offer is *conditional*.

Aid tied in this way has already attracted the attention of trade theorists. However, the emerging theory focuses on the particular case in which only private consumption goods are produced, consumed and traded; it is assumed, in effect, that the recipient government receives the aid, spends the required proportion on the donor's exports and the balance on other goods, then distributes to its households the basket of goods obtained in this way. Evidently, the recipient's households are somehow prevented from reselling on world markets. This seems to imply direct and unwelcome intervention by the recipient government in the decision-making of its households.

The root of the difficulty is that in the emerging theory only private consumption goods are recognized. In a context of foreign aid the assumption is unrealistic, for much of inter-governmental aid is in terms of dams, bridges and highways. In Chapter 12, therefore, I admit the possibility that commodities have a dual function: They can be privately consumed or they can be transformed into public consumption goods. In spite of this modification, however, one of the principal conclusions of the emerging theory remains intact: Even in a world of just two trading countries and two traded commodities, tied aid might benefit the donor and harm the recipient; that is, paradoxes might recur without the intervention of third or 'bystander' countries.

Chapters 13 to 16 are devoted to the much admired factor price equalization theorem, independently developed by Lerner and Samuelson. Each pioneer proved the theorem on the twin assumptions of convex production sets and perfectly competitive markets and confined his attention to a world of just two trading countries, two tradable products and two internationally immobile primary factors of production. It was soon understood that the theorem could be extended to more ample world economies, but even recent discussion of the theorem has continued to rely on the assumption of convex production sets and perfectly competitive markets. Indeed, some have sought to establish that those assumptions are *necessary* components of the theorem. The attempt was unsuccessful. For, as shown in Chapter 13, if the non-convexities flow from external economies associated with changes in world outputs, then the Lerner-Samuelson theory is already sufficiently general to accommodate the non-convexities; and, as shown in Chapter 14, the existing theory is already sufficiently general to cover any mixture of perfectly competitive and oligopolistic industries. Only a little re-interpretation is needed.

Chapters 15 and 16, on the other hand, establish the intuition of the Lerner-Samuelson theorem in the general context of $m$ factors, $n$ products and multiple equilibria, as well as sufficient conditions for the theorem to hold within each and every subset of two or more trading countries.

The final chapter of Part II was written to mark the hundredth anniversary of Bertil Ohlin's birth and considers the continuing role of Heckscher-Ohlin-Lerner-Samuelson theory in the training and equipment of trade specialists. It contains an early and succinct account of many of the points developed in more detail in other chapters.

In Part III the focus moves to the normative theory of international trade. That free trade is potentially beneficial to all members of a country is one of the oldest conjectures in the history of economic thought, going back to Montesquieu, writing in the first half of the eighteenth century. However, through most of its long history, the conjecture has lacked a general and complete proof; and, even during the last thirty years, when we have had access to widely accepted proofs, there has always been a picket line of those who purport to have found defects in the proofs. The objections are regularly put down, but the net cumulative effect of the continuing debate can only be to create doubts in the minds of non-specialists. Chapters 18 and 19 have been included for their benefit (and amusement), Chapter 19 disposing of several formal objections to the available proofs and Chapter 18 disposing of a popular but faulty argument in favour of trade gains.

Chapter 20 is a different cup of tea. It discusses, tentatively, two very serious and hitherto neglected barriers to complete reliance on existing proofs of the gainfulness of free trade. The best known of those proofs are based on simple modifications of the Arrow-Debreu theory of general equilibrium. (Essentially, the Arrow-Debreu theory is extended to accommodate several trading countries, some with schemes of lump sum compensation in place.) However, in finite trade models of Arrow-Debreu type it is implicitly assumed that each household is sufficiently ill-informed about the economy of which it is part, or sufficiently irrational, to believe that it has no market power. Such a pre-condition does not fit comfortably in a game to be played repetitively. If, instead, it is assumed that some households are both perfectly informed and perfectly rational, so that they are aware that they exercise market power wherever they buy or sell, the gainfulness of trade can be established only under additional assumptions. Sufficient additional assumptions have been provided. However, the additional assumptions have been imposed not on the exogenous parameters of the model but directly on the endogenous variables. They are not Arrow-Debreu assumptions.

The Arrow-Debreu trade model has been extended to accommodate overlapping generations, and a new proof of potential trade gains has been constructed on the basis of the extended model. However, the new proof does not allow for bequests or for inter-generational gifts *inter vivos*. When the possibility of bequests is allowed, so must be the strategic relationship in which the two pairs of parents-in-law find themselves. With that relationship is associated inefficient behaviour, bearing primarily on the rate of saving. Moreover, the opening of a country to free trade may so exacerbate the inefficiency of savings decisions as to negate the possibility of gainful trade. In any case, even with overlapping generations, each household is part of a finite economy and has market power, a fact that can be ignored only if the household is imperfectly informed or imperfectly rational or both.

In Chapters 21 and 22 we turn to the analysis of worldwide cooperative tariff reform. Chapter 21 focuses on the implications of multilateral GATT/WTO (General Agreement on Tariffs and Trade/World Trade Organization) negotiations subject to the most-favoured-nation clause and to the twin Paretian rules (that negotiations leave the trading world on its efficiency locus and each participating country in a preferred position). It is shown that the set of tariff reforms that satisfies both rules: (i) is always non-empty; (ii) might include no reforms that end in worldwide free trade; (iii) always includes reforms that are incompatible with free trade and involve revised tariffs one of which is positive, the other negative; and (iv) might include reforms that support a Pareto-optimal and Pareto-improving allocation of resources but also support other allocations with neither of those characteristics. The implications of (iv) for the implementability of tariff reform are spelled out. That matching any feasible international lump sum transfer there is an 'equivalent' pair of import duties, a proposition contributed by Wolfgang Mayer, plays a prominent role in the demonstration.

Chapter 21 focuses on several of the fundamental questions associated with tariff reform. However, all of the questions are handled in terms of the conventional Lerner-Samuelson two-by-two theory of international trade. Evidently, the questions might have been better posed in the broader context of $m$ countries and $n$ commodities ($m$, $n \geq 2$). In that broader context, some of our more important conclusions – that a free-trade agreement is not generally Pareto-improving and that if free trade is ruled out, then a Pareto-improving and Pareto-optimal outcome requires that, in one country, imports must be subsidized or exports taxed – survive in suitably modified form. All the same, matters cannot be left there. For in the broader $m$-by-$n$ context there is a new possibility – that a Pareto-improving and Pareto-optimal reform is not available. That possibility is discussed in Chapter 22. There it is shown by example that, for $m$-by-$n$ economies with only modest (Arrow-Debreu) restrictions on preferences and endowments (including endowments of market and technological information) and in the absence of arbitrary and possibly question-begging restrictions on equilibrium outputs and net exports, there does not generally exist an 'equivalent' tariff vector.

The final triplet of chapters forms a general technical appendix, the purpose of which is to make clear the methodological principles on which the earlier chapters are based. Thus Chapter 23 provides a brief exposition of the case against the assumption of a representative agent in general-equilibrium theory. That case is relied on in several of the earlier chapters. Chapter 24, on the other hand, draws attention to the fact that the Arrow-Debreu model of general equilibrium, which forms the basis of much of the descriptive theory of international trade and all of the static normative theory of trade, makes sense only if each household is incompletely informed about the environment of which it is part and/or incompletely rational. By implication, complete information and complete rationality on the part of some households can be combined with price taking only at the expense of internal consistency in a large part of trade theory. This point is central in the analysis of Chapter 20. Finally, Chapter 25 opposes the popular view that, to resolve policy issues it is often necessary to trade off realism for tractability. Against that view, it is suggested that, when policy advice is based on restrictive assumptions, the advisor must indicate how the advice would change if the assumptions were relaxed. Essentially, this chapter states the general case against relying on a single set of special assumptions without considering the implications of relaxing the assumptions.

Further methodological principles are developed along the way, especially in Chapter 7 (where dynamic Walrasian systems are shown to be internally inconsistent) and in Chapter 22 (where it is emphasized that designated endogenous variables must retain that status throughout an analysis).

Many of the chapters collected here are the product of collaborations within a small group of close friends: Geoffrey Fishburn, Yoshio Kimura, Nissan Liviatan, Masayuki Okawa, Koji Shimomura, Makoto Tawada and Henry Wan. I am grateful to them for their help and warm friendship extending over many years and, in the case of republished chapters, to Blackwell Science Limited, Edward Elgar Publishing Limited, Macmillan Press Limited, Springer-Verlag, the Centre for Studies in Social Sciences, Calcutta, and the Faculty of Business of City University of Hong Kong and Sweet and Maxwell Asia for permission to reprint. The collection was put together during 2006, while I was visiting Macquarie University, in Sydney. In preparing the chapters for publication, I have relied on the editorial and research skills of Ellen Young. Without Ellen's daily guidance the project would have soon foundered.

# Part I

# The classical theory of international trade

# 1 The Torrens-Ricardo Principle of Comparative Advantage

An extension

## 1.1 Introduction

Nearly two hundred years on, the Torrens-Ricardo Principle of Comparative Advantage is still widely admired within the profession, and appears prominently in many elementary textbooks and in most treatises on international trade. However, careful inspection of the Principle, either in the mildly disparate formulations of Torrens (1815: 264–5) and Ricardo (1817: 135) or in any later formulation, reveals that it relies on restrictive assumptions about preferences and technology in each trading country, assumptions that are always implicit, never explicit. Specifically, the Principle rests on the assumption that in autarkic equilibrium each country consumes all commodities, at least incipiently. Our purpose is to make good this claim and to reformulate the Principle in sufficient generality to accommodate alternative assumptions about preferences and technology. In our reformulation the emphasis is on marginal rates of substitution in consumption, not on the traditional ratios of marginal labour costs in production. Thus our restatement concerns not merely the proper display of the Torrens-Ricardo Principle but rather its essential content. It is shown in effect that, in existing formulations, the supply side is assigned a role that it cannot always sustain. That two classical economists overlooked this point can be understood, but the same indulgence cannot be extended to the authors of neo-classical textbooks.[1]

## 1.2 The standard formulation

In the usual textbook formulation, two countries, England and Portugal, produce, consume and trade two commodities, cloth and wine; there are no non-tradable commodities. Each commodity is produced by means of a single primary factor, homogeneous labour, under constant returns to scale. Within each country, but not necessarily across countries, all households are identical in all respects: size, age distribution, preferences, quality of labour and access to technical information.[2]

For England, the household and (by revision of quantity units) the economy-wide production possibility locus is represented in Figure 1.1(a) by the straight segment $Q_E Q'_E$, the slope of which is (minus) the ratio of the two marginal labour costs of production. In the absence of market distortions, a unique autarkic equilibrium is represented by point $C_E$, where a community indifference curve forms a tangent to the production possibility locus and where, for each commodity, (positive) consumption is equal to production. The equilibrium commodity price ratio is equal to the ratio of marginal labour costs. Similarly, the unique autarkic equilibrium of Portugal is represented in Figure 1.1(b) by point $C_P$.

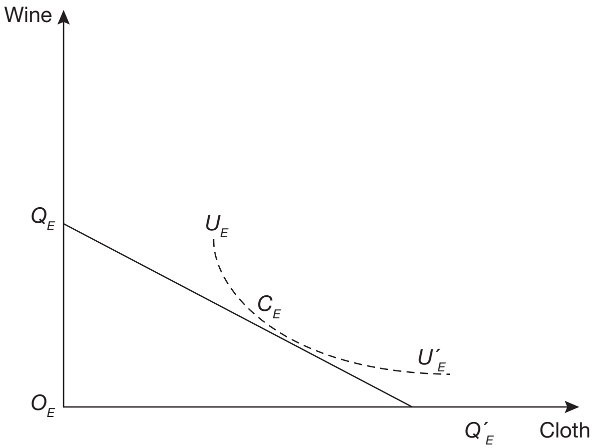


*Figure 1.1a* England's autarkic equilibrium, with incomplete specialization

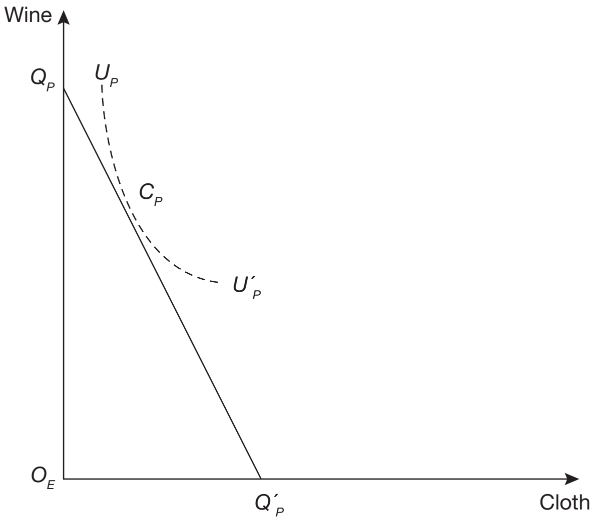


*Figure 1.1b* Portugal's autarkic equilibrium, with incomplete specialization

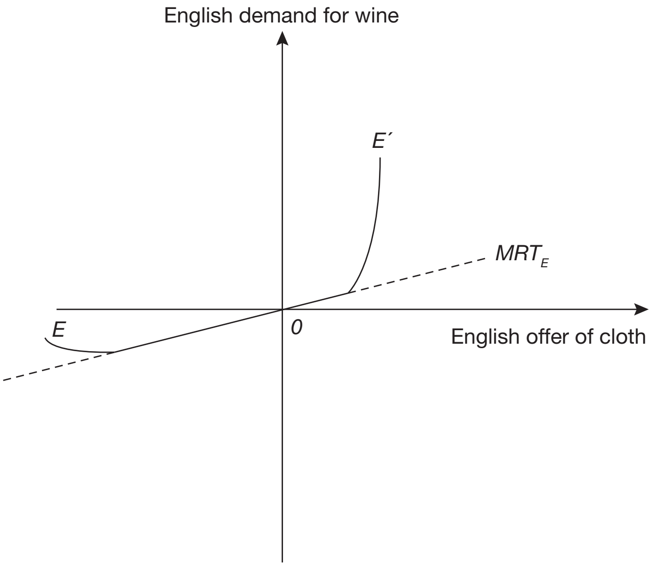


*Figure 1.2a* England's offer curve, with incomplete autarkic specialization

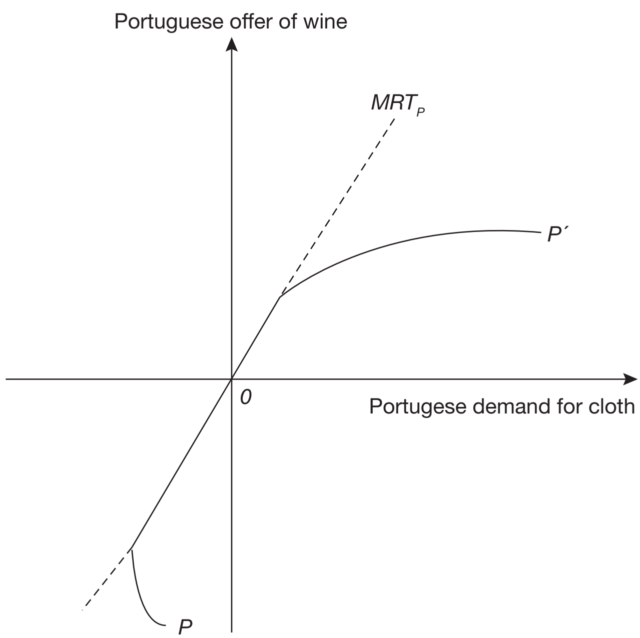


*Figure 1.2b* Portugal's offer curve, with incomplete autarkic specialization

Abandoning the assumption of autarky, let us pass in review all conceivable world price ratios. Given any particular price ratio, we can determine the profit-maximizing pair of English outputs, uniquely except when the hypothetical price ratio is equal to the equilibrium autarkic price ratio, and we can determine uniquely the utility-maximizing English consumption pair. Transferring that information to Figure 1.2(a), we obtain the offer curve $EOE'$ for England, where the straight segment is obtained from the production possibility locus of Figure 1.1(a), with marginal rate of transformation $MRT_E$. Similarly, Figure 1.2(b) displays the offer curve $POP'$ for Portugal. Finally, superimposing the Portuguese offer curve on Figure 1.2(a), we obtain Figure 1.3, in which the world equilibrium is represented by point $W$, where the world excess demands for cloth and wine are equated to zero by the equilibrium price ratio $Op$. The world equilibrium need not be unique. The Torrens-Ricardo result emerges easily from this construction.

*Proposition 1.1* The Torrens-Ricardo Principle of Comparative Advantage[3] If the two equilibrium autarkic price ratios differ then:

(a) The equilibrium autarkic commodity price ratios are determined by and are equal to the ratios of marginal labour costs, one ratio for each country.
(b) A non-trivial world trading equilibrium exists.

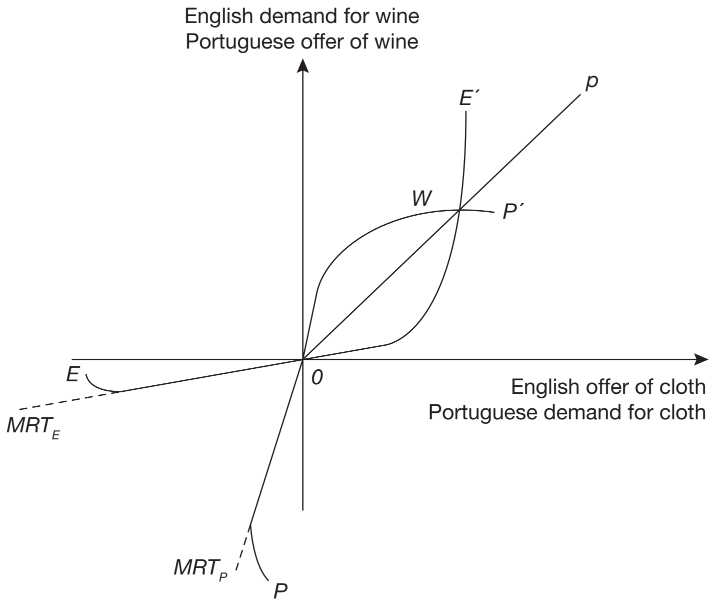


*Figure 1.3* The trading equilibrium, with incomplete autarkic specialization

(c) The equilibrium direction of world trade is determined solely by the two ratios of marginal labour costs.
(d) The equilibrium world price ratio (terms of trade) is bounded by the two autarkic price ratios.
(e) Neither country suffers from participation in international trade, and at least one country benefits.

Throughout the construction, however, it has been assumed that, in the autarkic equilibrium of each country, both commodities are consumed, at least incipiently. In the next section, we explore the implications of removing that assumption.

## 1.3 Specialized autarkic consumption

Suppose then that preferences are such that, in the autarkic equilibrium of each country, consumption is confined to that commodity for which the marginal labour cost is relatively lower than in the other country, that is, to cloth in England and wine in Portugal. The English autarkic equilibrium is depicted by point $C_E$ in Figure 1.4(a). At that point the English consumption of each commodity is equal to its domestic production. The slope of the dashed straight line through $C_E$ is (minus) the autarkic marginal rate of substitution, and the set of equilibrium commodity price ratios forms a continuum bounded by the marginal rate of substitution and the ratio of marginal labour costs. Similarly, the Portuguese autarkic equilibrium is depicted by point $C_P$ in Figure 1.4(b), where the Portuguese consumption of each commodity is equal to its domestic production. The continuum of equilibrium commodity price ratios in Portugal is bounded by the autarkic marginal rate of substitution

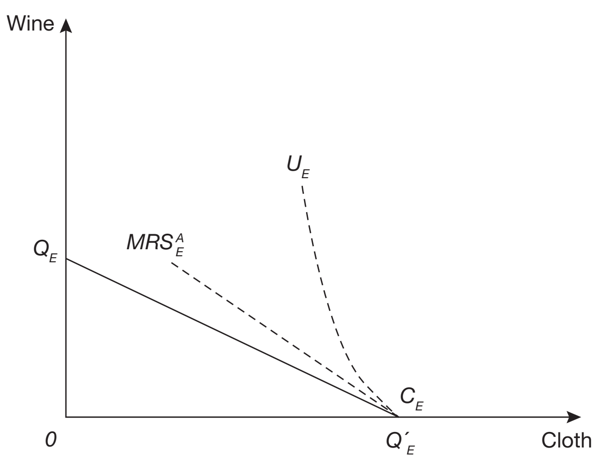


*Figure 1.4(a)* England's autarkic equilibrium with complete specialization

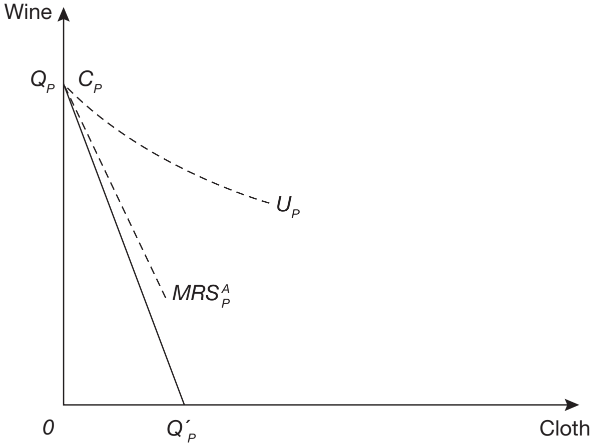


*Figure 1.4(b)* Portugal's autarkic equilibrium with complete specialization

(equal to minus the slope of the dashed straight line through $C_P$) and the ratio of marginal labour costs in Portugal.

Abandoning the assumption of autarky, we may consider, for each country, all conceivable world price ratios and, for each price ratio, the output-maximizing output pair and the utility-maximizing consumption pair. Transferring that information to Figures 1.5(a) and 1.5(b), we obtain the revised offer curves for England and Portugal. It is evident that for all hypothetical price ratios bounded by the autarkic marginal rate of substitution and the ratio of marginal labour costs, production and consumption remain specialized at their autarkic price levels. The new offer curves therefore have a kink at the origin. (They also have a less auspicious kink in the third quadrant.)

Finally, superimposing the revised Portuguese offer curve on Figure 1.5(a), we obtain Figure 1.6, where the world equilibrium is represented by point *W*, with the world excess demands for cloth and wine equated to zero by the equilibrium price ratio *Op.*

Comparing Figures 1.3 and 1.6, we see that the assumption of specialized autarkic consumption has brought about a change in attributes (a), (c) and (d) of the autarkic and world equilibria. In particular, the equilibrium world price ratio now must lie in the more restricted set defined by the two autarkic marginal rates of substitution; (d) is still true but it no longer conveys the whole truth. In the textbook case of Section 1.1, preferences played a role in choosing the equilibrium world price ratio from the set bounded by the two given ratios of marginal labour costs. In the present case, preferences play an additional role. They *determine* the least upper and the greatest lower bounds of the set of possible equilibrium price ratios.[4]

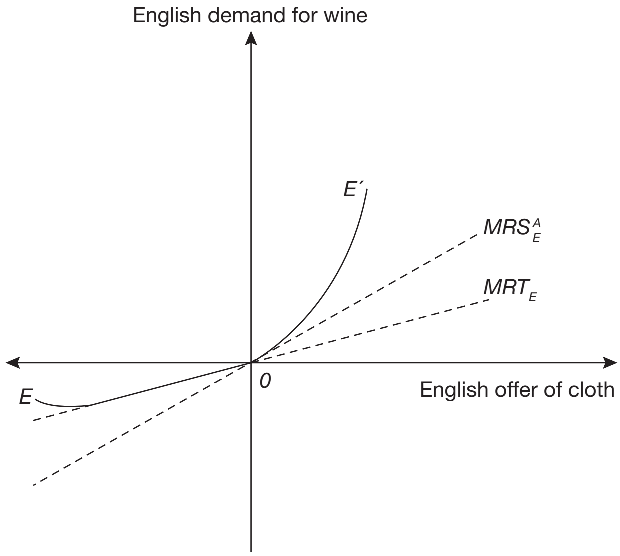


*Figure 1.5(a)* England's offer curve with complete autarkic specialization

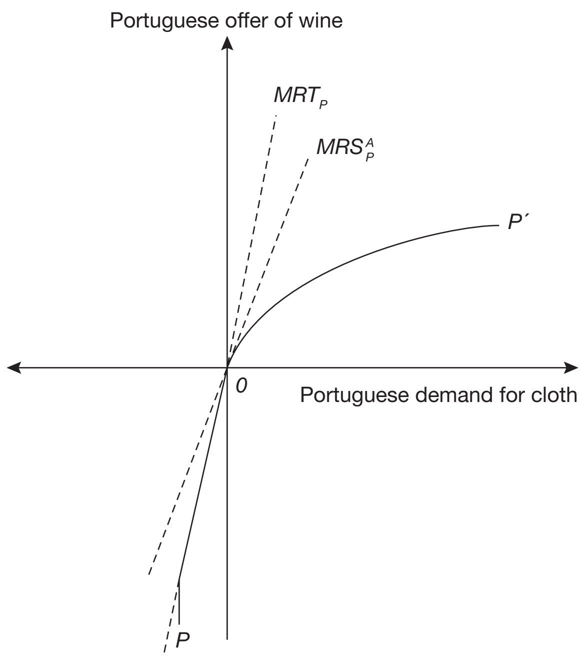


*Figure 1.5(b)* Portugal's offer curve with complete autarkic specialization

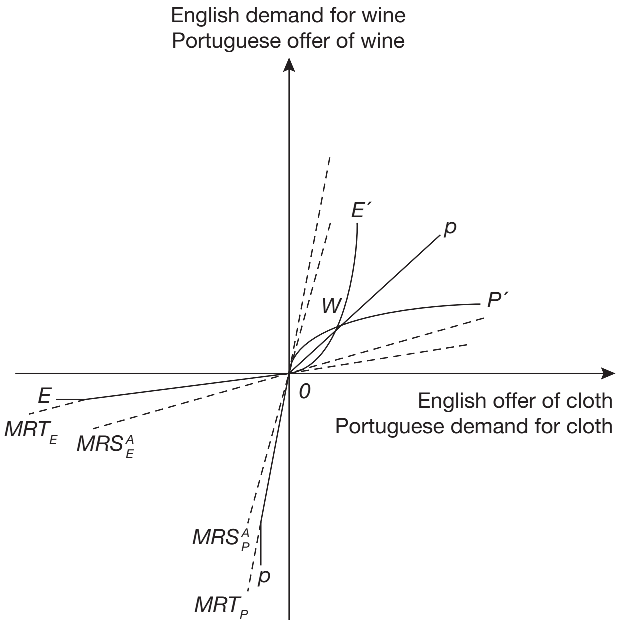


*Figure 1.6* The trading equilibrium, with complete autarkic specialization

Nor is that the end of the story. In Figure 1.6 the two marginal rates of substitution in consumption stand in the same relation to each other as do the two ratios of marginal labour costs. That is why a non-trivial world trading equilibrium still exists in spite of specialized autarkic consumption. However, it is also possible that the two marginal rates of substitution stand in a relationship to each other *opposite* to that of the two ratios of marginal labour costs. In that case, as Figure 1.7 makes clear, the unique world equilibrium is the trivial no-trade equilibrium, with the autarkic marginal rates of substitution preserved and the two sets of equilibrium autarkic price ratios replaced by a single set of equilibrium world price ratios (equal to the intersection of the two equilibrium autarkic price sets). In this extreme case, attributes (a), (b), (c) and (d) must be forgone: The Principle of Comparative Advantage fails altogether. There exists a unique world allocation, without trade and without gains from trade, even when the two equilibrium autarkic price ratios differ from each other; and there exists an interval of price ratios any one of which will support that allocation. Since there are no costs of transporting commodities between countries and no artificial impediments to trade, the same price ratio must prevail in each country. It remains to be noted that this extreme outcome might emerge with specialized autarkic consumption in one country only. For that outcome it is necessary and

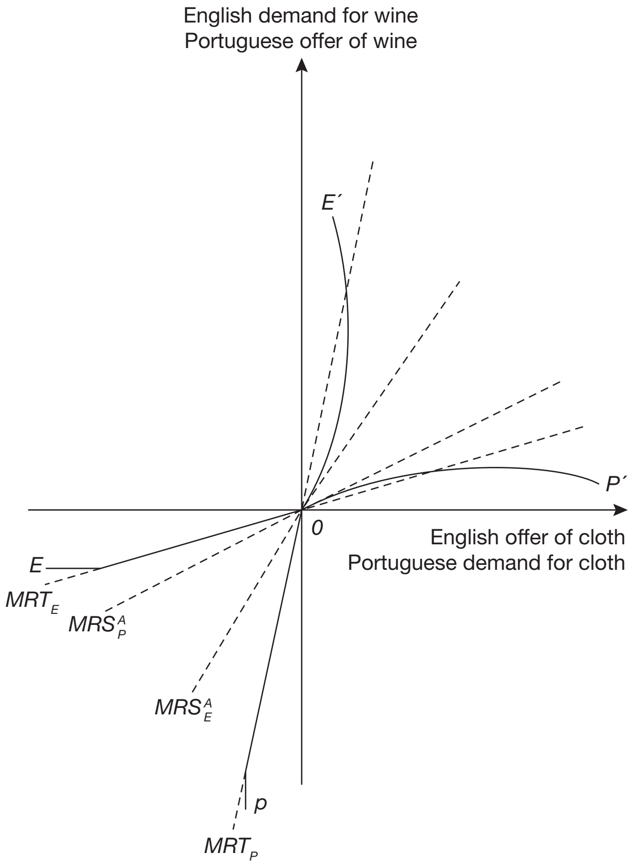


*Figure 1.7* The no-trade trading equilibrium

sufficient that the two marginal rates of transformation and the two autarkic marginal rates of substitution do not stand in the same relationship to each other.

Thus we arrive at our first extension of the standard result.

*Proposition 1.2* First Extension of the Torrens-Ricardo Principle

Suppose that, in the autarkic equilibrium of each country, consumption is confined to that commodity for which the marginal labour cost is relatively lower than in the other country. Then if and only if the two autarkic marginal rates of substitution in consumption stand in the same relationship to each other as do the two ratios of marginal labour costs, a non-trivial world trade equilibrium exists and possesses the following properties:

(i) The equilibrium direction of trade is determined solely by the two autarkic marginal rates of substitution in consumption.
(ii) The equilibrium world price ratio lies between the two autarkic marginal rates of substitution.
(iii) Neither country suffers from its participation in trade, and at least one country benefits from its trade.

Notice that the extended Principle is valid whether or not both commodities are consumed in the free-trade equilibrium.

Notice also that, in this section, we have not ruled out the possibility that, in autarkic equilibrium, the marginal rate of substitution of a country is equal to its marginal rate of transformation so that, incipiently, it consumes both commodities. In particular, we have not excluded limiting Torrens-Ricardo examples in which both countries incipiently consume both commodities and for which the First Extension of the Torrens-Ricardo Principle reduces to the Torrens-Ricardo Principle.

Notice finally that we have not ruled out the possibility that preferences are identical across countries. However, preferences that are both homothetic and identical across countries ensure that the two autarkic marginal rates of substitution in consumption stand in the same relationship to each other as do the two ratios of marginal labour costs.

## 1.4 Further analysis

Let us now reverse the assumptions of Section 1.3 by supposing that in each country autarkic consumption is confined to that commodity for which the marginal labour cost is relatively *higher* than in the other country, that is, to wine in England and cloth in Portugal.

Under the revised assumption, the two autarkic marginal rates of substitution necessarily bear the same relationship to each other as do the two marginal rates of transformation. Hence we need consider only one case. Without entering into the details of its construction, we simply display Figure 1.8, the counterpart to Figure 1.6. However, we do note that the horizontal and vertical linear segments of the offer curves may vanish or merge into non-linear segments; it all depends on the properties of the household and community utility functions.

It is clear from Figure 1.8 that, under our present assumptions, we regain most of the characteristics of the Torrens-Ricardo world.

*Proposition 1.3* Second Extension of the Torrens-Ricardo Principle

Suppose, that in the autarkic equilibrium of each country, consumption is confined to that commodity for which the marginal labour cost is relatively higher than in the other country. Then a non-trivial world trade equilibrium exists and possesses the following properties:

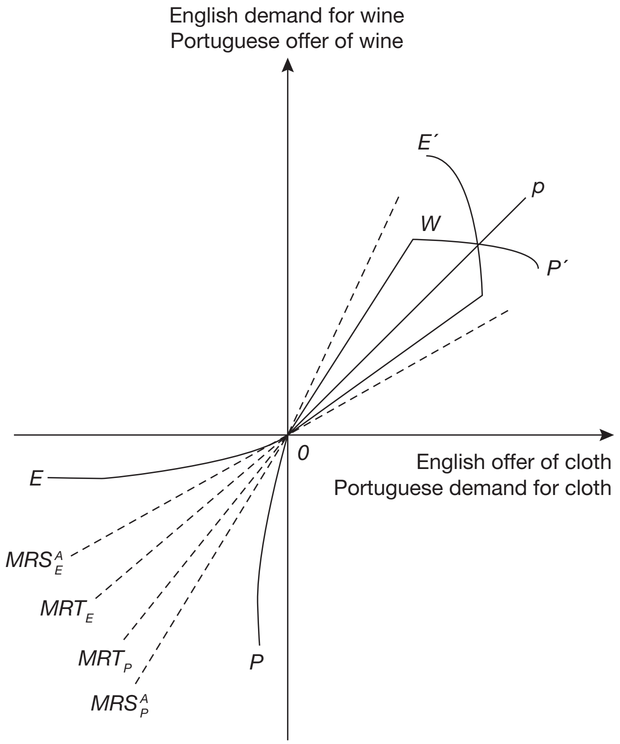


*Figure 1.8* Torrens and Ricardo restored

(i′) The equilibrium direction of trade is determined solely by the two marginal rates of transformation in production.
(ii′) The equilibrium world price is bounded by the two marginal rates of transformation in production.
(iii′) Neither country suffers from its participation in trade and at least one country benefits from its trade.

There remains the mixed case in which, in autarky, both countries specialize in the consumption of the same commodity. In that case, as the reader may easily verify, there are two possibilities. *Either* the marginal rates of transformation and the autarkic marginal rates of substitution stand in the same relationship to each other *or* they do not stand in the same relationship. In the former case conclusions (i)–(iii) of the First Generalized Principle remain valid. In the latter case, the unique world equilibrium is again the trivial no-trade equilibrium; the Principle of Comparative Advantage again fails altogether.

## 1.5 Final remarks

The many-commodities extension of the Torrens-Ricardo Principle, which we owe to von Mangoldt (1871), Edgeworth (1894b), Marshall (1923), Haberler (1929) and, in its continuum-of-commodities form, to Dornbusch, Fisher and Samuelson (1977), also relies on the assumption that autarkic consumption is non-specialized and must be qualified if the assumption is relaxed.[5] Even if all commodities but one are consumed in autarky, the 'chain of comparative advantage' may be broken; thus, except in the very special two-commodities case, our critique does not rely on the complete specialization of consumption.

Let us focus on the simplest case, in which just two countries (England and Portugal) produce and trade in $n$ ($n > 2$) commodities; and let us denote by $a_i$ (respectively, $b_i$) the labour cost of producing a unit of commodity $i$, $i = 1, 2, \ldots, n$, in England (respectively, Portugal). Without loss we may suppose that

$$b_1 / a_1 > b_2 / a_2 > \ldots > b_n / a_n \tag{*}$$

Let $w \equiv w_a / w_b$, where $w_a$ (respectively, $w_b$) is the wage rate in England (respectively, Portugal). For any trade to take place it is necessary that

$$b_1 / a_1 > w > b_n / a_n \tag{**}$$

If (**) is satisfied and if in each country all $n$ commodities are consumed in autarky, then England will alone produce every commodity $i$ such that $b_i / a_i > w$ and Portugal will alone produce every commodity such that $b_i / a_i < w$ . It follows that if $w$ increases towards $b_1 / a_1$, then England will eventually relinquish the production of some commodities to Portugal. However, if in autarky either country fails to consume all $n$ commodities and if the relevant marginal rates of substitution and transformation are sufficiently different, then the order in which England relinquishes the production of commodities will not be as indicated by (*).

In this paper we have focused on a major weakness common to all expositions of the Principle of Comparative Advantage. There is another weakness, which we must at least mention. Beginning with those of Torrens and Ricardo, all expositions of the Principle are based on the assumption, implicit or explicit, that, in each country, households are identical. However, as Kemp and Shimomura (1995) have noted, identical agents who know themselves to be identical would not play a non-cooperative game. They would cooperate; in particular, they would cooperate to manipulate world prices. The two countries, each treated as a single decision-making entity, would not engage in free trade but would engage in a tariff or quota war. The post-war world equilibrium would be distinct from the Torrens-Ricardo free-trade equilibrium. In the post-war equilibrium, at least one country must be worse off than in the Torrens-Ricardo equilibrium; possibly both countries would be worse off. However, both countries would be better off than in autarky. In fact, the post-war equilibrium would share all of the properties (a)–(e) of the Torrens-Ricardo equilibrium. In sections 1.3 and 1.4 we ignored these matters, to allow a clear focus on the destructive implications of specialized autarkic consumption.

# 2 Gottfried Haberler's Principle of Comparative Advantage

## 2.1 Introduction

Breaking away from classical one-factor models of international trade, Gottfried Haberler (1930, especially Section 3) noted the necessity of working henceforth with non-linear production frontiers. He also noted that the relative opportunity costs of producing autarkic equilibrium quantities determine both the direction of free international trade and the manner in which the gains from trade are shared by the trading countries. Thus, in a single article, Haberler freed both descriptive and normative trade theory from more than a century of classical inhibitions. In particular, he transformed the linear Torrens-Ricardo Principle of Comparative Advantage into a more general principle that accommodates non-linear production frontiers.

Like the Torrens-Ricardo Principle, however, Haberler's Principle rests on the implicit assumption that, in autarkic equilibrium, each country produces and consumes all commodities, at least incipiently. Without that assumption, both principles must be substantially qualified.

In a companion paper (Kemp and Okawa 2006), the necessary qualifications have been attached to the Torrens-Ricardo Principle. In the present paper, a similar service is performed for Haberler's Principle. For the most part, we follow Haberler in focusing on just two countries, each potentially producing the same pair of commodities by means of two primary factors of production; however, we do briefly consider the many-commodities case. Neither the primary factors nor the technologies need be the same for each country, but it will be convenient to pretend that the same primary factors are available everywhere.

## 2.2 Analysis

In each of England and Portugal, cloth and wine are non-jointly produced by labour and land under constant returns to scale, with one commodity (not necessarily the same in each country) relatively labour-intensive at all wage:rental ratios. In each country and in each industry, both factors are essential to production. For the time being it will be assumed that, in autarkic equilibrium, production and therefore consumption is completely specialized in each country on a country-specific commodity; for concreteness only, it will be assumed that, in autarkic equilibrium, England produces and consumes cloth only while Portugal produces and consumes wine only. Finally, throughout our analysis, the preferences of each country are those of a price taking representative agent.[1] On the other hand, no special restrictions are placed on the utility functions of the two representative agents; specifically, they are not necessarily homothetic, nor need they rule out inferiority.

In Figure 2.1(a), $Q_E Q'_E$ is the English production frontier and $U_E C_E$ is a single English indifference curve. The English autarkic equilibrium occurs at point $C_E$. At that point, the English marginal rate of transformation might differ from the English marginal rate of substitution ($MRT^A_E$) might differ from the English marginal rate of substitution ($MRS^A_E$). If $MRT^A_E = MRS^A_E$, then the market-clearing price ratio is equal to $MRS^A_E$; otherwise, the equilibrium price ratio can be anywhere in the continuum bounded by $MRT^A_E$ and $MRS^A_E$. We shall refer to the cone defined by ($MRT^A_E$, $MRS^A_E$) as the English *autarkic price cone*, not excluding the extreme case in which $MRT^A_E = MRS^A_E$.

Similarly, the Portuguese autarkic equilibrium is represented in Figure 2.1(b) by point $C_P$, where the Portuguese marginal rate of transformation ($MRT^A_P$)might differ from the Portuguese marginal rate of substitution and where the market-clearing price ratio must lie in the Portuguese *autarkic price cone* ($MRT^A_P$, $MRS^A_P$), not excluding the extreme case in which $MRT^A_P = MRS^A_P$.

Abandoning the assumption of autarky, we may consider for each country all conceivable world price ratios; and for each price ratio we may consider

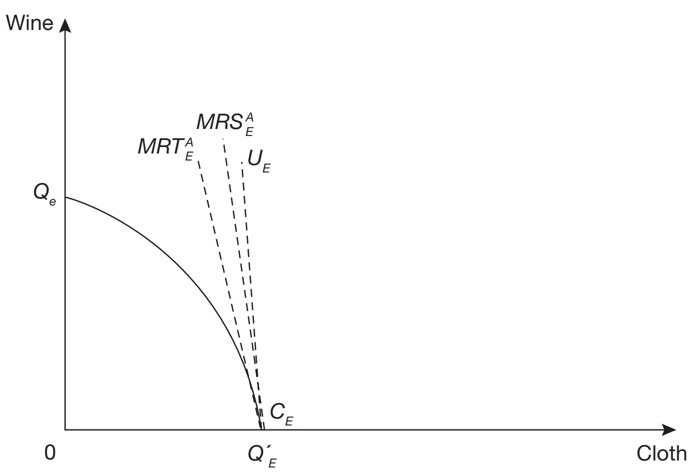


*Figure 2.1(a)* England's autarkic equilibrium, with complete specialization

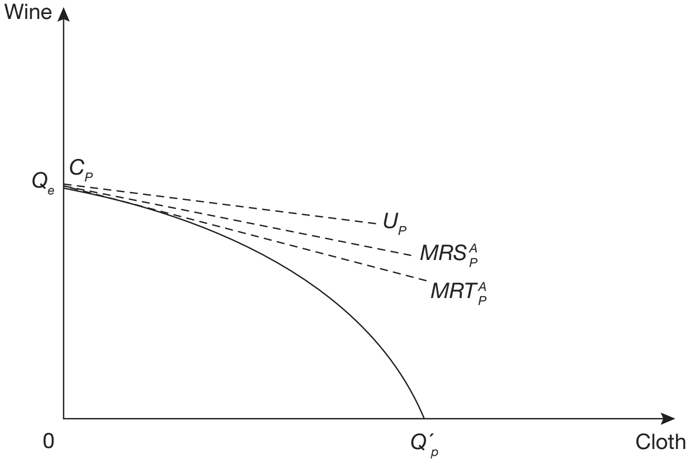


*Figure 2.1(b)* Portugal's autarkic equilibrium, with complete specialization

the profit-maximizing pair of outputs and the utility-maximizing consumption pair of the price taking representative agent. From that information can be derived the English and Portuguese offer curves. It is apparent that, for all hypothetical price ratios in a country's autarkic price cone, production and consumption remain specialized at the autarkic level for that country. If and only if $MRT_j^A \neq MRT_j^A$, country $j$'s offer curve has a kink at the origin as displayed in Figures 2.2(a) and 2.2(b) for $j = E$ and $j = P$, respectively.

We can now move forward to consider the central questions of the paper. Suppose, first, that the two autarkic price cones have no points in common. Rotating the Portuguese offer curve through 180° and then superimposing it on Figure 2.2(a), we obtain Figure 2.3(a) or Figure 2.3(b), depending on the relative positions of the price cones. In each figure the world equilibrium is represented by point $W$, where the world excess demands for cloth and wine are equated to zero by the unique price ratio $Op$. Close scrutiny of the two parts of Figure 2.3 reveals that they differ in an important detail: In Figure 2.3(a), the equilibrium price ratio lies within the close embrace of the two autarkic marginal rates of transformation whereas, in Figure 2.3(b), it lies within the close embrace of the two autarkic marginal rates of substitution. On the other hand, each part of Figure 2.3 brings the same glad tidings: In spite of kinks in their offer curves, each country benefits from free international trade.[2]

Suppose alternatively that the two autarkic price cones intersect. Then, instead of Figure 2.3(a) or 2.3(b), we obtain Figure 2.4(a) or 2.4(b), depending on the relative positions of the two (intersecting) cones. In each case, equilibrium world trade is zero; hence neither country benefits from free trade.

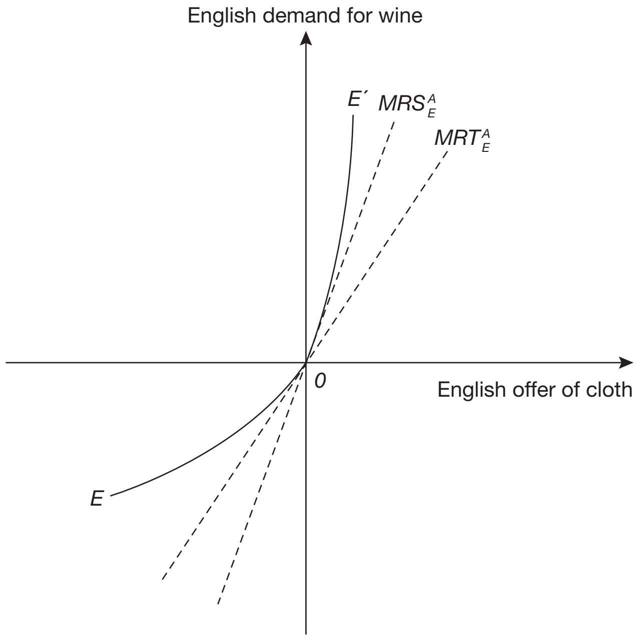


*Figure 2.2(a)* England's offer curve, with complete autarkic specialization

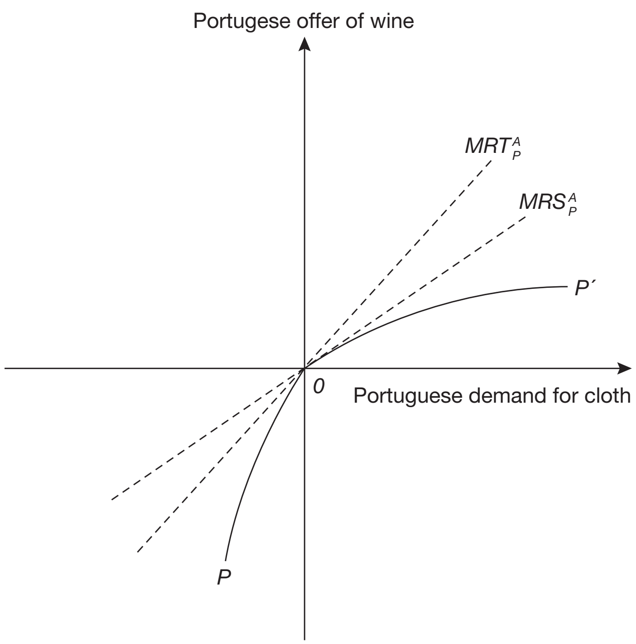


*Figure 2.2(b)* Portugal's offer curve, with complete autarkic specialization

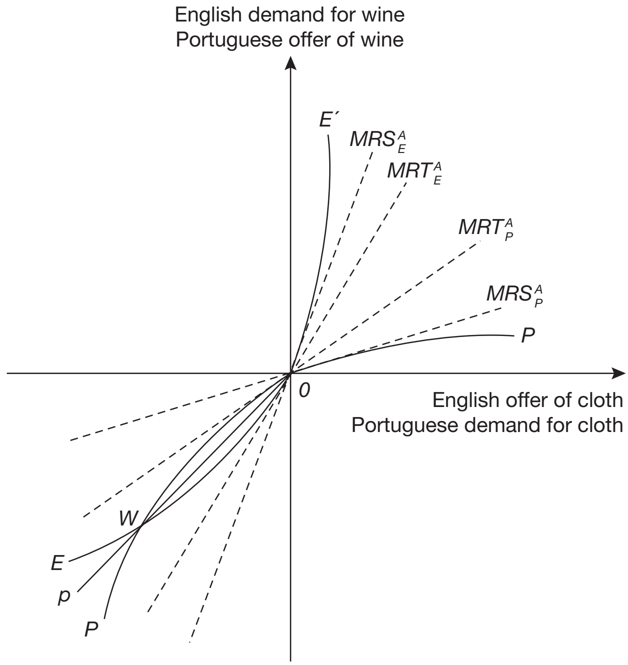


*Figure 2.3(a)* The trading equilibrium, with complete autarkic specialization and non-intersecting price cones, Case (a)

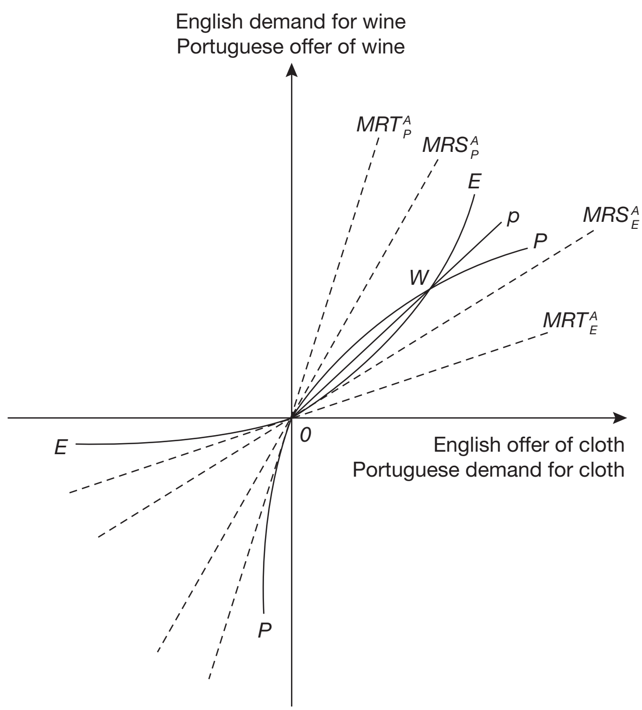


*Figure 2.3(b)* The trading equilibrium, with complete autarkic specialization and non-intersecting price cones, Case (b)

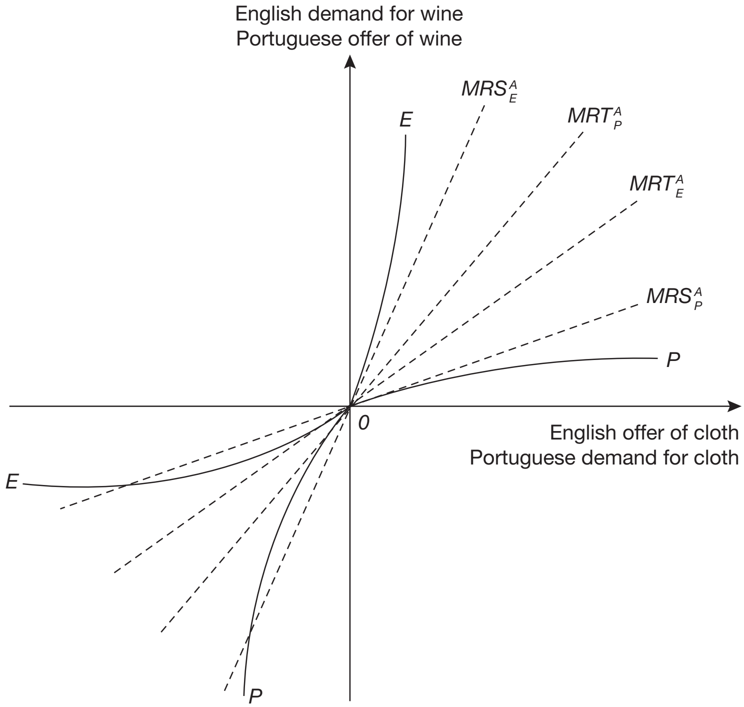


*Figure 2.4(a)* The trading equilibrium, with complete autarkic specialization and intersecting price cones, Case (a)

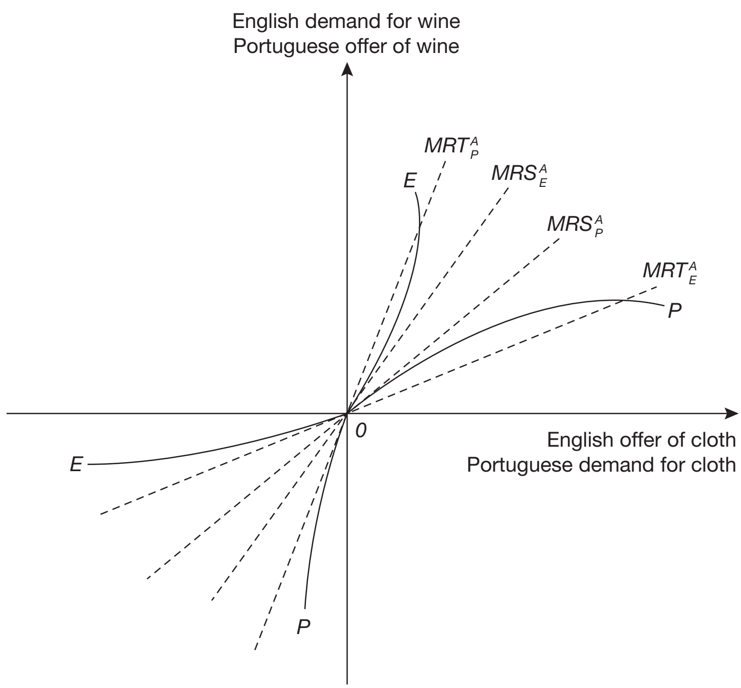


*Figure 2.4(b)* The trading equilibrium, with complete autarkic specialization and intersecting price cones, Case (b)

*Proposition 2.1* Suppose that under autarky each country specializes in the production and consumption of a different commodity. If and only if the two autarkic price cones have no points in common, free trade is beneficial to each country; moreover, each country exports the commodity in the production of which it is relatively more efficient under autarky. If the cones intersect, equilibrium world trade is zero.

In deriving Proposition 2.1 it was convenient to focus on the case in which, under autarky, each country specializes in the production and consumption of a particular country-specific commodity. However, the proposition is valid without that assumption; that is, it is valid even if, under autarky, the two countries specialize in producing and consuming the same commodity and even if, under autarky, only one country specializes. Thus we may confidently conclude that Haberler's Principle of Comparative Advantage survives if and only if the two autarkic price cones are discrete, with no points in common.

In this section we have followed Haberler in focusing on the familiar two-countries, two-commodities case. We now turn our attention to more ample world economies with more than two member countries.

## 2.3 More than two countries

Suppose that England and Portugal are joined by France, each country capable of producing cloth and wine but under autarky completely specializing in the production and consumption of cloth and therefore possessing its own autarkic price cone. Three cases will be considered in detail:

1 No two of the autarkic price cones intersect.
2 Two of the autarkic price cones intersect.
3 Each autarkic price cone intersects at least one of the other cones.

*Case 1* This case is illustrated by Figure 2.5. It is not difficult to see that any equilibrium world price ratio must lie in the cone ($MRT_E^A$, $MRS_F^A$), which we will call the *world price cone*, for any other price ratio would fail to induce a positive net supply of each commodity.

Suppose next that the equilibrium world price ratio lies in the sub-cone ($MRT_E^A$, $MRS_P^A$). At that price England imports cloth from Portugal and France in exchange for wine; there is no trade between Portugal and France. Similarly,

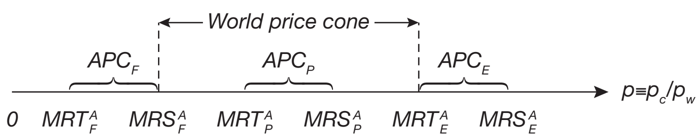


*Figure 2.5* Non-intersecting autarkic price cones

if the equilibrium world price ratio lies in the sub-cone ($MRS_F^A$, $MRT_P^A$), France imports wine from England and Portugal in exchange for its exports of cloth; there is no trade between England and Portugal. In each sub-case all three countries gain from trade.

So far, there are no surprises. If, however, the equilibrium world price ratio lies in the remaining sub-cone ($MRT_P^A$, $MRS_P^A$), the outcome is quite different. For that sub-cone coincides with Portugal's autarkic price cone, implying that, in the world equilibrium, Portugal does not trade. England and France, on the other hand, trade in English wine and French cloth. Thus, although the three autarkic price cones have no points in common, only two countries gain from trade. Evidently our proposition needs modification to accommodate an additional country.

*Case 2* This case is illustrated by Figure 2.6, in which the autarkic price cones of Portugal and France intersect. Any equilibrium world price ratio must lie in the new world price cone ($MRS_F^A$, $MRT_E^A$). If the equilibrium price ratio falls in the sub-cone ($MRS_P^A$, $MRT_E^A$), England exports wine to Portugal and France in exchange for cloth. Portugal and France do not trade with each other. All countries gain from trade. If, on the other hand, the equilibrium world price ratio lies in the sub-cone ($MRS_F^A$, $MRS_P^A$) and therefore in Portugal's autarkic price cone, Portugal does not trade with England or France. The latter countries trade with each other in English wine and French cloth, to the advantage of each country.

Thus in Cases 1 and 2, we encounter essentially the same list of possible outcomes. No new possibilities are created by Case 2's limited intersection of autarkic price cones.

*Case 3* This case is illustrated by Figures 2.7(a) and 2.7(b). In Figure 2.7(a), the English and French autarkic price cones intersect the Portuguese cone but do not intersect each other; in Figure 2.7(b) the three autarkic price cones have a common intersection that coincides with the French cone. In the sub-case depicted in Figure 2.7(a), any equilibrium world price ratio must lie in the world price cone ($MRT_E^A$, $MRS_F^A$); any other price ratio can be ruled out because no two countries would trade on opposite sides of the market. Since any equilibrium price ratio lies in Portugal's autarkic price

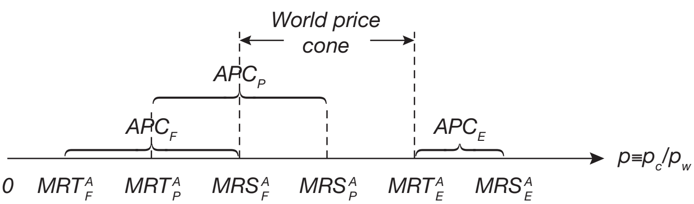


*Figure 2.6* Two autarkic price cones intersect

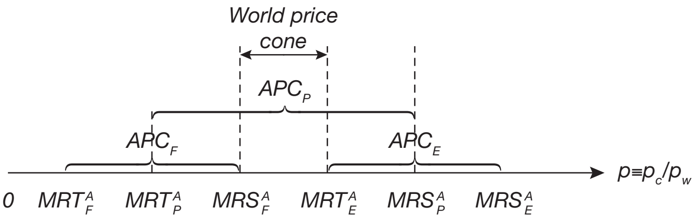


*Figure 2.7(a)* Multiple intersections of autarkic price cone

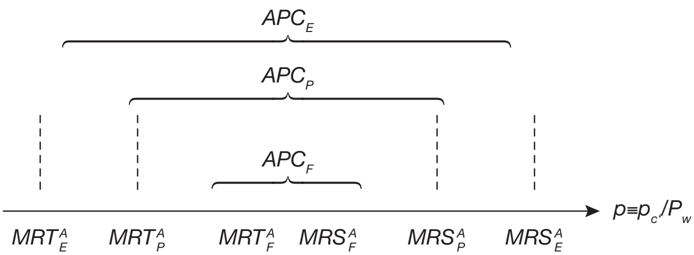


*Figure 2.7(b)* Multiple intersections of autarkic price cones with no world price cone

cone, Portugal does not trade. England exports wine to France in exchange for cloth, to the benefit of each country.

In the sub-case depicted in Figure 2.7(b), on the other hand, there is no possibility of trade. At each imaginable price ratio, either no country wishes to trade or those countries willing to trade are all on the same side of the market.

Summarizing, in a world of three countries with non-intersecting autarkic price cones, the opening of trade might benefit all countries or it might benefit only the 'extreme' countries, that is, those countries with autarkic price cones in terms of the largest and smallest marginal rates of substitution and transformation. This remains true if some but not all of the autarkic price cones intersect. Only if the three autarkic price cones have a common intersection is all trade ruled out, as in our proposition.

In each of our three cases, it has been assumed that under autarky all countries specialize in cloth production. However, that assumption does not rule out mutually profitable trade. In fact, all of our conclusions can be derived without that assumption.

Finally, we note that the two-dimensional figures employed in this section can be readily extended to accommodate more than three countries.

# 3 Trade between countries with radically different preferences

## 3.1 Introduction

In the standard Heckscher-Ohlin model of international trade, two trading countries share a common constant returns technology and common homothetic preferences but differ in their factor endowment ratios, which are therefore seen as determining the pattern of trade. In that model, the equilibrium free-trade commodity price ratio is non-negative and, since the preferences of each country are homothetic, bounded by the two equilibrium autarkic price ratios. These properties persist whether or not production is joint.

In the present paper we construct a model that differs from the standard model in important details. First, the preferences of the trading countries differ radically, in the sense that any commodity that yields satisfaction in one country yields dissatisfaction in the other; that is, any 'good' in one country is a 'bad' in the other country. Such a disparity in preferences might in turn be based on differences in climate and/or religious belief. One thinks of India and Pakistan, beef and pork, and of Singapore and Iceland, refrigeration and space heaters. Second, the two countries share a technology in which the final goods are jointly produced, and that allows each country to dispose of its 'bad' by sacrificing some of its 'good'. (Under free trade the 'bad' can also be disposed of by export.) Finally, there is a single primary factor of production, available in the same amount to each family. Thus, our model is modified Ricardian; there is, therefore, no role for international disparities in relative factor endowments in determining the pattern of trade. Nor is there a role for international disparities in technical information or for international differences in infrastructure. Each of the Ricardian and Heckscher-Ohlin models focuses on one of those determinants; we focus on international disparities in preferences.

It is characteristic of the model that, whether or not preferences are uniformly homothetic in each country, the equilibrium autarkic price ratios are unique and negative, and there is a unique positive equilibrium free-trade price ratio but possibly several negative equilibrium free-trade price ratios. It follows that the positive equilibrium free-trade price ratio is not bounded by the equilibrium autarkic price ratios. This finding contrasts sharply with the familiar Torrens-Ricardo Principle of Comparative Advantage and with Haberler's counterpart. Even when there are multiple free-trade equilibria, with the additional free-trade price ratios negative, it remains true that no equilibrium free-trade price ratio is strictly bounded by the equilibrium autarkic price ratios. Moreover, it turns out that Marshallian local stability analysis, commonly preferred by trade theorists to the alternative Walrasian analysis, is inapplicable at free-trade equilibria with negative prices. On the other hand, the central proposition concerning trade gains remains intact, in the sense that at least one country is better off and no country is worse off under free trade rather than in autarky.

The jointness of production suggests the possibility of undesirable by-products the disposal of which is costly. This in turn suggests the possibility of negative equilibrium prices, at least in a closed economy. What is surprising is that this latter possibility survives in a world economy in which no commodity is everywhere undesirable and in which the national economies are of approximately the same size.

The proposed international differences in preferences are extreme. However, the preferences of each country are restricted only in the minimal Arrow-Debreu manner. Moreover, equilibria of the type displayed in Section 3.3 could be obtained, but not so easily, with less extreme assumptions.

## 3.2 Autarkic equilibria

There are two countries, the home country and the foreign. In each country, two commodities, 1 and 2, are jointly produced by a single primary factor of production, labour, with one unit of labour yielding one unit of commodity 1 and $a$ units of commodity 2. The home country is endowed with $L$ units of labour, the foreign country with $L^*$ units.

In the home country, households view commodity 1 as a good, commodity 2 as a bad; the converse view is held in the foreign country.

Commodities 1 and 2 are disposable, at a cost. To dispose of one unit of commodity $i$, $b_{ij} > 0$ units of commodity $j$ are needed ($i, j = 1, 2$; $i \neq j$). For convenience only, it will be assumed that $b_{12} < a < 1/b_{21}$; all of our qualitative conclusions remain valid under alternative assumptions. Each country chooses the method of disposal appropriate to its preferences. Thus the relevant home production possibility locus is $AB$ in Figure 3.1(a), where the slope of $AB$ is max. $\{b_{12}, 1/b_{21}\}$ and the length of $OA$ is proportional to $L$; and the relevant foreign production possibility locus is $A^*B^*$ in Figure 3.1(b), where the slope of $A^*B^*$ is min. $\{b_{12}, 1/b_{21}\}$ and the length of $O^*A^*$ is proportional to $L^*$.

In Figure 3.1, $II$ is a typical Scitovsky community indifference curve of the home country and $I^*I^*$ is a typical Scitovsky curve of the foreign country. Points $E$ and $E^*$ represent the unique autarkic equilibria of the home and foreign countries, with negative equilibrium price ratios indicated by (minus) the slopes of $AB$ and $A^*B^*$. As indicated by the figure, each country chooses to rid itself of some but not all of its 'bad'.

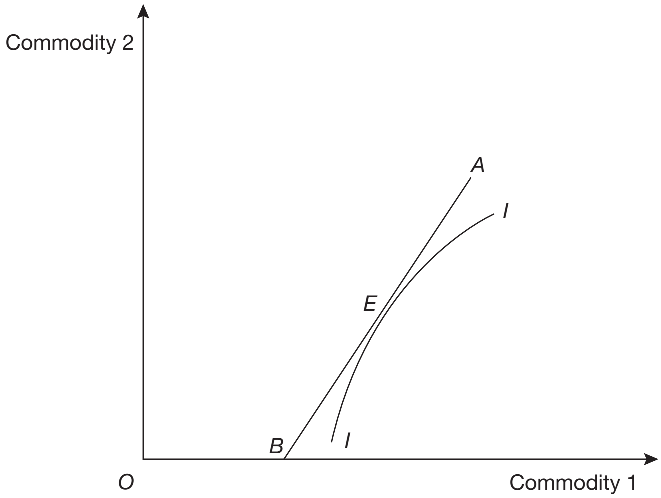


*Figure 3.1(a)*

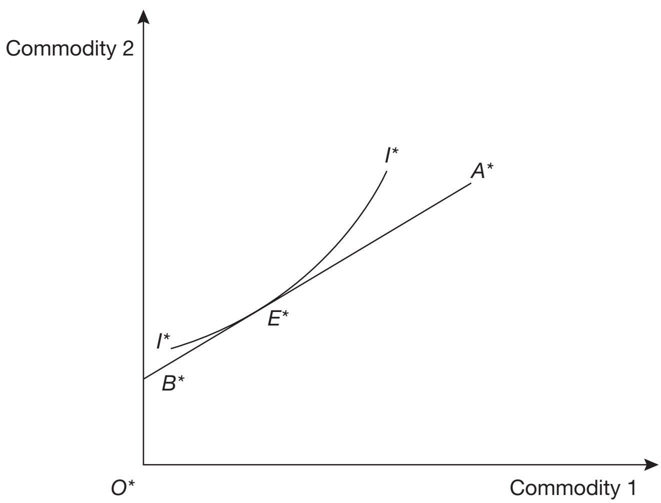


*Figure 3.1(b)*

## 3.3 Free-trade equilibria

Given the assumptions introduced in Section 3.2, we can derive the home and foreign offer curves. These are displayed in Figures 3.2 and 3.3. Spinning Figure 3.3 through 180° and superimposing it on Figure 3.2, we obtain Figure 3.4. In the unique equilibrium marked by *E*, each country imports its

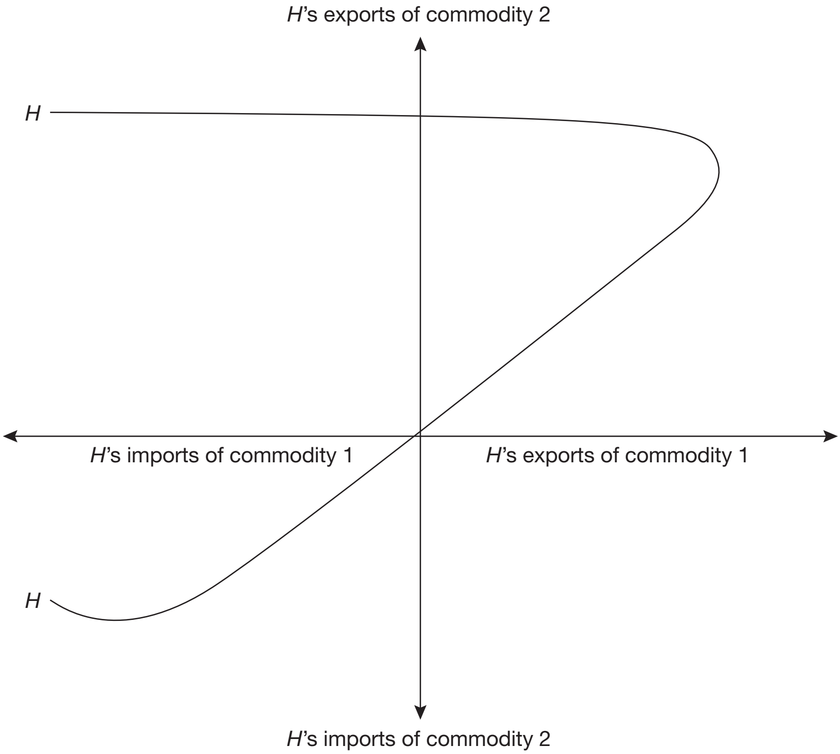


*Figure 3.2*

preferred commodity, implying that the equilibrium terms of trade are positive. Since the autarkic price ratios are negative, they do not bound the world price ratio. Thus the familiar Torrens-Ricardo and Haberler propositions have no counterpart in the present model. Moreover it can be verified that this is so even when preferences are uniformly homothetic in each trading country.

Nevertheless multiple equilibria are possible and can be obtained by varying the details of the common technology and of the country-specific preferences. However, the additional equilibria always appear in the first and/or third quadrants, as in Figure 3.5. In those quadrants, one country exports both commodities, implying that the equilibrium world price ratio is negative. Moreover, this possibility emerges even though both commodities are consumed in the autarkic equilibria. Thus it remains true that there is a unique positive price ratio and that it is not bounded by the two equilibrium autarkic price ratios. In fact, it remains true that no equilibrium free-trade price ratio, positive or negative, is strictly bounded by the equilibrium autarkic price ratios.

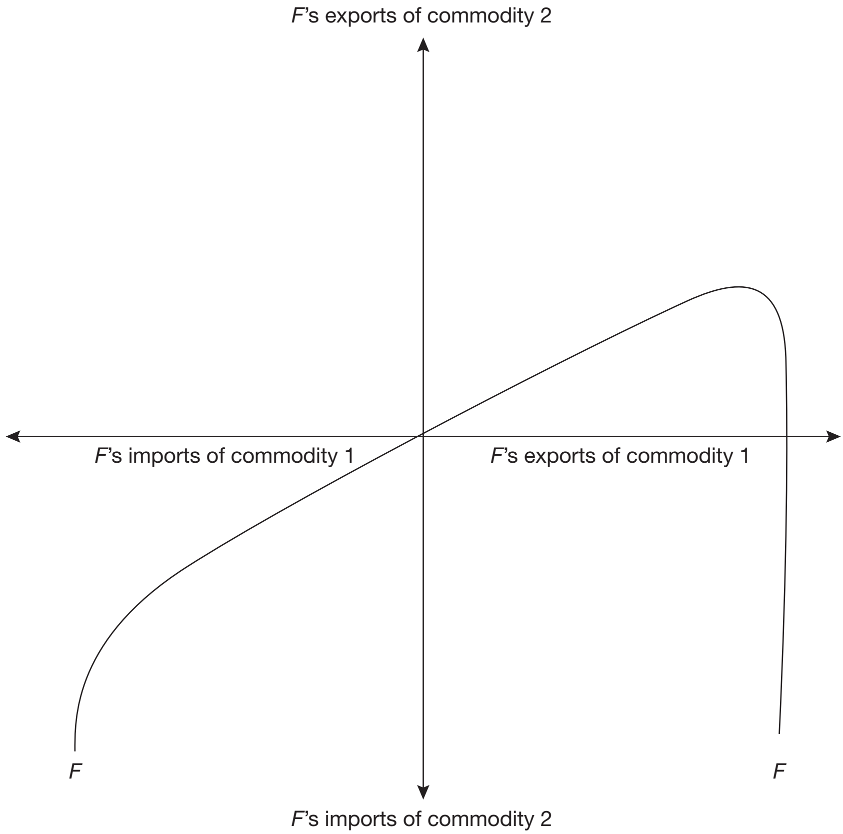


*Figure 3.3*

In spite of the novel features of the world equilibria, each country gains from trade, in the weak sense that it is not harmed. A proof may be based on McKenzie's (1959, 1981) demonstration of the existence of a competitive general equilibrium, extended to accommodate two trading countries and one or more schemes of lump sum compensation. Thus if the free-trade equilibrium is represented by point $E$ in Figure 3.4 or Figure 3.5, then each country is clearly better off than in autarky; and if the free-trade equilibrium is represented by $E_1$, $E_2$, $E_3$ or $E_4$ in Figure 3.5, then at least one country is better off (the other not worse off) than in autarky.

## 3.4 Final remarks

The present paper is companion to an earlier paper (see Kemp and Okawa 2006). In the earlier paper it was shown that the Torrens-Ricardo Principle of Comparative Advantage is dependent on the hidden assumption that in each country all goods are consumed in autarkic equilibrium. In the present paper it has been shown that the Principle depends also on preferences being sufficiently alike across countries.

The analysis has been entirely static. Nothing has been said about the stability or instability of the equilibria; in particular, nothing has been said about the stability of the free-trade equilibria. In conclusion, we seek to make good this oversight. In the simple case depicted in Figure 3.4, there is a single equilibrium and the equilibrium price is positive. It is straightforward to confirm that the equilibrium is locally stable, both in the sense of Walras and in the sense of Marshall. In the case depicted in Figure 3.5, however, matters are more complicated. The unique equilibrium in quadrant 2 is characterized by a positive price ratio and local stability, both in the sense of Walras and in the sense of Marshall. The remaining equilibria, in

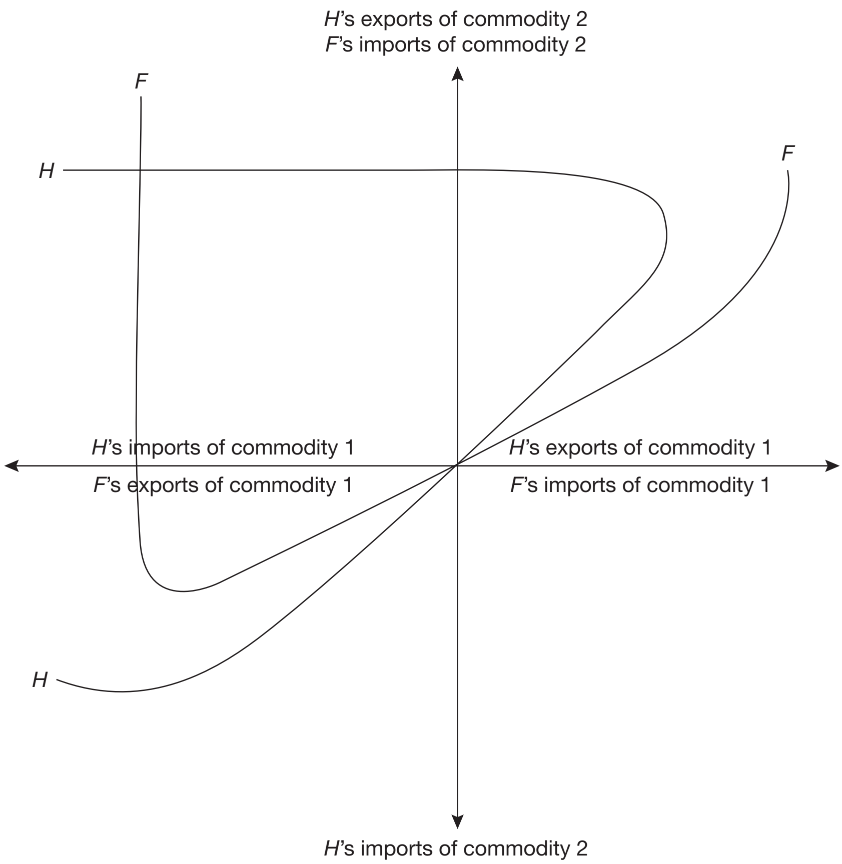


*Figure 3.4*

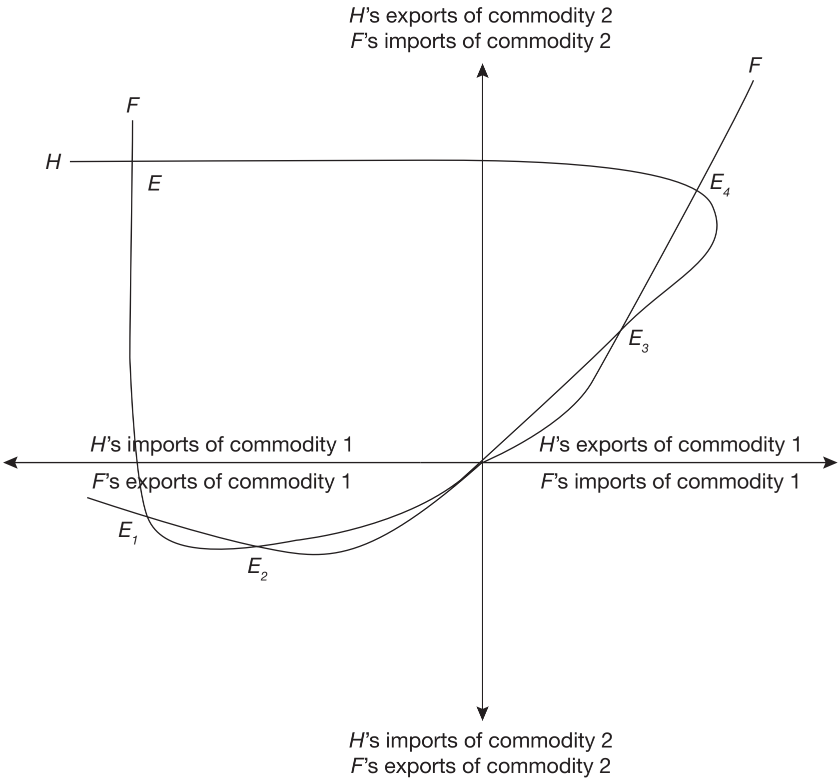


*Figure 3.5*

quadrants 1 and 3, are characterized by a negative price ratio and alternating Walrasian stability and instability, beginning with stability at $E_1$. However, it is not possible to follow the Marshallian approach. For, when both commodities are exported by the same country, there is no single 'supply price' associated with an equilibrium.

# 4 Production and trade patterns under uncertainty

## 4.1 Introduction

What happens to the propositions of trade theory when the traditional assumption of certainty is relaxed? In the present paper we direct this question to the classical or Torrens-Ricardo version of the theory. The classical theory is a most suitable guinea pig since the strong assumptions on which it is based yield implications for trade and production patterns that, compared to those of the more general Heckscher-Ohlin theory, are unambiguous.

Of course, we cannot take over the classical theory just as it is. It must be suitably modified so that uncertainty may be grafted in a non-trivial way. The modifications we have chosen relate to the timing and short-run reversibility of decisions concerning the allocation of labour to competing activities.

Suppose then that:

(i) There are just two trading countries, each composed of identical consuming-producing units that may, however, differ from country to country.
(ii) Each country is capable of producing two tradable commodities. With each commodity there is associated a no-joint-products activity vector in which a single primary factor, labour, and possibly the other commodity appear as inputs. The commitment of labour to each activity must be made one period before production takes place; in the period during which production takes place, labour may be withdrawn from an activity, but it cannot be transferred to the competing activity. In short, *ex ante* labour is mobile between activities, *ex post* it is immobile. However, actual input and output are contemporaneous.
(iii) Each country produces in addition a purely domestic or non-tradable commodity. The output of this commodity is, like Cournot's spring water, exogenously determined, beyond the control of the individual consuming-producing units, and does not absorb labour or either tradable good. The supply of this commodity to consumers is random and affects their choices concerning the two producible commodities. This is how uncertainty enters the model.

(iv) Labour is internationally immobile and in fixed aggregate supply.
(v) Either all product markets are spot, that is, all contracts to buy and sell are contemporaneous with the exchange itself, or all product markets are forward, that is, all contracts are made one period before the exchange takes place.

The assumptions are not quite those of Torrens and Ricardo. In particular, we have departed from the strict letter of classical trade theory in recognizing both purely domestic and intermediate goods and in introducing a lag between the commitment of labour and the associated output of each of the two tradable commodities. Nevertheless, in the absence of uncertainty (when the supply of the non-tradable commodity is constant) our assumptions yield all the familiar classical conclusions.

In particular, the relevant (long-run) production possibility frontiers are straight lines based on free mobility of labour between activities. Moreover, comparative advantage in production in the long run means the same thing and plays the same role as in classical theory.

We shall show, however, that under conditions of uncertainty and imperfect mobility the implications of the classical theory are no longer valid. In particular, long-run comparative advantage in production has little predictive value concerning the pattern of trade or specialization. To dramatize our analysis we shall show that the pattern of trade and specialization under uncertainty may be just the reverse of the pattern of trade and specialization under certainty. This will be shown to be so whether markets are organized on a spot or a futures basis.

From these results we infer that one should not generally expect to be able to explain empirical patterns of trade in terms of some crude notion of comparative advantage, as was the case for example in the controversy concerning the Leontief Paradox. We also infer that planned patterns of trade and specialization cannot properly be based on static considerations alone.[1]

## 4.2 Equilibrium of a single country in isolation: spot markets

We begin by drawing a distinction between short-run and long-run production possibilities. Consider Figure 4.1. If the entire labour force were allocated to the production of the first tradable commodity, the net output of that commodity would be $\bar{X}_1$, and the (negative) net output of the second tradable commodity would be $\underline{X}_2$, where $-\underline{X}_2$ is the amount of the second commodity needed to produce $\bar{X}_1$ of the first commodity. The vector $OA_1$ then represents the first activity normalized on the total labour force. Similarly, if the entire labour force was devoted to the second activity, the net outputs of the two commodities would be represented by the point $(\underline{X}_1, \bar{X}_2)$ and the second activity by the vector $OA_2$. For an open economy, the long-run locus of net production possibilities is the straight-line segment $A_1A_2$; for a closed economy, the long-run locus is that part of $A_1A_2$ cut off by the axes, that is, $B_1B_2$. During any particular time period, however, the allocation of labour between industries is fixed by decisions of the preceding period. Labour cannot be immediately transferred from one activity to another; at most it can be withdrawn from an activity and allowed to stand idle, thus ensuring some saving of the intermediate or produced input. Suppose that one-half of the labour force has been committed to each activity, so that the long-run production point is $P^0$. Then, for an open economy, the locus of short-run production possibilities is $E_1P^0E_2$, where $E_1P^0$ is parallel to $OA_2$ and $E_2P^0$ is parallel to $OA_1$; for a closed economy, the locus is $D_1P^0D_2$. Of course, there is a different locus of short-run production possibilities for each allocation of labour, that is, for each choice of $P^0$ on $A_1A_2$.

We consider now the equilibrium of a single country in isolation, say country $A$. Since all consuming-producing units in $A$ are alike, we may introduce a single utility function $U(C_1,C_2;\ C_3)$ and interpret $C_i$ either as aggregate consumption of the $i$th commodity or as a constant multiple of consumption by the typical unit. The function $U$ is assumed to be not separable with respect to $C_3$. Under conditions of technological certainty we may set $C_3 = X_3^0$, where $X_3^0$ is the exogenously-given constant output of the purely

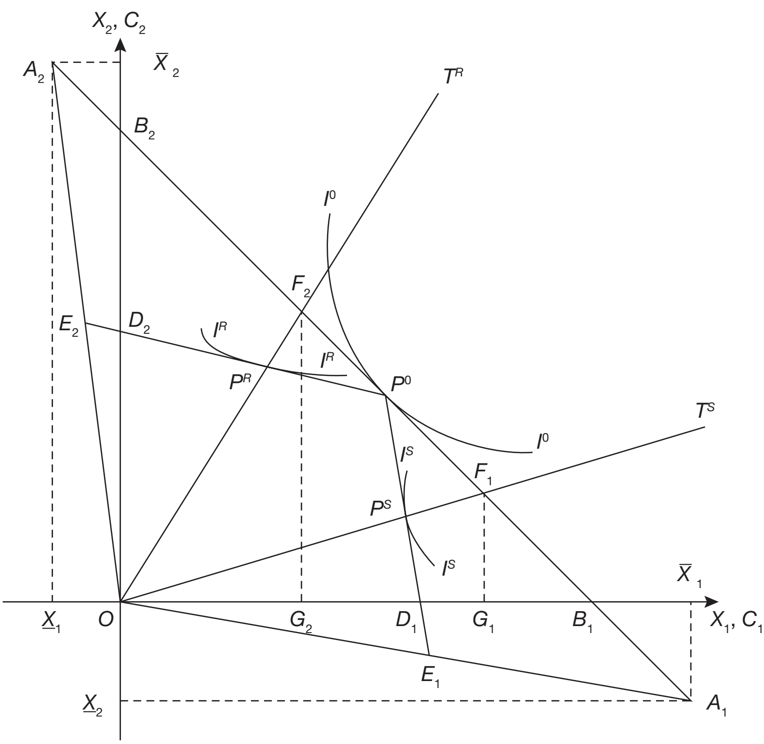


*Figure 4.1*

domestic good, and thus obtain a 'partial' utility function of the amounts consumed of the two tradable commodities, that is, $U(C_1, C_2; X_3^0)$. Then, superimposing a family of indifference curves on Figure 4.1, we find that the community reaches a production-consumption equilibrium at $P^0$, where the locus of long-run production possibilities $B_1B_2$ forms a tangent to the indifference curve $I^0I^0$. The loci of short-run production possibilities play no role in the determination of an uncertainty-free equilibrium.

In order to maintain comparability of the certainty and uncertainty models, we introduce uncertainty in such a way as to leave the production technology of the two tradable commodities unaffected. For this purpose we suppose that the exogenously given supply of the third commodity is random, taking the values $X_3^R$ and $X_3^S$ ($R$ and $S$ for 'Rain' and 'Shine') with probabilities $W^R$ and $W^S$ $(= 1 - W^R)$, where $0 < W^R < 1$. Utility may then be written as $U(C_1, C_2; C_3^j)$ where $C_3^j = X_3^j$, and $j = R, S$. If $X_3 = X_3^R$, we have a partial indifference map consisting of curves such as $I^RI^R$ in Figure 4.1 while, if $X_3 = X_3^S$, the entire indifference map changes and we have curves such as $I^SI^S$.

Given the choice of $P^0$ by the economic units, the locus of short-run production possibilities in the following period is $D_1P^0D_2$ (Figure 4.1). If, in the following period, $X_3 = X_3^R$, the equilibrium production-consumption point is $P^R$ on $D_2P^0$ and $p^R$, the equilibrium price of the first commodity in terms of the second is proportional to the slope of $D_2P^0$. If, alternatively, $X_3 = X_3^S$, the equilibrium production-consumption point is $P^S$ on $D_1P^0$, and the equilibrium price $p^S$ is indicated by the slope of $D_1P^0$. Let $OT^R$ be the Engel curve or expansion path defined by $p^R$ and $C_3 = X_3^R$; and let $OT^S$ be the Engel curve defined by $p^S$ and $C_3 = X_3^S$. (For simplicity, the Engel curves are taken to be linear.) The locus of long-run production possibilities $A_1A_2$ intersects $OT^R$ and $OT^S$ at $F_2$ and $F_1$, respectively. Then it is clear that, for any $P^0$ between $F_1$ and $F_2$, the equilibrium price ratio is $p^R$ ($p^S$) with probability $W^R(W^S = 1 - W^R)$. Henceforth we shall focus on $P^0$ within those bounds; in constructing an example of trade reversal, we shall work with values of our parameters that ensure that $P^0$ satisfies that restriction.

How then should economic units choose $P^0$? We suppose that each economic unit seeks to maximize its expected utility. Let $U^*(p^j, P^0)$ be the maximum utility for given $P^0$ and $p^j, j = R, S$. Then the problem facing the typical economic unit is to find

$$\max_{P^0} \{W^R U^*(p^R, P^0) + W^S U^*(p^S, P^0)\}. \tag{1}$$

By a suitable choice of $U$ we can ensure that a solution value of $P^0$ lies in the interior of $F_1F_2$. We then have: a stationary equilibrium characterized by the clearance of all markets; a stable and known probability distribution of prices, with the probability of $p^R$ being $W^R$ and the probability of $p^S$ being $W^S$; the maximization by each economic unit of its utility subject to the prevailing price ratio and to the locus of short-run production possibilities defined by the solution to (1). We note that, for a single economy containing identical economic units, the 'spot' equilibrium just described is the same as the Arrow-Debreu 'futures' equilibrium, and that both equilibria are Pareto-optimal.

## 4.3 Trading equilibrium: spot markets

Let us now introduce a second country, say *B,* and the possibility of free trade in the first and second commodities. To keep the argument as simple as possible, we suppose that country *B* is small in relation to *A,* in the sense that *B* could sell any part of its maximum output of either commodity at the *A*-price ratio $p^R$ or $p^S$. The technology of country *B* is displayed in Figure 4.2. Denoting by $s(P_iP_j)$ the absolute value of the slope of any straight-line segment, referred to the horizontal axis, we assume that

$$s(\hat{E}_2\hat{P}^0) < p^R = s(E_2P^0) < s(A_1A_2) < s(\hat{A}_1\hat{A}_2) < s(E_1P^0) \\ = p^s < s(\hat{E}_1\hat{P}^0). \qquad (2)$$

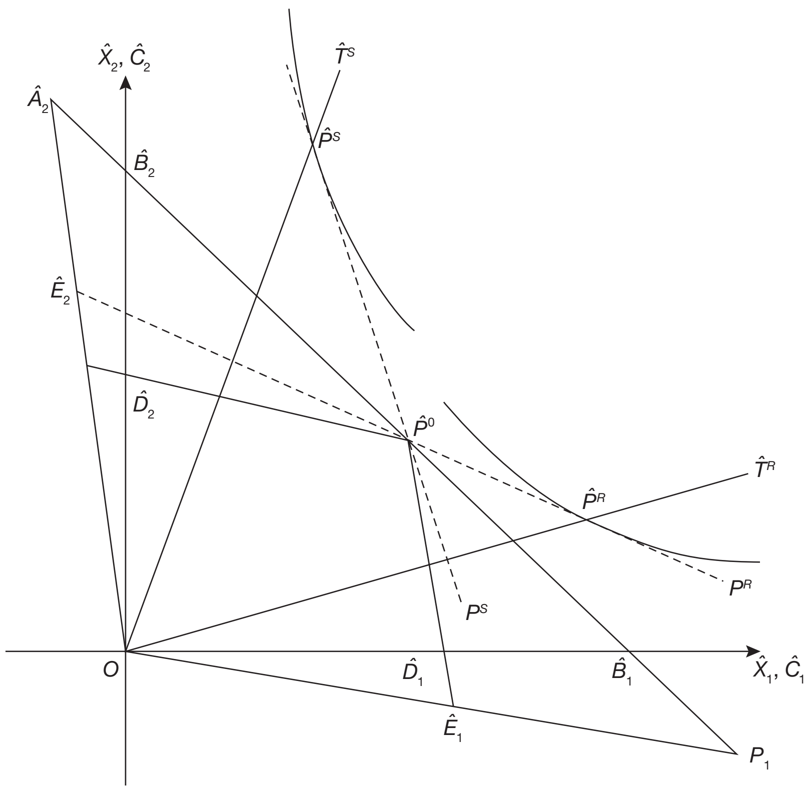


*Figure 4.2*

(A circumflex distinguishes the quantities relating to Country *B*.) It follows that the price ratio facing country *B* fluctuates above and below the level indicated by $s(\hat{A}_1\hat{A}_2)$

As a further simplification, we suppose that in country *B* there is no technological uncertainty at all. (Of course *B,* a price taker, must still cope with uncertainty concerning prices.) The partial utility function of *B* is then $\hat{U}(\hat{C}_1, \hat{C}_2; \hat{X}_3^0) = \hat{U}(\hat{C}; \hat{X}_3^0)$, with $\hat{X}_3^0$ a constant. If $\hat{E}_1\hat{P}^0\hat{E}_2$ is the locus of short-run production possibilities in country *B* then at prices $p^R$ and $p^S$ the equilibrium consumption vectors are indicated by points $\hat{P}^R$ and $\hat{P}^S$, respectively. The equilibrium points change if the commitment of labour changes. In the long-run equilibrium the small country chooses its $P^0$ to maximize expected utility at the $p^j$ price ratios determined in the large country. (It may be noted that for the small country this stationary equilibrium with trade is different from the one that would prevail in an Arrow-Debreu model of contingent markets.)

In the case of certainty we know that the relevant production slopes are $s(A_1A_2)$ and $s(\hat{A}_1\hat{A}_2)$ and that if the latter is larger then country *B* will specialize in producing the second commodity. The same will be true under uncertainty if both $p^R$ and $p^S$ are smaller than $s(\hat{A}_1\hat{A}_2)$. However, if $p^j$ fluctuates above and below $s(\hat{A}_1\hat{A}_2)$ then country *B* will ordinarily diversify its production between the two industries. Moreover its production need not be dominated by its 'comparative advantage' in the second commodity. To take an extreme case, it is even possible that country *B* will completely specialize in the first commodity (in which it has a comparative disadvantage under certainty).[2] That outcome will be referred to as *complete trade reversal.*

## 4.4 An example

In the present section we develop a numerical example of complete trade reversal. Let the utility function of country *A* be

$$U(C_1,C_2;C_3) = \log C_1 + \delta(C_3)\log C_2 \tag{3}$$

$$\delta(C_3) > 0, \quad \frac{d}{dC_3}\delta(C_3) > 0, \quad \frac{d^2}{dC_3^2}\delta(C_3) < 0 \tag{4}$$

so that *U* is strictly concave, displaying relative risk aversion in all of its arguments. The equation of the locus of long-run production possibilities is, say,

$$X_2^* = \alpha - \beta X_1^* \tag{5}$$

and the budget constraint is

$$\begin{aligned} C_2^R + p^R C_1^R &= X_2^* + p^R X_1^* \text{ when } X_3 = X_3^R \\ C_2^S + p^S C_1^S &= X_2^* + p^S X_1^* \text{ when } X_3 = X_3^S \end{aligned} \tag{6}$$

The demand functions derived from (3), (5) and (6) are

$$C_1^R = \frac{\alpha + (p^R - \beta)X_1^*}{p^R[1+\delta(X_3^R)]},\; C_2^R = \delta(X_3^R)p^R C_1^R \tag{7a}$$

and

$$C_1^S = \frac{\alpha + (p^S - \beta)X_1^*}{p^S[1+\delta(X_3^S)]},\; C_2^S = \delta(X_3^S)p^S C_1^S. \tag{7b}$$

Substituting from (7) into (3) and taking account of (5), we obtain

$$E\{U\} = \sum_{j=R,S} W^j \left[ \left(1+\delta^j\right) \log \frac{\alpha + \left(p^j - \beta\right) X_1^*}{p^j \left(1+\delta^j\right)} + \delta^j \log\left(\delta^j p^j\right) \right] \tag{8}$$

$$\equiv V(X_1^*)$$

Hence

$$\frac{d}{dX_1^*} V(X_1^*) = \sum_j \left[ \frac{W^j (1+\delta^j)(p^j - \beta)}{\alpha + (p^j - \beta) X_1^*} \right] \tag{9}$$

and it can be verified that $d^2V/dX_1^{*2} < 0$. At an interior maximum, expression (9) is equal to zero; hence

$$\text{opt}\, X_1^* = -\frac{\alpha \sum_j W^j (1+\delta^j)(p^j - \beta)}{(p^R - \beta)(p^S - \beta)\sum_j W^j (1+\delta^j)}. \tag{10}$$

We assume, as in Figure 4.1, that $p^R < \beta < p^S$; hence the denominator of (10) is negative. We also assume, as in Figure 4.1, that $p^R$ and $p^S$ correspond to the slopes of the locus of short-run production possibilities and that the slopes of $OT^R$ and $OT^S$ are given, by (7), as $\delta^R p^R$ and $\delta^S p^S$, respectively. To ensure the existence of a feasible solution for the economy as a whole, we require that

$$\delta^S p^S < \frac{X_2^*}{X_1^*} < \delta^R p^R. \tag{11}$$

Applying (5) and (10) to (11), we obtain

$$\delta^S p^S < \frac{W^R(1+\delta^R)(p^R-\beta)(-p^S)+W^S(1+\delta^S)(p^S-\beta)(-p^R)}{W^R(1+\delta^R)(p^R-\beta)+W^S(1+\delta^S)(p^S-\beta)} < \delta^R p^R. \tag{12}$$

Suppose now that

$$W^S = 1 - W^R = 0.83,\ \delta^R = 6,\ \delta^S = 1,\ p^S = 3,\ \beta = 2. \tag{13}$$

It may be verified that these values are consistent with (12), yielding $\delta^S p^S = 3 < 4.06 < 6 = \delta^R p^R$. They therefore yield an internal equilibrium of country *A* between the two expansion paths.

Let us turn to the small country *B*. The utility function of *B* is assumed to be

$$U(C_1, C_2; C_3) = \log C_1 + \delta \log C_2. \tag{14}$$

For B, $\delta^R = \delta^S = \delta$; hence the counterpart to (10) is

$$\text{opt}\, X_1^* = -\frac{\hat{\alpha}\sum_j W^j(p^j-\hat{\beta})}{(p^R-\hat{\beta})(p^S-\hat{\beta})} = -\frac{\hat{\alpha}(\bar{p}-\hat{\beta})}{(p^R-\hat{\beta})(p^S-\hat{\beta})} \tag{15}$$

where $\bar{p} \equiv \sum_j W^j p^j$. As in Figures 4.1 and 4.2, we require that

$$p^R < \beta < \hat{\beta} < p^S. \tag{16}$$

Suppose further that neither productive process involves intermediate inputs, so that the locus of long-run production possibilities in country *B* is confined to the non-negative quadrant and the maximal value of $X_1^*$ is

$$\hat{X}_{1m}^* = \frac{\hat{\alpha}}{\hat{\beta}}. \tag{17}$$

For a corner solution at $\hat{X}_{1m}^*$ it is necessary and sufficient that opt $\hat{X}_1^* \geq \hat{X}_{1m}^*$ or, in view of (15) and (17), that

$$-\frac{\hat{\beta}(\bar{p}-\hat{\beta})}{(p^R-\hat{\beta})(p^S-\hat{\beta})} \geq 1. \tag{18}$$

From (13) we calculate that $\bar{p} = 2.66$. Suppose now that $\hat{\beta} = 2.1$. The left-hand side of (18) is then 1.18 and the inequality is satisfied. Since $p^R = 1$

and $\beta = 2$, $\hat{\beta} = 2.1$ also satisfies (16). Hence country $B$ specializes completely in the production of the first commodity, in spite of the fact that $\hat{\beta} > \beta$.

## 4.5 Alternative model incorporating futures markets

The foregoing analysis rested on the assumption that, while the allocation of labour is determined under conditions of uncertainty, trade itself is conducted *ex post* or spot, under conditions of complete certainty. In the present section we swing to the other extreme and assume that all contracts to exchange the two traded commodities are entered into *ex ante* and involve the Arrow-Debreu type of contingent claims on future goods.[3] In our earlier model we allowed trade within given states of nature but not across states of nature. We now introduce the latter possibility by defining commodities 1 and 2 in terms of their physical characteristics, location and state of nature and by assuming that all contracts to buy and sell are entered into before the actual production of the commodity. The third commodity is now supposed to be not subject to exchange.

The assumptions of Arrow and Debreu are, of course, unrealistic. Nevertheless they are attractive because they are simple and because, from a welfare point of view, they represent an idealization of the real world. Indeed, since the risk model of Arrow and Debreu preserves so many features of the model of risk-free competitive equilibrium, one might suppose that it is incompatible with trade reversal of the type discussed earlier.

We begin by studying $A$ (the large country) in isolation. For simplicity we consider the extreme case in which neither commodity 1 nor commodity 2 is needed as an intermediate input in the production of the other commodity. (This assumption can be relaxed.) Then the locus of short-run production possibilities is rectangular, as in Figure 4.3. We denote by $p_i^R$ and $p_i^S$ the prices contracted now to be paid next period for the delivery next period of a unit of the $i$th commodity in the two alternative states of nature. Then $p_i^R + p_i^S$ is the price to be paid for the certain delivery of a unit of the $i$th commodity. Every economic unit is endowed with the same utility function $V(C^R, C^S, C_3^R, C_3^S)$, which, we suppose, takes the special von Neumann form

$$V = W^R U(C^R; C_3^R) + W^S U(C^S; C_3^S). \tag{19}$$

($C^j$ denotes the vector $[C_1^j, C_2^j]$.) If today the economic unit sells $X_i^*$ on the futures market, it receives tomorrow an income of

$$I = \sum_{i=1}^{2} (p_i^R + p_i^S) X_i^* \tag{20}$$

where $X_2^* = \alpha - \beta X_1^*$ along the locus of long-run production possibilities ($A_1 A_2$ in Figure 4.3). We consider now the problem of maximizing $V$ with respect to $C^R$ and $C^S$ for given $X_1^*$ and, to this end, introduce the Lagrangean

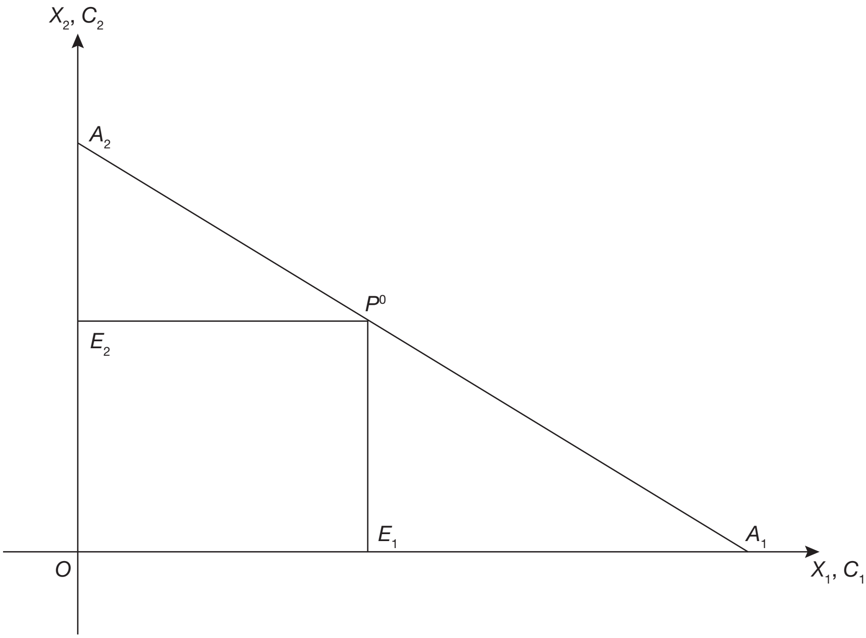


*Figure 4.3*

$$L = W^R U(C^R; X_3^R) + W^S U(C^S; X_3^S) - \lambda \left[ \sum_{i=1}^{2} (p_i^R C_i^R + p_i^S C_i^S) - I \right]. \tag{21}$$

The first order conditions for an interior maximum are

$$\frac{U_1^R}{U_2^R} = \frac{p_1^R}{p_2^R} \cdot \frac{U_1^S}{U_2^S} = \frac{p_1^S}{p_2^S} \cdot \frac{U_1^R}{U_1^S} = \frac{p_1^R}{p_1^S} \cdot \frac{W^S}{W^R} \tag{22}$$

where $U_1^R \equiv \partial U(C_1^R, C_2^R; X_3^R)/\partial C_1^R$, etc. Differentiating the optimal value of $L$ with respect to $X_1^*$ and applying the appropriate envelope theorem, we obtain

$$\frac{\partial}{\partial X_1^*}(\text{opt } L) = \lambda \frac{\partial I}{\partial X_1^*} = \lambda[(p_1^R + p_1^S) - \beta(p_2^R + p_2^S)]. \tag{23}$$

Thus for an interior solution it is necessary that the prices satisfy $\partial I/\partial X_1^* = 0$ or

$$\beta = \frac{p_1^R + p_1^S}{p_2^R + p_2^S} \tag{24}$$

a condition with an obvious counterpart under certainty. Finally, the equilibrium of the economy is determined by (22), (24) and the market clearing conditions

$$C_i^R = C_i^S = C_i^* \qquad i = 1, 2. \tag{25}$$

Including (5), we have altogether nine equations and nine variables: $C_i^R$ and $C_i^S$ (four variables), $X_i^*$ (two variables) and three price ratios. (In sections 4.2 and 4.3, because of the separation of the states of nature, it was possible to choose two separate numeraires; here we may choose only one.)

Let us now re-introduce country $B$. Since $B$ is small in relation to $A$, all prices are determined in the manner just described. For $B$ we retain our earlier assumption that each commodity is needed in the production of the other. Thus $\hat{a}_{ij}$ is the amount of good $i$ required per unit of good $j$. The loci of short-run production possibilities are typified therefore by $\hat{E}_1\hat{P}^0\hat{E}_2$ in Figure 4.4, where $s(\hat{E}_2\hat{P}^0) = \hat{a}_{21}$ and $s(\hat{E}_1\hat{P}^0) = \frac{1}{\hat{a}_{12}}$. The figure is drawn on the further assumption that

$$0 < \frac{p_1^R}{p_2^R} < \hat{a}_{21} < \frac{1}{\hat{a}_{12}} < \frac{p_1^S}{p_2^S} < \infty. \tag{26}$$

Let the equation of the locus of long-run production possibilities be

$$\hat{X}_2^* = \hat{a} - \hat{\beta} X_1^* \tag{27}$$

and denote by $\hat{X}_{2m}^R$ the value of $\hat{X}_2$ at $\hat{E}_2$ and by $\hat{X}_{1m}^S$ the value of $\hat{X}_1$ at $\hat{E}_1$. It can be calculated that

$$\hat{X}_{1m}^R = \frac{\hat{a}_{12}}{1 - \hat{a}_{12}\hat{a}_{21}}[-\hat{\alpha} + (\hat{\beta} - \hat{\alpha}_{21})\hat{X}_1^*]$$

$$\hat{X}_{2m}^R = \frac{1}{1 - \hat{a}_{12}\hat{a}_{21}}[\hat{\alpha} - (\hat{\beta} - \hat{a}_{21})\hat{X}_1^*]$$

$$\hat{X}_{1m}^S = \frac{\hat{a}_{12}}{1 - \hat{a}_{12}\hat{a}_{21}}\left[\hat{\alpha} - \left(\hat{\beta} - \frac{1}{\hat{a}_{21}}\right)\hat{X}_1^*\right]$$

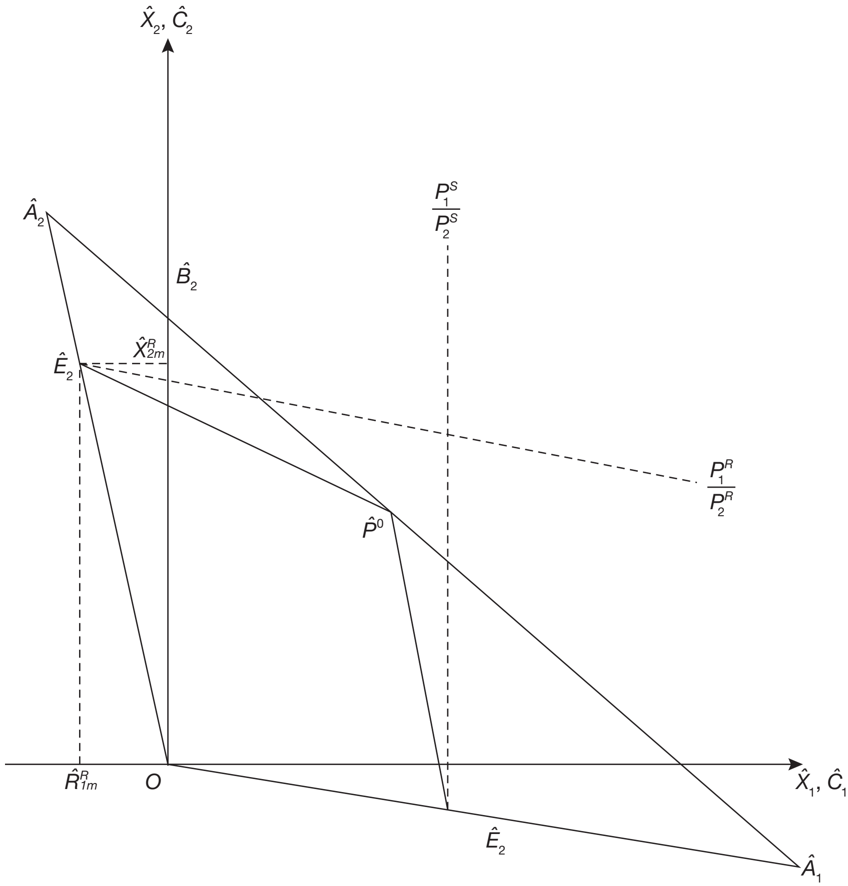


*Figure 4.4*

$$\hat{X}_{2m}^S = \frac{\hat{a}_{12}\hat{a}_{21}}{1-\hat{a}_{12}\hat{a}_{21}}\left[-\hat{\alpha}+\left(\hat{\beta}-\frac{1}{\hat{a}_{12}}\right)\hat{X}_1^*\right]. \tag{28}$$

The income of country $B$ is

$$\hat{I} = \sum_{i=1}^{2}\left(p_i^R \hat{X}_i^R + p_i^S \hat{X}_i^S\right). \tag{29}$$

(The short-run transformation function containing $\hat{X}_i^R$ and $\hat{X}_i^S$ depends, of course, on $\hat{X}_i^*$.)

Suppose now that the utility function of $B$ is of the same general form as that of $A$, that is, $\hat{V}(\hat{C}^R, \hat{C}^S, \hat{C}_3^R, \hat{C}_3^S)$, and let us maximize $\hat{V}^R$ subject to given

$\hat{X}_i^*$. This can be done in stages: First, $\hat{I}$ is maximized; then $\hat{V}$ is maximized, given $\hat{I}$. From the first stage we obtain $\hat{I}(\hat{X}_1^*)$, and from the second $\hat{V}(\hat{I})$. Thus

$$\frac{\partial \hat{V}}{\partial \hat{X}_1^*} = \frac{d\hat{V}(\hat{I})}{d\hat{I}} \cdot \frac{d\hat{I}(\hat{X}_1^*)}{d\hat{X}_1^*}. \tag{30}$$

Now $d\hat{V}(\hat{I})/d\hat{I}$ is obviously positive; hence the economic unit will continue to increase (decrease) $\hat{X}_i^*$ as long as $d\hat{I}(\hat{X}_i^*)/d\hat{X}_i^*$ is positive (negative). We therefore may concentrate on the second stage of maximization.

We note that the maximization of $\hat{I}$ with respect to $\hat{X}_i^R$ and $\hat{X}_i^S$, given $\hat{X}_1^*$, involves two separable constraints, which may be written in general form as

$$\begin{aligned} \Phi(\hat{X}^R; \hat{X}_1^*) &= 0 \\ \Phi(\hat{X}^S; \hat{X}_1^*) &= 0. \end{aligned} \tag{31}$$

It follows that if $\hat{I}$ is maximized with respect to $\hat{X}_i^R$ and $\hat{X}_i^S$, then $\sum p_i^R \hat{X}_i^R$ must be maximized subject to $\Phi(\hat{X}^R; \hat{X}_1^*) = 0$ and $\sum p_i^S \hat{X}_i^S$ must be maximized subject to $\Phi(\hat{X}^S; \hat{X}_1^*) = 0$. Thus, given $\hat{X}_1^*$, we may independently maximize the income components associated with each of the two states of the world. From Figure 4.4 it is clear that, in the first state, the optimal production point is $\hat{E}_2$ and that, in the second state, the optimal point is $\hat{E}_1$. The net amounts produced are given by (28) above. Substituting from (28) into the expression for income, we obtain

$$\begin{aligned} \max_{\hat{X}_i^R, \hat{X}_i^S} \hat{I} &= \text{const.} + \frac{1}{1-\hat{a}_{12}\hat{a}_{21}}\left[\left(\hat{\beta}-\hat{a}_{21}\right)\left(p_1^R\hat{a}_{12}-p_2^R\right)+\left(\hat{\beta}-\frac{1}{\hat{a}_{12}}\right)\left(p_2^S\hat{a}_{21}-p_1^S\right)\hat{a}_{12}\right]\hat{X}_1^* \\ &= \text{const.} + k\hat{X}_1^* \end{aligned} \tag{32}$$

say, where, from the construction of Figure 4.4,

$$\hat{\beta}-\hat{a}_{21} > 0,\ p_1^R\hat{a}_{12}-p_2^R < 0,\ \hat{\beta}-\frac{1}{\hat{a}_{12}} < 0,$$
$$p_2^S\hat{a}_{21}-p_1^S < 0 \text{ and } 1-\hat{a}_{12}\hat{a}_{21} > 0.$$

We note that, since $k$ is constant, the small country must always specialize completely, as in the classical model (but in contrast to the conclusions of Section 4.3). We note also that the sign of $k$ depends not only on $\hat{\beta}$ and the prices but also on the parameters $\hat{a}_{ij}$ of the locus of *short-run* production possibilities. Thus the direction of specialization in $B$ depends partly on the ease of short-run adjustment in that country and is not simply dependent on the relative magnitudes of $\beta$ and $\hat{\beta}$. It follows that an example of trade reversal can be constructed.

We now provide such an example. Suppose that $\hat{\beta} > \beta$, so that, under certainty, country *B* specializes in the production of the second commodity. Suppose further that there exists in country *A* an interior equilibrium, so that $\beta = (p_1^R + p_1^S)/(p_2^R + p_2^S)$. Then

$$\hat{\beta} > \frac{p_1^R + p_1^S}{p_2^R + p_2^S}. \tag{33}$$

In addition, from Figure 4.4,

$$\frac{p_1^R}{p_2^R} < \hat{a}_{21} < \hat{\beta} < \frac{1}{\hat{a}_{12}} < \frac{p_1^S}{p_2^S}. \tag{34}$$

Now if country *B* is to specialize in the production of the *first* commodity then $k$ must be positive, that is,

$$\left(\hat{\beta} - \hat{a}_{21}\right)\left(p_1^R \hat{a}_{12} - p_2^R\right) + \left(\hat{\beta} - \frac{1}{\hat{a}_{12}}\right)\left(p_2^S \hat{a}_{21} - p_1^S\right)\hat{a}_{12} > 0. \tag{35}$$

Thus the problem of constructing an example of trade reversal reduces to that of finding positive values of $\hat{\beta}$, $\hat{a}_{ij}$, $p_i^R$, and $p_i^S$ that satisfy (33), (34) and (35). The following values meet that requirement:

$$p_1^R = 1.9, p_2^R = 1, \hat{a}_{21} = 3.5, \hat{\beta} = 4, \frac{1}{\hat{a}_{12}} = 5, p_1^S = 6, p_2^S = 1. \tag{36}$$

## 4.6 Concluding remarks

In conclusion we note that, simply by considering $X_3$ as a random preference parameter, it is possible to interpret our models in terms of uncertainty about tastes. On this interpretation, however, the plausibility of (1) as a criterion of country *A*'s welfare is much reduced, and we do not wish to emphasize the possibility.

# Part II

# The neo-classical theory of international trade

# 5 International trade without autarkic equilibria

Macroeconomic implications

## 5.1 Introduction[1]

It is possible that even a wealthy trading country has no autarkic equilibrium. It may lack the climate, land area, soil fertility and technology needed for subsistence food production. One thinks immediately of Holland, Belgium, Ireland, Singapore, Hong Kong, and even Japan and the United Kingdom. Of course, observations of this sort become precise only after the size of the population and its distribution by age have been specified. Allowances must be made for the possibility that a country lacks the technology or raw materials needed for subsistence medical care, that is, medical care compatible with pre-assigned population size and life expectancies. One now thinks of many more countries, especially those in the early stages of industrial development.

Each country now lacking an autarkic equilibrium was once able to survive without international trade but with historical population characteristics. Indeed, given the difficulties of transportation and communication for many ancient societies, there may have been virtually no alternative to autarky. However, over the years, natural resources may have been depleted or degraded, and trade-based wealth may have induced substantial increases in population size and in life expectancies.

Current theories of international trade (Ricardian, Heckscher-Ohlin and others) presuppose the existence of autarkic equilibrium for all countries and, on the basis of that supposition, seek to provide sufficient conditions for worldwide trading equilibria, whether perfectly competitive, imperfectly competitive, or oligopolistic; and they seek to compare the trading equilibria with autarkic alternatives, in terms of national output and consumption patterns and in terms of levels of national well-being. But if not all countries have access to autarkic equilibria, what are we to make of these theories? Do they survive without the crutch of autarkic equilibrium?

If a country has no autarkic equilibrium, and if there are just two traded goods, the offer curve of that country consists of two disjoint segments, each in its own quadrant; or it reduces to the origin. This in turn suggests that, if there are just two countries, the offer curves of those countries may fail to intersect. Clearly, the usual textbook depiction of a worldwide free-trade equilibrium in terms of an intersection of continuous offer curves must be revised. Similarly, the common belief that any closed distortion-free Arrow-Debreu or McKenzie economy must benefit from the opening of its frontiers, whatever the characteristics of other economies, must be reconsidered. Finally, econometric estimates of the *structure* of open economies, the reliability of which is conditional upon the existence of market-clearing equilibria at each point of time, must be viewed with some scepticism.

My first purpose in preparing the present article is to extend conventional static trade theory to accommodate the possibility of missing autarkic equilibria. Conditions necessary and sufficient for the existence of a worldwide free-trade equilibrium, and for the existence of gains from free trade for individual countries, are provided. Then, second, the scope for paradoxical comparative macro-statics is illustrated, by showing that an increase in the foreign demand for a country's exports, which under conventional assumptions would improve that country's well-being, can no longer be relied on to have that effect. Additional macro-statical paradoxes will be briefly noted, as will be the implications of such paradoxes for the theory of open-economy fiscal policy. Finally, it will be noted that the Torrens-Ricardo Principle of Comparative Advantage must be reformulated in a new *conditional* form.

## 5.2 Existence

Let there be just two countries, England and France, each producing and consuming two commodities, wheat and medicine. Figure 5.1 depicts the locus of English production possibilities $Q_E Q_E'$ and the Scitovsky social indifference curve $U_E U_E'$ associated with the subsistence level of well-being for each English household. It can be seen that $U_E U_E'$ forms the lower boundary of the households' joint consumption set.[2] In Figure 5.1, $U_E U_E'$ lies wholly outside $Q_E Q_E'$ indicating that England cannot survive in a state of autarky. If $U_E U_E'$ lies wholly in region I, England cannot survive even under free trade. Leaving aside that uninteresting case, we suppose that $U_E U_E'$ lies partly in each of regions II and III, indicating that England can survive at some world prices but not at others. At hypothetical world price ratios indicated by the slopes of $ee'$ and $EE'$, and given in each case the appropriate scheme of lump sum compensation, the subsistence level of well-being is achieved; at price ratios outside the range defined by $ee'$ and $EE'$, and given in each case an appropriate scheme of lump sum compensation, England enjoys greater than subsistence well-being; and at any price ratio within that range, England cannot survive. The English offer curve consists of two disjoint segments, such as $R_E R_E'$ and $R_E'' R_E'''$ in Figure 5.2. Readers may imagine the English subsistence trade indifference curve tangential to $ee'$ at $R_E'$ and to $EE'$ at $R_E''$.

Suppose that France has a distortion-free economy, satisfying Arrow and Debreu's (1954) or McKenzie's (1954) sufficient conditions for autarkic

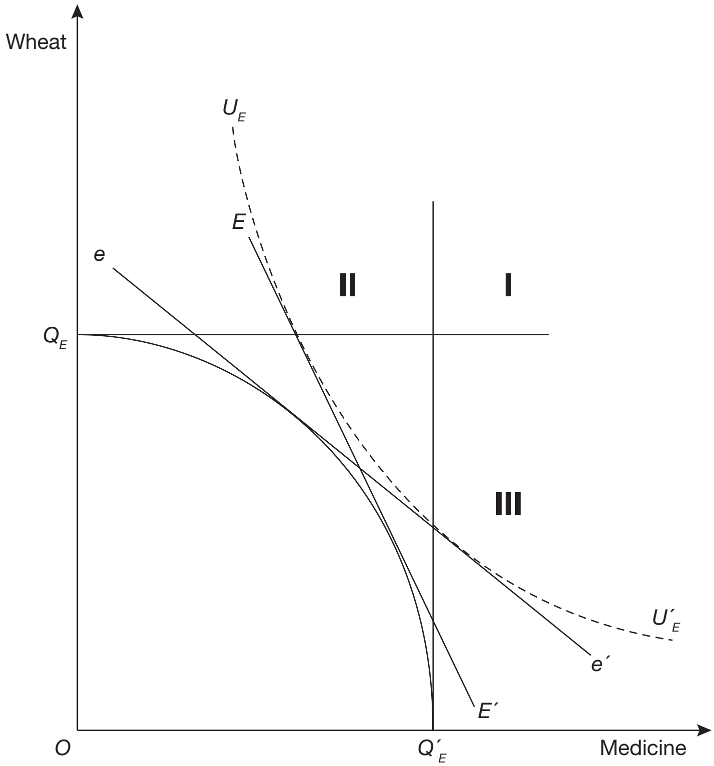


*Figure 5.1*

existence, so that the French offer curve is conventionally continuous. Clearly, the two offer curves may fail to intersect, *either* because the slope of the French offer curve at the origin is greater than the slope of $EE'$ and the offer curve intersects $EE'$ to the left of $R''_E$, *or* because the slope of the French offer curve at the origin is less than the slope of $EE'$ but greater than the slope of $ee'$, *or* because the slope of the French offer curve at the origin is less than the slope of $ee'$ but the offer curve intersects $ee'$ above $R'_E$. In each case there is no worldwide equilibrium embracing both countries. If there is no worldwide equilibrium, it is because the French demand for medicine is insufficient to pull England out of its below-subsistence consumption region. If there is a worldwide equilibrium, it is because the French demand for medicine suffices to extract England from its below-subsistence region.

Going a step further, let us suppose that for neither country is there an autarkic equilibrium. Figures 5.3 and 5.4 illustrate the construction of the French offer curve, which consists of the two disjoint segments $R_F R'_F$ and $R''_F R'''_F$. The undrawn French subsistence trade indifference curve can be imagined tangential to $ff'$ at $R'_F$ and to $FF'$ at $R''_F$.

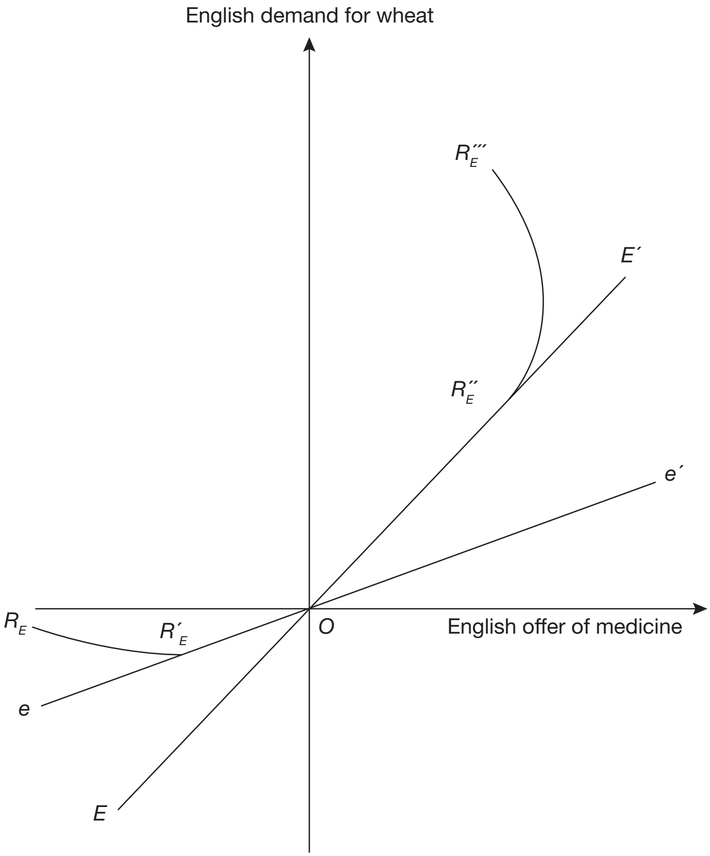


*Figure 5.2*

Superimposing Figure 5.4 on Figure 5.2 to obtain Figure 5.5, we see at once that there can be no worldwide equilibrium if $EE'$ is steeper than $FF'$. If $EE'$ is not steeper than $FF'$, the segments $R''_E R'''_E$ and $R''_F R'''_F$ may or may not intersect, implying that there may or may not exist a worldwide free-trade equilibrium; whether the segments intersect depends partly on the scheme of lump sum compensation adopted by England.

Rephrasing our conclusions, we may say that, for the existence of a worldwide free-trade equilibrium, it is necessary that the sets of price ratios compatible with survival should overlap. For England, the relevant price ratios lie in either of the cones $XOE'$ and $YOe$; for France, the relevant price ratios lie in either of the cones $WOf'$ and $ZOf$. In the case illustrated by Figure 5.5, therefore, the condition is met only by price ratios in the intersection $E'OF'$ of $WOF'$ and $XOE'$. If, within the intersection, the two offer curves intersect, then there exists a worldwide free-trade equilibrium.

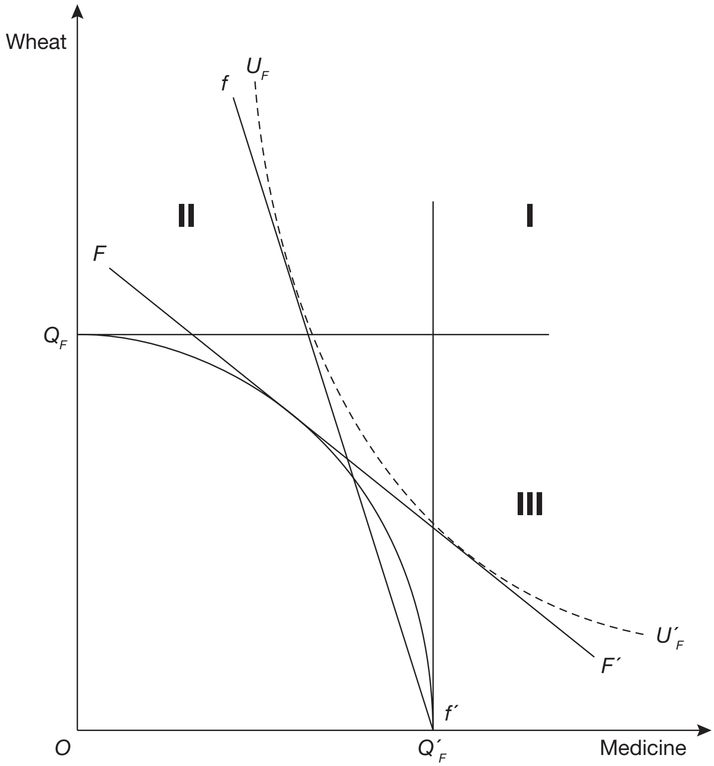


*Figure 5.3*

*Proposition 5.1* For the existence of a worldwide free-trade equilibrium, it is necessary and sufficient that the possibly discontinuous offer curves of the two countries intersect, and for such an intersection it is necessary that the two sets of price ratios compatible with survival also intersect. If one (or both) countries has an autarkic equilibrium then the relevant offer curve(s) is (are) continuous and the relevant set(s) of price ratios can be represented by the entire first quadrant.

The proposition has been demonstrated only for the symmetrical case in which the subsistence curve $U_E U'_E$ lies partly in each of regions II and III of Figure 5.1 and $U_F U'_F$ lies in each of regions II and III of Figure 5.3. However, it can be easily verified that the proposition remains valid even when each indifference curve lies in just one of the two relevant regions. Similarly, the proposition has been proved only for the case in which the English and the French production sets are strictly convex. However, the proposition is valid without that assumption; in particular, it is valid in the Ricardian single-factor case in which the production sets have linear upper boundaries.

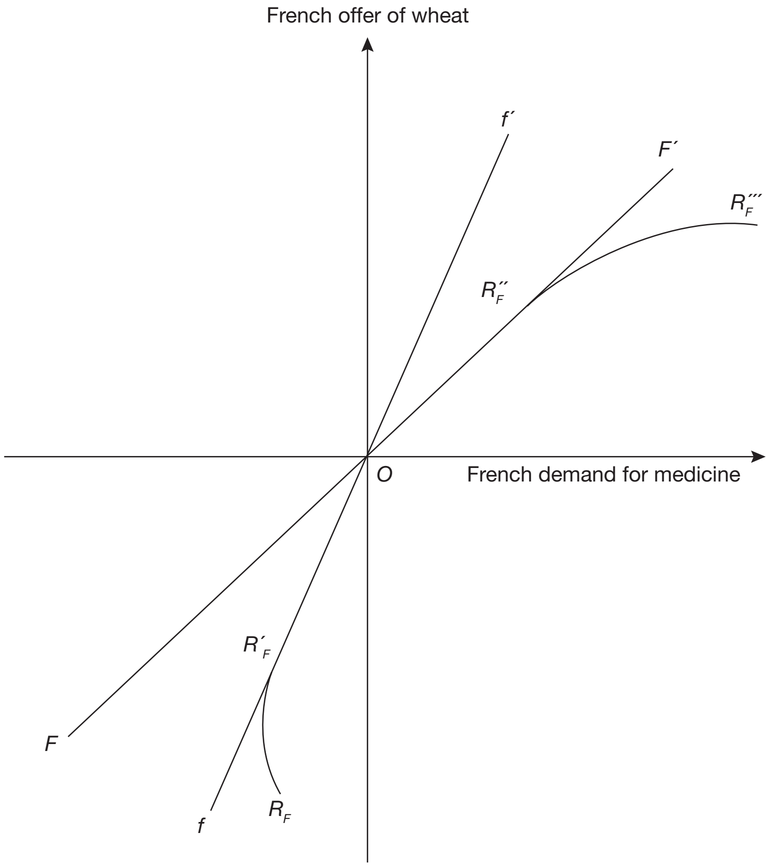


*Figure 5.4*

Proposition 5.1 extends readily to accommodate any finite number of countries and any finite number of dated commodities, and therefore to accommodate international borrowing and lending. It also extends to accommodate preferences and technologies that are widely disparate internationally. Thus, suppose that there are $n$ countries, $n > 2$. For a worldwide equilibrium it is necessary and sufficient that the intersection of the $n$ sets of price ratios compatible with survival be non-null and that, in the intersection, there be a price ratio that equates world demand and supply for each commodity. However, even if there does not exist a worldwide equilibrium, there may yet be an equilibrium embracing some subset or 'club' containing $m$ countries, $m < n$, with each of the $n - m$ excluded countries lacking an autarkic equilibrium. Indeed there may be two or more *alternative* clubs, with distinct but overlapping memberships $m_i$ ($m_i < n$; $i = 1, 2 \ldots$). We can even admit

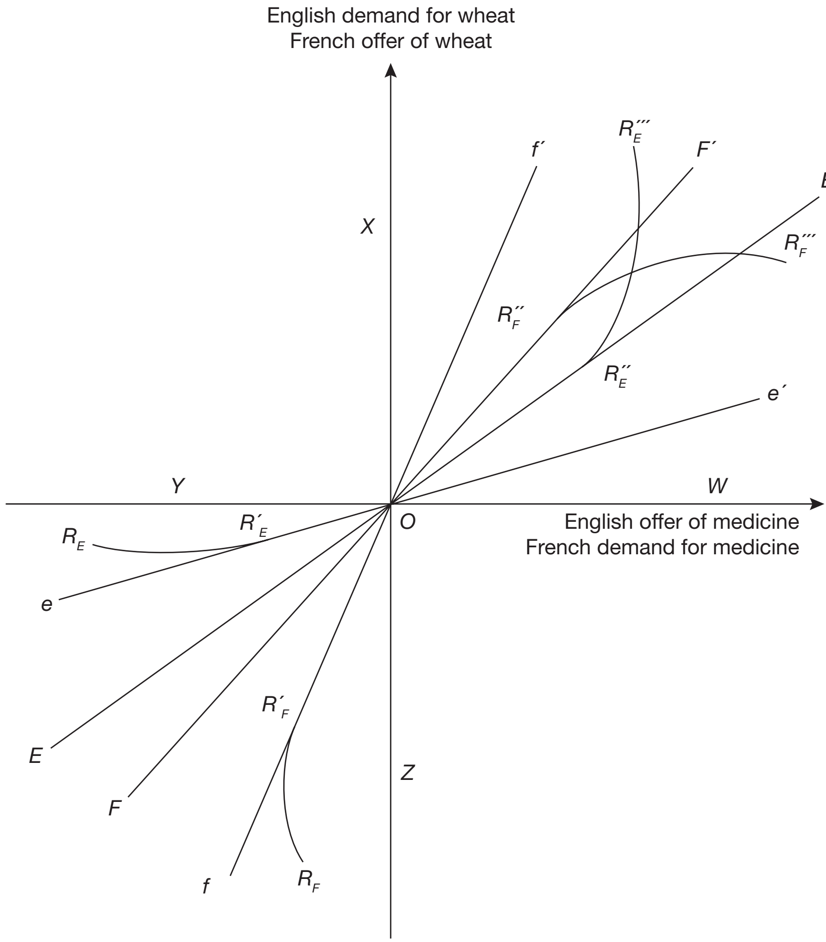


*Figure 5.5*

the (remote) possibility of two clubs with non-overlapping memberships that *coexist* with common prices for internationally traded goods but zero inter-club trade.[3]

Suppose, for example, that initially the world contains only England and France, and that there exists a free-trade equilibrium. A third country, say Portugal, makes its entry. Then, whether or not there exists a Portuguese autarkic equilibrium, three possibilities must be recognized: (i) there is a worldwide equilibrium in which each pair of countries engages in trade; (ii) there is a worldwide equilibrium in which Portugal trades with England or France but not both; and (iii) there is no worldwide equilibrium. Thus, the emergence of a third country may create trade between two initially non-trading countries, or it may destroy a free-trade equilibrium, leaving no trade.

Almost needless to say, the non-existence of autarkic equilibria for some countries complicates the existence conditions not only for free-trading clubs but also for preferential trading clubs such as free trade associations and customs unions.

*Proposition 5.2* Any member of an $m$-country equilibrium benefits from free trade. However, the sense in which the country benefits depends on whether the economy of that country has an autarkic equilibrium. If the economy does have an autarkic equilibrium, then the country gains from trade in the conventional sense that, after compensation, each of its residents is better off than in the autarkic equilibrium. If the economy lacks an autarkic equilibrium, then it can be affirmed only that, after compensation, each individual is better off than under bare subsistence.

## 5.3 Macroeconomic implications

In both the Ricardian and the Heckscher-Ohlin contexts, a shift in foreign excess demand in favour of a particular country's exported commodity will turn the terms of trade in favour of that country and hence improve its well-being. However, this simple and apparently reliable proposition is generally valid only if each country possesses an autarkic equilibrium. In the present section we explore some of the implications of relaxing that condition.

Let us return to the simplest model of Section 5.2, in which two countries produce, consume and trade two commodities, each produced with the aid of two primary factors of production. Initially, the world economy is in equilibrium at point $R_E''$ in Figure 5.6, at which the French offer curve is truncated. In that equilibrium, England exports medicine to France in exchange for wheat, at a price indicated by the slope of $0F'$. At that price, France enjoys only the subsistence level of well-being.

The initial equilibrium is disturbed by a change in French preferences. The change is treated as autonomous, induced by factors such as climate or environmental pollution that are of no direct interest to us. In Figure 5.7, which corresponds to our earlier Figure 5.3, the new French subsistence indifference curve $U_F''U_F'''$ is assumed to lie uniformly outside the old curve $U_FU_F'$, as though they are level curves of a single homothetic function. Evidently the French demand for English medicine at any price ratio consistent with post-disturbance French subsistence must increase. In terms of Figure 5.6, the truncated French offer curve shifts to $\bar{R}_F''\bar{R}_F'''$, implying that there is no post-disturbance worldwide equilibrium. Not only France but also England, the presumed 'beneficiary' of the change in French preferences, is harmed; both economies disappear.

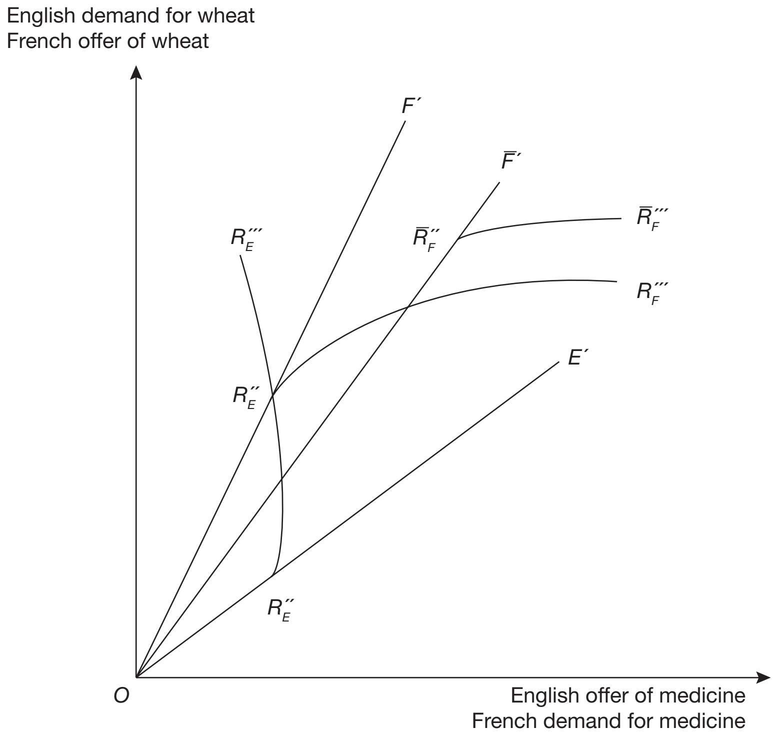


*Figure 5.6*

The demonstration of this possibility has been based on an example that is very special, indeed singular in the sense that the pre-disturbance English offer curve is truncated precisely at its intersection with the French offer curve. That feature of the example has been convenient in removing the need to specify the magnitude of the change in French preferences. However, it will be clear that the intersection may be supposed to occur away from the point of truncation, how far away depending on the magnitude of the change in preferences.

The above example suggests that a widely respected macroeconomic principle should be applied with caution. However, the example is purely static. It was therefore assumed that if both autarkic and worldwide equilibria are absent, then countries simply disappear. In a more dynamic analysis, other less dramatic outcomes might be admitted; for example, the gradual decline of population (through death and migration) and in life expectancies (as the result of a reduction in the availability of medical care) might end with the emergence of a new worldwide equilibrium.

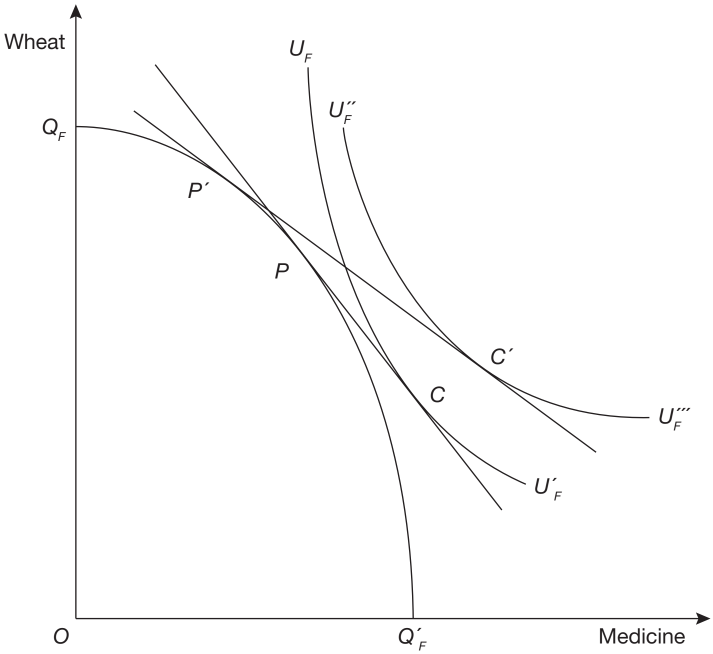


*Figure 5.7*

On the other hand, the macroeconomic principle highlighted in this section is only one of several that might have been considered. The principle on which we focused attention is a ‘supply side’ principle. In a companion paper, the focus is on a pair of ‘demand side’ results; see Chapter 6. Long ago, Edgeworth (1894a, 1899) showed that a free-trading country might be impoverished by its own technical improvements if the latter are confined to that country’s export industry and if no produced good is inferior in consumption. More recently, it has been shown that improvements confined to the country’s import-competing industry can never be impoverishing if in that country no commodity is inferior in consumption; see Kemp (1964: 87). However, in all available proofs of the propositions, it has been assumed that for each country there exists an autarkic equilibrium. It is shown in the companion paper that, without that assumption, both ‘supply side’ propositions must be severely qualified. Specifically, improvements in the import-competing industry can impoverish a progressive country even in the absence of inferiority.

## 5.4 The Torrens-Ricardo Principle of Comparative Advantage

The Torrens-Ricardo Principle of Comparative Advantage is one of the oldest and most widely respected propositions in economic theory. After nearly two hundred years, it survives in essentially the form given it by Ricardo in 1817. However, in that form, it rests on the assumed existence of an autarkic equilibrium for each country. Whether it survives the relaxation of that assumption has not, to my knowledge, been considered. Suppose then that one country or more cannot survive under autarky but that, nevertheless, there exists a worldwide free-trade equilibrium. Does the Principle survive under the revised conditions, or must it be abandoned or cast in some other form?

In the Ricardian model, there is just a single primary factor of production, implying that the production possibility curve of each country is linear. Figures 5.2, 5.4 and 5.5 remain unchanged, but we now find it useful to add to Figure 5.4 (Figure 5.5) a positively sloped straight line $G_E G'_E$ ($G_F G'_F$), passing through the origin with slope equal to minus that of England's (France's)

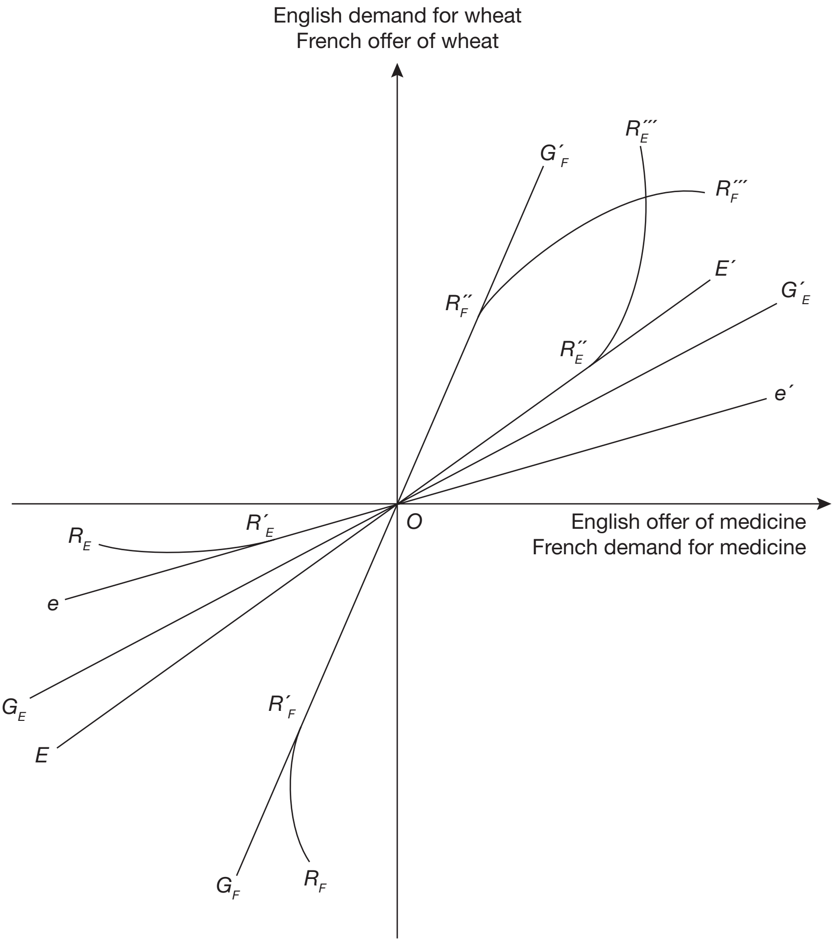


*Figure 5.8*

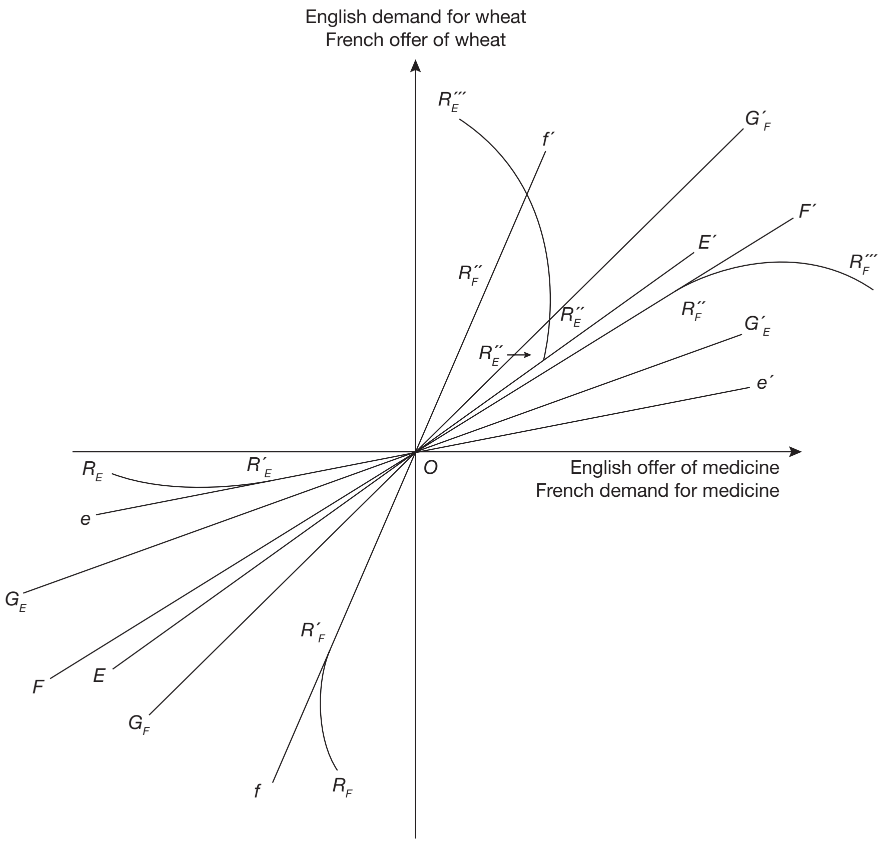


*Figure 5.9*

production possibility curve. As drawn, $G_E G'_E$ is less steep than $G_F G'_F$, implying that England has a comparative advantage in producing medicine. If there is no autarkic equilibrium in England (France), $G_E G'_E$ ($G_F G'_F$) must lie strictly between *ee′* and *EE′* (*ff′* and *FF′*) in Figure 5.2 (Figure 5.4); otherwise, it will form part of England's (France's) continuous offer curve.

Suppose that France, but not England, has an autarkic equilibrium and that there exists a worldwide free-trade equilibrium. In the free-trade equilibrium, depicted in Figure 5.8, each country exports that commodity in the production of which it has a comparative advantage. As Figure 5.9 reveals, the same conclusion emerges when both countries lack an autarkic equilibrium. To this point, the most relevant component of the Torrens-Ricardo Principle is confirmed.[4]

However, it must be borne in mind that the Principle of Comparative Advantage is now conditional on the existence of a worldwide free-trade equilibrium. If in Figure 5.8 $EE'$ is steeper than $G_F G_F'$ or if in Figure 5.9 $EE'$ is steeper than $FF'$, each possibility compatible with the assumption that $G_E G_E'$ is less steep than $G_F G_F'$, then there is no worldwide equilibrium and the Principle does not come into play.

## 5.5 A final remark

Arrow and Debreu imposed constraints on individual households to ensure that each household would survive with or without access to free markets; in particular, they required that each household's endowment vector lies within its consumption set. In the present paper, on the other hand, I have worked with some countries that cannot survive without the opportunity to trade with other countries and have examined the circumstances under which some or all of them will survive under free trade.

# 6 Impoverishing technical and preferential improvements

## 6.1 Introduction

Edgeworth (1894a, 1899), building on the earlier thoughts of J.S. Mill (1854), showed that a free-trading country might be harmed by its technical improvements. Specifically, he showed that a country might suffer if factor-neutral improvements are confined to that country's export industry and if in that country no commodity is inferior in consumption.[1] Later, it was shown by Kemp (1964: 87) that a free-trading country can never be harmed by factor-neutral technical improvements confined to its import-competing industry if in that country no commodity is inferior in consumption. The two propositions remain basic components of the normative comparative statics of technical improvements in open economies. In the present paper, we draw attention to a new possibility, showing by example that a free-trading country not only might be impoverished by a factor-neutral improvement confined to its import-competing industry but might be left stranded below its subsistence consumption. Recycling and redefining Edgeworth's own colourful term, we describe such improvements as 'damnifying'. Indeed it is shown that an improvement might damnify not only the improving country but also that country's trading partner.

In constructing the example, we follow Edgeworth in confining attention to a free-trading competitive world of two commodities and in ruling out inferiority of consumption in the progressive country. However, we depart from Edgeworth (and from those, such as Johnson 1955, Kemp 1955, 1964 and Bhagwati 1958, who followed Edgeworth) in allowing for the possibility that a country might lack an autarkic equilibrium. The plausibility of that possibility, and not only in small island economies, has been argued elsewhere (Kemp 2003a); evidently economies that lack an autarkic equilibrium do not satisfy all of the Arrow-Debreu sufficient conditions for the existence of a competitive equilibrium. Moreover we depart from Mill and Edgeworth in confining improvements to the import-competing industry.

## 6.2 A first example: technical improvements

Two countries, England and France, produce (under constant returns to scale) and consume two commodities, medicine and wheat. In England, the progressive country, neither commodity is inferior in consumption. Neither country can survive under autarky; however, both countries can survive under free trade and the initial technologies. Figure 6.1 contains England's initial or pre-improvement production possibility curve $Q_1Q_2$ and its subsistence indifference curve $uu'$ (the Scitovsky community indifference curve, at each point of which every English household enjoys subsistence consumption and a common marginal rate of substitution). At the terms of trade represented by the slope of *PC,* England survives by producing at *P*, consuming at *C*, importing wheat and exporting medicine. At world prices more favourable to medicine, England enjoys a higher standard of living; and, at terms of trade less favourable to medicine, England disappears from our static vision. Thus England's offer curve is defined only for terms of trade not inferior to those represented by the slope of *PC*. The (truncated) English offer curve is displayed in Figure 6.2.[2] The straight lines $OT_E$ of Figure 6.2 and *PC* of Figure 6.1 have slopes equal in magnitude but of opposite sign. Similar figures can be drawn for France; see Figures 6.3 and 6.4.

Combining Figures 6.2 and 6.4, we obtain Figure 6.5. From that figure we learn that the pre-improvement free-trade equilibrium is represented by *G*, the singular point at which France's offer curve is truncated. At *G* the equilibrium terms of trade are equal to the slope of $OT_F$.

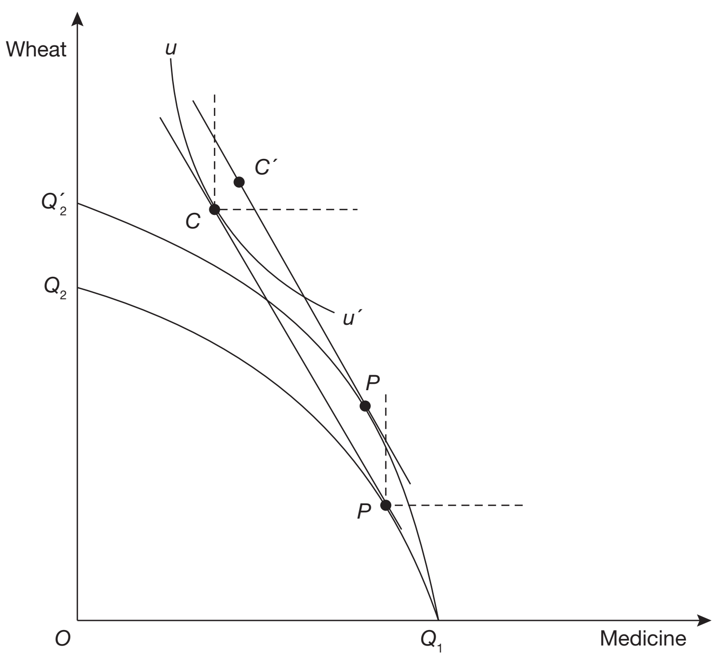


*Figure 6.1*

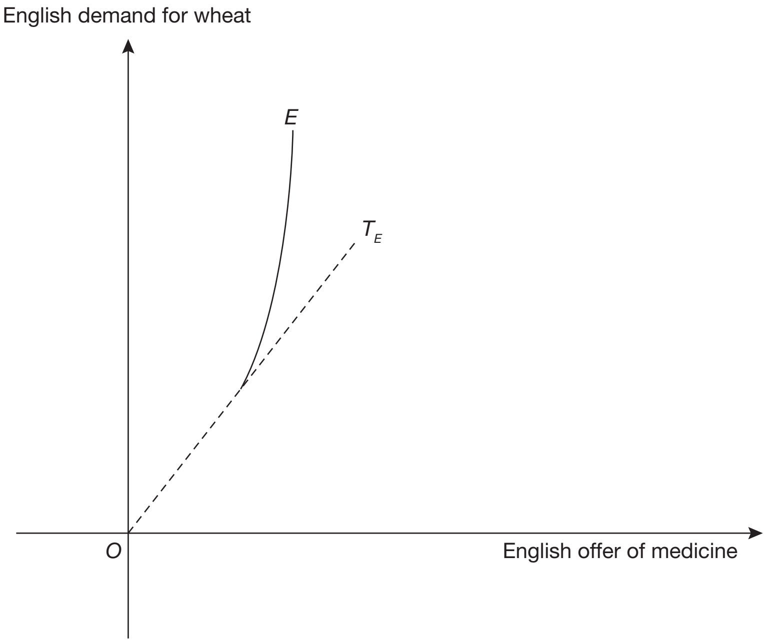


*Figure 6.2*

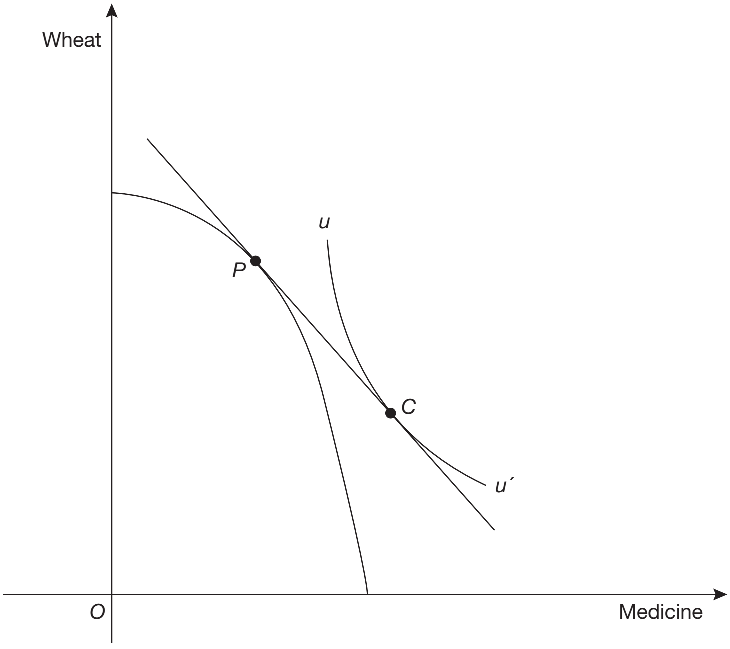


*Figure 6.3*

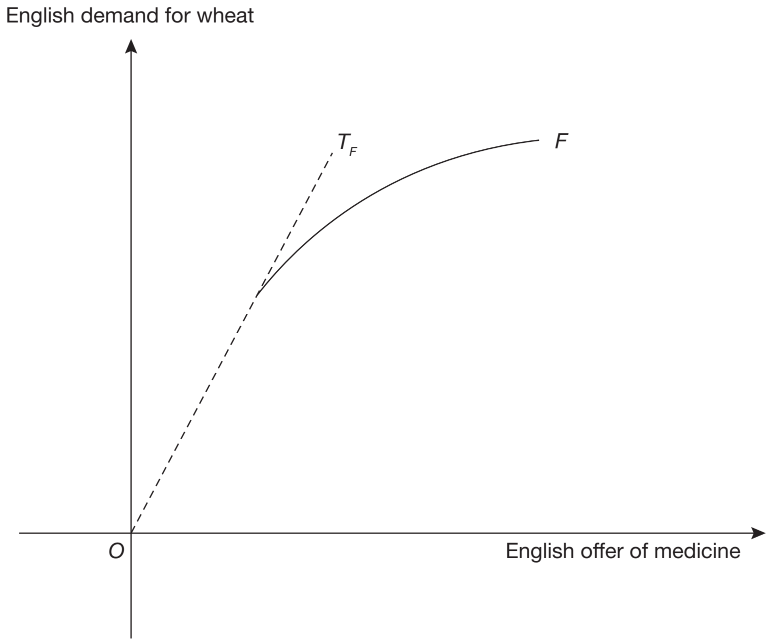


*Figure 6.4*

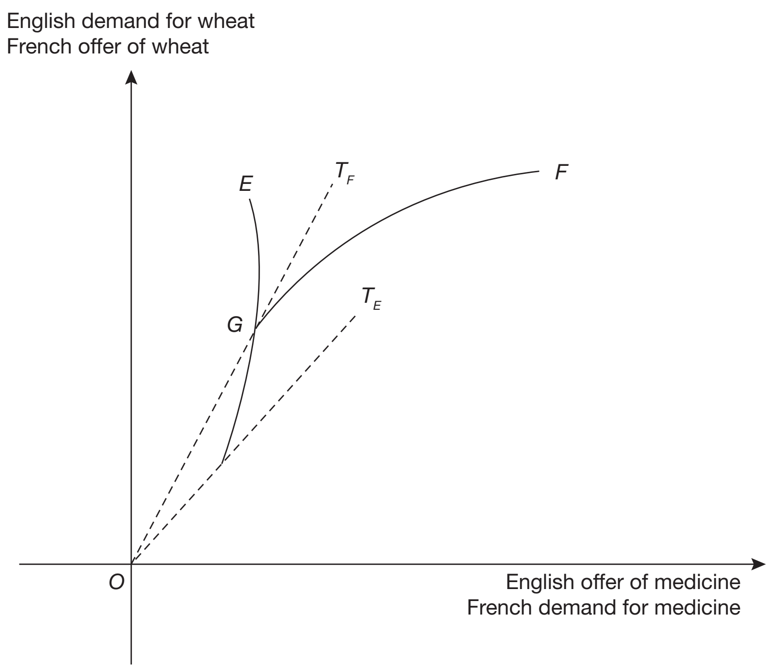


*Figure 6.5*

Consider now the implications of a factor-neutral improvement in the English import-competing wheat industry. The new English production possibility curve is $Q_1Q_2'$ in Figure 6.1, still incompatible with autarkic equilibrium. At the pre-improvement world equilibrium terms of trade, England must produce more wheat and less medicine, implying that the new production point $P'$ must lie on $Q_1Q_2'$, north-west of $P$; and, since we have ruled out inferiority in English consumption, the post-improvement English consumption point $C'$ must lie to the east of the pre-improvement consumption point $C$. It follows that, at the pre-improvement terms of trade, England must offer less medicine than before the improvement. We may be sure, therefore, that the post-improvement English offer curve lies inside (to the left of) the (unchanging) French offer curve. Hence there is no post-improvement world equilibrium and, since there are no autarkic equilibria, both countries disappear.

That completes construction of the example. The latter is special, in the sense that the pre-improvement English offer curve passes through the point at which the French offer curve is truncated. That feature of the example has been convenient in removing the need to specify the magnitude of the improvement. However, it will be clear that the intersection may be supposed to occur away from the point of truncation, how far away the intersection may occur depending on the magnitude of the improvement.

## 6.3 Further remarks on the example

It was assumed in Section 6.2 that the progressive country is free-trading, both before and after a technical improvement. Needless to say, the progressive country could not be damnified (or even impoverished) if, after the improvement, it had in place an optimal import duty. In fact, in Section 6.2 we based our analysis on social utility functions, which might have been justified in terms of representative households, either given by nature or contrived by Scitovsky compensation. If to the assumption of representative households is added the companion assumption that each agent is aware of its representative status, we may apply the logic of Kemp and Shimomura (1995) and conclude that in the progressive country agents will cooperate to impose an optimal tariff. If after the improvement *each* country imposes an optimal tariff, then neither country could be damnified but either (or both) might be impoverished.

In Section 6.2 it was also assumed that technical progress is confined to the import-competing industry. That extreme assumption can be relaxed by admitting technical progress shared by the two industries. However, it cannot be replaced by the equally extreme alternative assumption of Mill and Edgeworth (that progress is confined to the export industry) without excluding the possibility of damnification.

On the other hand, the analysis of Section 6.2 can be extended to accommodate more than two commodities and/or more than two countries. As in the two-by-two case, a carefully chosen factor-neutral improvement confined to a single industry in a single country might cause all or some countries to disappear, leaving any surviving countries in a new and still gainful trading equilibrium. Indeed there may be several such equilibria, each with its own set of survivors.

Like all earlier contributors to the field, we have typically assumed that all economies are perfectly competitive. However, Okawa (2005) has recently varied the traditional assumption by allowing one of the two industries to be Cournot-duopolistic, with one producer in each country, and has shown that the results of Edgeworth and Kemp must be qualified, especially when technical progress is confined to the duopolistic industry.

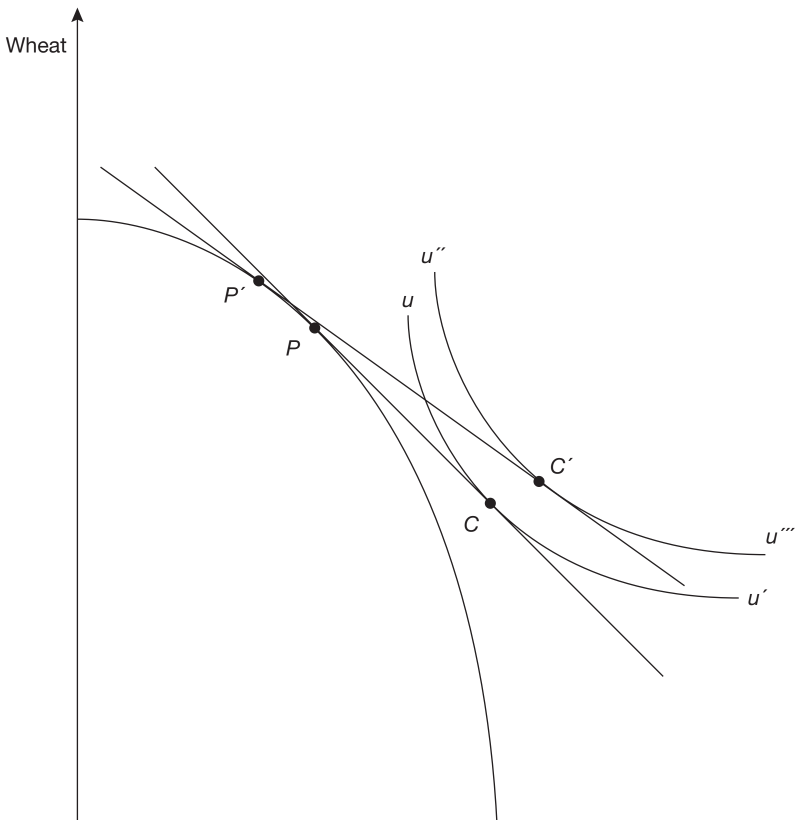


*Figure 6.6*

## 6.4 Further examples: preferential improvements

We have shown in Section 6.2 that one of two well-known supply-side propositions must be substantially revised if the existence of an autarkic equilibrium is denied. The same is true of other established propositions, including at least two from the demand side: (i) A shift in foreign excess demand in favour of a particular country's exported commodity will turn the terms of trade in favour of that country and improve its well-being. (ii) The optimal tariff on a country's imported commodity is equal to the inverse of the elasticity of the foreign import demand; see, for example, Kemp (1964: 171).

Consider Figure 6.6, which corresponds to our earlier Figure 6.1 but illustrates a change in the preferences of France rather than a change in the technology of England. Initially, a subsistence level of French well-being is

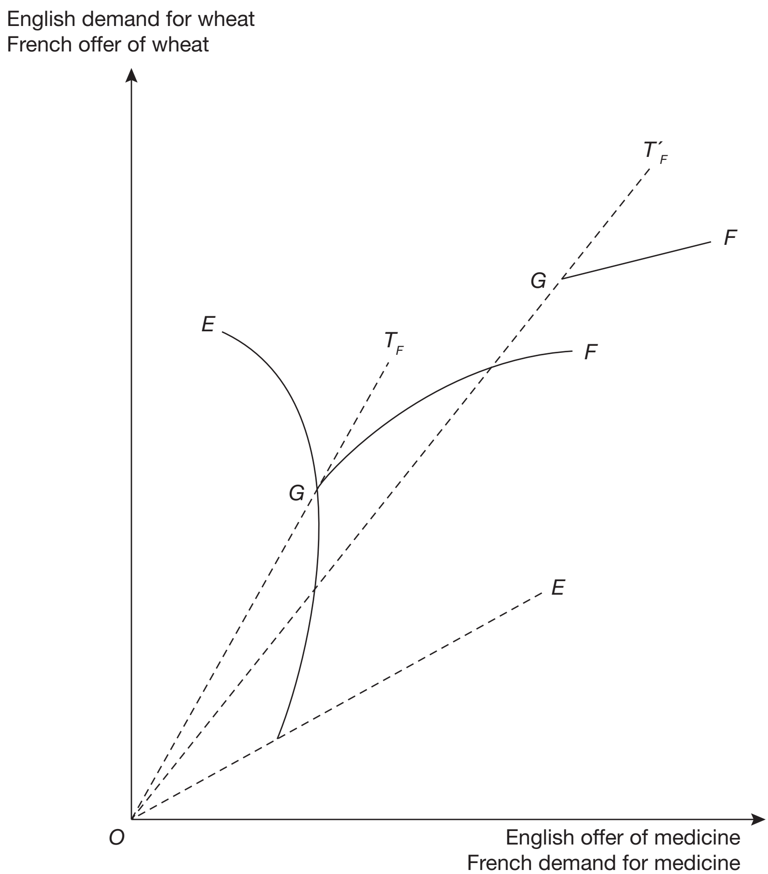


*Figure 6.7*

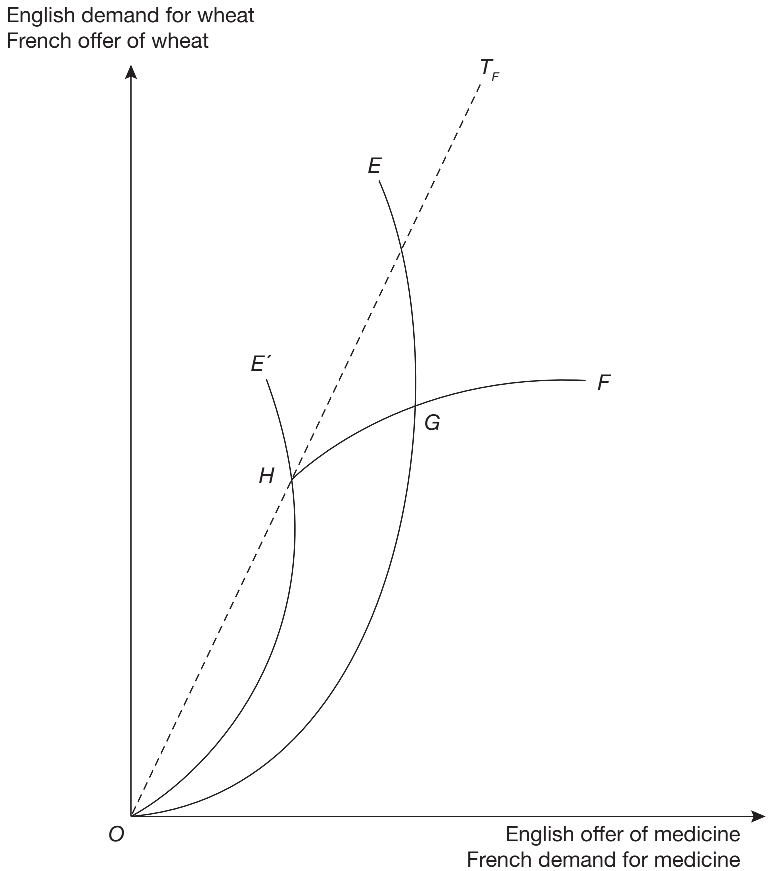


*Figure 6.8*

achieved on $uu'$ by producing at $P$ and consuming at $C$. Preferences then change – possibly as the result of purely psychological adjustments – so that the subsistence level of well-being can be attained only on $u''u'''$, where $uu'$ and $u''u'''$ are assumed to bear the same relationship to each other as two level curves of the same homothetic function. Let the initial, pre-disturbance equilibrium be represented by point $G$ in Figure 6.7 and let $OT_F$ ($OT'_F$) represent the same terms of trade as $PC$ ($P'C'$) in Figure 6.6. Then, given that the French output of medicine is smaller at $P'$ than at $P$ and given the restrictions imposed on $u''u'''$, the French demand for imported medicine at the world prices indicated by $OT'_F$ must be greater after the disturbance than before. As Figure 6.7 makes clear, there can be no post-disturbance world equilibrium and England is damnified; indeed, both countries are damnified. This possibility does not rest on the exclusion of inferiority in consumption.

Let us turn now to proposition (ii). Suppose that England, but not France, can survive under autarky and that there exists a free-trade equilibrium at point $G$ in Figure 6.8, where England's tariff-free offer curve $OGE$ intersects France's truncated offer curve $HGF$. The optimal English tariff ensures that England's tariff-distorted offer curve $OE'$ passes through $H$. However, in general, the English trade indifference curve through $H$ will not be tangential to $HGF$ at that point; it will be steeper than $HGF$, implying that the optimal tariff is of smaller magnitude than the elasticity of the French import demand. A higher tariff would destroy trade between the two countries and drive France below the subsistence level of well-being.

# 7 A dynamic Heckscher-Ohlin model

## The case of costly factor reallocation[1]

### 7.1 Introduction

It is well known that competitive and market-clearing general equilibrium models of the Arrow-Debreu-McKenzie kind yield almost no descriptive comparative static propositions.[2] In their everyday general equilibrium work, therefore, economists have retreated to special versions of the Arrow-Debreu-McKenzie models. In particular, in many branches of their subject economists now rely on the Heckscher-Ohlin model and on its descriptive comparative statics, packaged as the Stolper-Samuelson, Rybczynski, Factor Price Equalization, Heckscher-Ohlin and Hicks-Ikema propositions. However, the comparative-static manipulations of the Heckscher-Ohlin model have relied on the rarely mentioned assumption that the reallocation of factors in response to any external disturbance is without cost. This assumption is implausible. Moreover, it rules out the construction of a sensible, dynamic, market-clearing version of the model. For if reallocation is costless but occurs at a finite rate, then factor owners have an incentive to adjust even faster. To overcome this difficulty, we must introduce resource-using, and therefore costly, adjustment and allow the speed of adjustment to be optimally chosen by factor owners. In short, equilibrium dynamics is possible if and only if costly reallocation of factors is accommodated.[3]

In the present paper we consider a small open economy with costly reallocation of factors. It is shown that in such a context the standard descriptive comparative statics of the Heckscher-Ohlin model, which presuppose that each factor earns the same amount in each industry, generally have no comparative steady-state counterparts; however, normative exceptions to the rule are noted. A new (non-)correspondence principle is proposed: the introduction of market-clearing and of resource-using dynamics fails to sharpen (and may even blunt) the associated comparative statics.

For pioneering studies of the costly reallocation of a single primary factor of production, readers are referred to Kemp and Wan (1974), Long (1978) and Mussa (1978). However, the focus of those early papers is on questions quite different from ours.

## 7.2 Analysis

Consider a small and fully employed open economy that potentially produces two final goods (labelled 1 and 2) with the aid of two factors of production (labour and land, available in amounts $L$ and $T$). In the absence of costs of reallocation, the cost-minimizing labour:land ratio would differ from industry to industry. The *GNP* function for the economy is defined as

$$Y(p, L_1, T_1) = Y^1(L_1, T_1) + pY^2(L - L_1, T - T_1) \qquad (1)$$

where $p$ is the given and constant relative price of the second commodity, $L_1(T_1)$ is the amount of labour (land) allocated to the first industry and $Y^i$ is the output of industry $i$, $i = 1, 2$. It is assumed that each production function is increasing, strictly quasi-concave, homogeneous of degree one in the two factor inputs and satisfies the Inada conditions.

To move a factor of production from one industry to the other requires inputs of commodities 1 and 2. The production functions are defined by

$$\left|\dot{X}_1\right| = G(y_{1X}, y_{2X}) \quad X = L, T \qquad (2)$$

where $\dot{X}_1$ is the (positive or negative) rate of reallocation of factor $X$ from industry 2 to industry 1, $y_{iX}$ is the amount of commodity $i$ ($i = 1, 2$) employed in reallocating factor $X$, and the production function $G(\cdot, \cdot)$ satisfies the same conditions as the production function $Y^i(\cdot, \cdot)$. Formulation (2) is special. The production function $G$ is the same for each factor of production, and it is symmetric (the same for outgoing and incoming factor movements). Nevertheless, our general conclusions do not depend on that formulation.[4] Given (2), the minimum cost, in terms of the first commodity, of effecting a pair of inter-industrial factor movements $(\dot{L}_1, \dot{T}_1)$ is

$$\alpha(p)\left[\left|\dot{L}_1\right| + \left|\dot{T}_1\right|\right] \qquad (3)$$

where the unit cost function $\alpha(p)$ is a positive, increasing and concave function of $p$.

All households are identical: at all times they have the same preferences, own the same amounts of the two factors of production and allocate those amounts in the same way. In other words, each household is a representative agent in the narrow modern or post-Marshallian sense, with the indirect utility function

$$u[Y(p, L_1, T_1) - \alpha(p)(|\dot{L}_1| + |\dot{T}_1|), p]$$

Given the further assumption of perfect foresight, and with the number of households normalized to unity, the competitive market may be viewed as solving the control problem

$$\max_{\dot{L}_1, \dot{T}_1} \int_0^\infty \exp(-\rho t) u\left[Y(p, L_1, T_1) - \alpha(p)\left(|\dot{L}_1| + |\dot{T}_1|\right), p\right] \mathrm{d}t$$

$$\text{subject to} \quad 0 \leq L_1 \leq L, \quad 0 \leq T_1 \leq T, \qquad (4)$$
$$L_1(0),\ T_1(0) \quad \text{given,}$$

where $\rho$ is the given, constant and positive rate of time preference. It is assumed that $u\ [\cdot]$ is increasing and strictly concave in income and that it satisfies the Inada condition as consumption goes to zero. It is shown in Appendix 7.1 (at the end of this chapter) that problem (4) possesses a unique optimal solution. The phase diagram in $(L_1, T_1)$-space is constructed in Appendix 7.1 and will shortly bear the burden of our analysis. First, however, we must explain the loci that play key roles in the construction of the phase diagram.

Consider the box diagram, Figure 7.1. Inscribed in the diagram is the heavy contract locus $O_1EO_2$ (based on the immaterial assumption that the first industry is relatively land-intensive) and the two dashed loci $Y_T \equiv \partial Y/\partial T_1 = 0$ and $Y_L \equiv \partial Y/\partial L_1 = 0$ intersecting at the unique long-run equilibrium $E$ defined by the given $p$. That the three loci occupy the depicted relationship to each other is not hard to show. Already we know from Kemp *et al*. (1977)

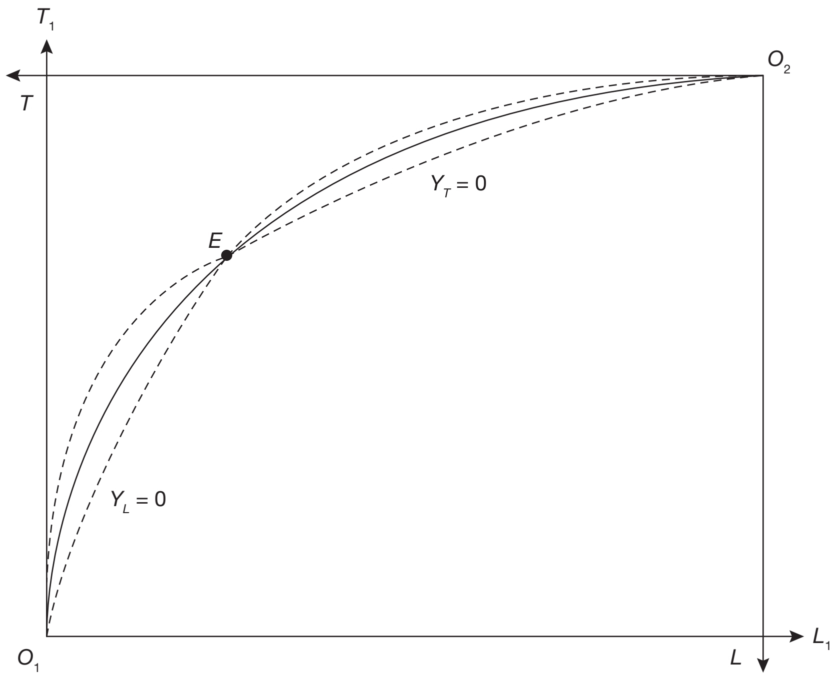


*Figure 7.1*

that they bear that relationship in a sufficiently small neighbourhood of $E$. The global relationship then follows from the fact that each of the dashed curves can intersect the contract locus only once.

We now add to the box the additional loci $Y_L = \rho\alpha(p)$, $Y_L = -\rho\alpha(p)$, $Y_T = \rho\alpha(p)$, $Y_T = -\rho\alpha(p)$, $Y_L = Y_T$ and $Y_L = -Y_T$. However, to avoid overcrowding, we omit the contract locus and focus on a neighbourhood of $E$, containing the 'diamond' *HJMR*; see Figure 7.2. With the exception of the locus $Y_L = -Y_T$, all loci are positively sloped; but nothing is known of their curvatures (nor is such information needed). A detailed derivation of the locus $Y_L = Y_T$ may be found in Appendix 7.1. We note that the size and shape of the diamond depend on $p$ and that the diamond is completely formed only if $\rho\alpha\ (p)$ is sufficiently small.

Given the scaffolding formed by these loci, we can construct the complete phase diagram; see Figure 7.3. Inspection of the figure reveals that, if the locus $Y_L + Y_T = 0$ is everywhere negatively sloped, then, starting from *any* point outside the diamond *HJMR*, the optimal trajectory of factor allocations

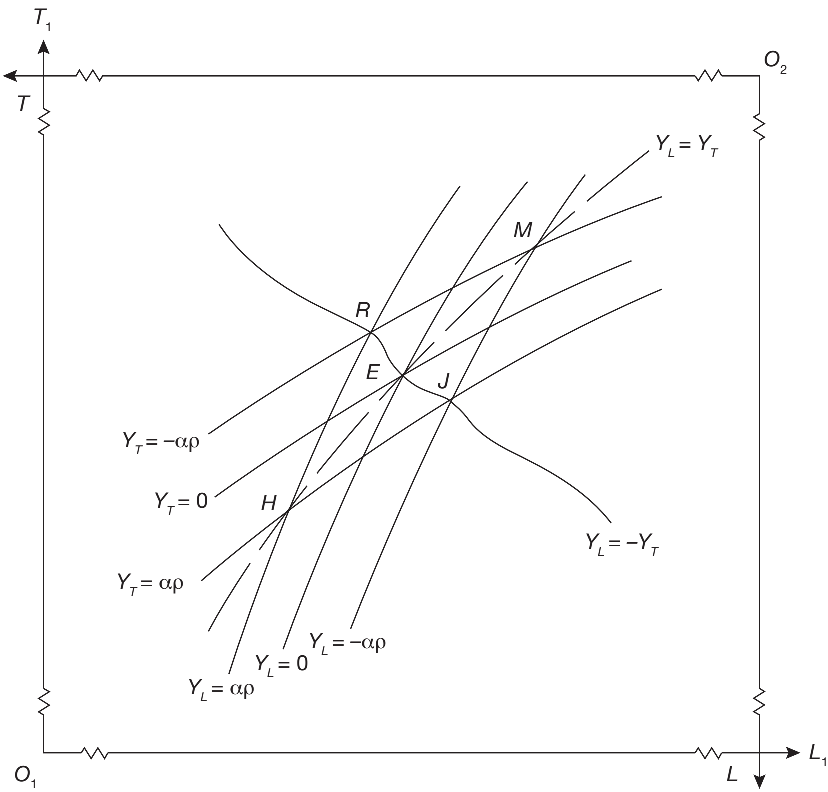


*Figure 7.2*

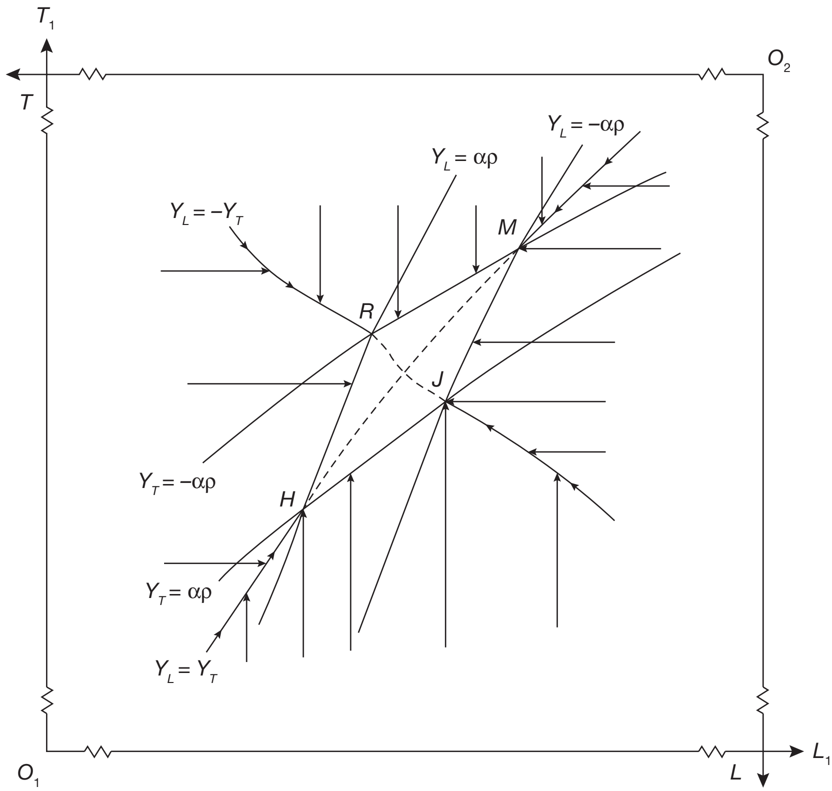


*Figure 7.3*

and outputs weakly monotonically converges to a point on the boundary of the diamond. We note that however close the initial point is to *HJMR*, the journey to the boundary of the diamond takes infinite time.

A clear interpretation of this result is available. Consider any point on the diamond *HJMR*, where

$$-\rho\alpha \leq Y_L^1 - pY_L^2 \leq \rho\alpha$$

$$-\rho\alpha \leq Y_T^1 - pY_T^2 \leq \rho\alpha$$

or

$$Y_L^1 - \rho\alpha \leq pY_L^2 \text{ and } pY_L^2 - \rho\alpha \leq pY_L^1$$

$$Y_T^1 - \rho\alpha \leq pY_T^2 \text{ and } pY_T^2 - \rho\alpha \leq pY_T^1$$

Any unit of labour employed in industry 2 earns the wage $pY_L^2$. If it moves to industry 1, it can earn the wage $Y_L^1$. In the absence of any cost of reallocation, it will move to industry 1 if $Y_L^1 > pY_L^2$ . In the stationary state, however, the movement of one unit of labour costs $\rho\alpha$ units of the numeraire. Thus, the net income that labour can earn by moving is $Y_L^1 - \rho\alpha$. To justify the move, therefore, it is necessary that $Y_L^1 - \rho\alpha \geq pY_L^2$. Similarly, to justify the movement of a unit of land from industry 2 to industry 1, it is necessary that $Y_T^1 - \rho\alpha \geq pY_T^2$.

## 7.3 Comparative steady states

In the absence of costs of reallocation, each factor of production earns the same value of marginal product in each industry, and on that foundation rest the familiar comparative-static propositions of the Heckscher-Ohlin theory. The introduction of costs of reallocation ruptures the equality of marginal value products and thus removes the foundation of those propositions listed in Section 7.1.

Nevertheless, it is clear from Figure 7.3 that the smaller is $\rho$, the smaller are the diamonds associated with alternative values of $p$ and that, for any given $\Delta p$, the smaller is $\rho$, the more nearly accurate is the Stolper-Samuelson proposition, considered now as a comparative steady state proposition. Indeed, one may go a step further.

*Revised Stolper-Samuelson Proposition* Given any constant-returns production functions and any initial and final values of p, say p′ and p″, which, in the absence of costs of reallocation, are compatible with the incomplete specialization of production, there exists a unique and positive ρ, say ρ(p′, p″), such that, if and only if ρ ≤ ρ (p′, p″), the Stolper-Samuelson conclusions remain valid as statements about any initial steady state and the associated final steady state.

Similar remarks can be made about the robustness of the remaining comparative-static propositions. A formal proof of the Revised Stolper-Samuelson Proposition can be found in Appendix 7.2 at the end of this chapter.

It must be added that it is the comparative-static results of the *descriptive* Heckscher-Ohlin theory that are vulnerable to costly reallocation: the comparative statics of the *normative* Heckscher-Ohlin theory are not vulnerable. For example, the central gains-from-free-trade proposition is quite independent of costs of reallocation; and the same is true of the Kemp-Wan proposition about customs unions, and of Samuelson's two-countries proposition about the welfare implications of international transfers.[5] However, it is shown elsewhere that the comparative statics of the normative Heckscher-Ohlin theory are vulnerable to the relaxation of the popular representative-agent assumption; see Kemp and Shimomura (2002c).

## 7.4 Final remark

We have shown that the leading comparative-static propositions of the descriptive Heckscher-Ohlin theory survive the recognition of costly reallocation only if they are interpreted as comparative steady-state propositions, and even then only in attenuated form. It will be evident that this lack of robustness is characteristic of all descriptive comparative statics; we have merely illustrated a general failing in terms of one particular model. Thus, we have stumbled upon a general (*non-*)*correspondence* between the comparative statics and dynamics of reallocation. One quite naturally recalls the Correspondence Principle (henceforth, CP) announced by Paul Samuelson (1941); see also Samuelson (1947, especially Part II). However, the old CP and the new non-CP are distant cousins. Thus, Samuelson's dynamics are of the *tâtonnement* variety, absorb no resources, and are not derived from the optimizing choices of individual agents. They yield stability conditions that sometimes sharpen the comparative statics associated with *given* stationary equilibria. In our model, on the other hand, markets always clear, and the dynamics are resource-using and also are rooted in the choices of individual resource-owners. However, the introduction of market-clearing and resource-using dynamics fails to sharpen comparative-static calculations; since the set of stationary equilibria depends on the dynamics, the latter have the effect of blunting the comparative statics. In view of these disparities, the new non-CP seems to be immune to the criticisms of the old CP by, for example, Arrow and Hahn (1971) and Fisher (1983).

## Appendix 7.1 Derivation of phase diagram

Associated with problem (4) is the current-value Hamiltonian

$$H \equiv u\,[Y\,(p, L_1, T_1) - \alpha\,(p)(|\dot{L}_1|+|\dot{T}_1|),\, p] \;+\; \mu_L \dot{L}_1 + \mu_T \dot{T} \qquad \text{(A1)}$$

Along an optimal trajectory, the Hamiltonian is maximized for given state and co-state variables, $(L_1, T_1)$ and $(\mu_L, \mu_T)$, respectively. To help characterize the optimal trajectory, we offer several preliminary lemmas.

*Lemma 7.1* Along any solution path, and for $X = L, T$, if $\mu_X \geq 0$ ($\mu_X \leq 0$) then $\dot{X}_1 \geq 0$ ($\dot{X}_1 \leq 0$).

*Proof* If $\dot{L}_1 \geq 0$ along the solution path then

$$\frac{\partial H}{\partial \dot{L}_1} = -\alpha(p)u'\Big[Y(p, L_1, T_1) - \alpha(p)\big(|\dot{L}_1| + |\dot{T}_1|\big), p\Big] + \mu_L = 0$$

or $\mu_L = \alpha u' > 0$; that is, if $\dot{L}_1 > 0$, then $\mu_L > 0$ ($u'$ denotes the partial derivative of $u$ with respect to the income term). Similarly, if $\dot{L}_1 < 0$ along the optimal path then

$$\frac{\partial H}{\partial \dot{L}_1} = \alpha(p)u'\left[Y(p, L_1, T_1) - \alpha(p)\left(-\dot{L}_1 + \left|\dot{T}_1\right|\right), p\right] + \mu_L = 0$$

or $\mu_L = -\alpha u' < 0$; that is, if $\dot{L}_1 < 0$, then $\mu_L < 0$. Finally, at $\dot{L}_1 = 0$ the costs of reallocation, and therefore $H$, are not differentiable with respect to $\dot{L}_1$. For each $L_1$, therefore, there is an interval containing positive and negative $\mu_L$-values such that $\dot{L}_1 = 0$.

A parallel argument can be made for $\dot{T}_1$.

*Lemma 7.2* Along any solution path, and for $\{X, Z\} = \{L, T\}$, $X \neq Z$, if $|\mu_X| > |\mu_Z|$ then $\dot{Z}_1 = 0$.

*Proof* If $\mu_X > \mu_Z \geq 0$, then from Lemma 7.1, both $\dot{X}_1$ and $\dot{Z}_1$ are non-negative. Hence[6]

$$0 \geq \frac{\partial H}{\partial \dot{X}_1} = -\alpha u' + \mu_X > -\alpha u' + \mu_Z = \lim_{\dot{Z}_1 \to +0} \frac{\partial H}{\partial \dot{Z}_1},$$

which implies that $\dot{Z}_1 = 0$. Similarly, if $0 \geq \mu_Z > \mu_X$, then, from Lemma 7.1, both $\dot{X}_1$ and $\dot{Z}_1$ are non-positive. Hence

$$\frac{\partial H}{\partial \dot{X}_1} = \alpha u' + \mu_X < \alpha u' + \mu_Z = \lim_{\dot{Z}_1 \to -0} \frac{\partial H}{\partial \dot{Z}_1} \leq 0,$$

which implies that $\dot{Z}_1 = 0$. Next, if $\mu_X \geq 0 \geq \mu_Z$ and $|\mu_X| > |\mu_Z|$, then, again from Lemma 7.1, $\dot{X}_1 \geq 0$ and $\dot{Z}_1 \leq 0$. Hence

$$0 \geq \frac{\partial H}{\partial \dot{X}_1} = -\alpha u' + \mu_X > -\alpha u' - \mu_Z = \lim_{\dot{Z}_1 \to -0} \frac{\partial H}{\partial \dot{Z}_1},$$

which implies that $\dot{Z}_1 = 0$. Finally, if $\mu_X \leq 0 \leq \mu_Z$ and $|\mu_X| > |\mu_Z|$, then, from Lemma 7.1, $\dot{X}_1 \leq 0$ and $\dot{Z}_1 \geq 0$. Hence

$$\frac{\partial H}{\partial \dot{Z}_1} = -\alpha u' - \mu_Z < -\alpha u' - \mu_X = \lim_{\dot{Z}_1 \to 0} \frac{\partial H}{\partial \dot{Z}_1}$$

which implies that $\dot{Z}_1 = 0$.

*Lemma 7.3* Along any solution path, and for $X = L, T$, if $|\mu_X| < \alpha(p)\, u'$ $[Y(p, L_1, T_1)p]$ then $|\dot{X}_1| = 0$.

*Proof* Suppose that $\dot{L}_1 > 0$. Then, from the assumed inequality and since $u'' < 0$,

$$\frac{\partial H}{\partial \dot{L}_1} = -\alpha(p)u'\left[Y(p, L_1, T_1) - \alpha(p)\left(\dot{L}_1 + |\dot{T}_1|\right), p\right] + \mu_L$$
$$< -\alpha(p)u'\left[Y(p, L_1, T_1), p\right] + \mu_L$$
$$\leq 0,$$

a contradiction. Similarly, if $\dot{L}_1 < 0$, then, from the assumed inequality, and since $u'' < 0$,

$$\frac{\partial H}{\partial \dot{L}_1} = \alpha(p)u'\left[Y(p, L_1, T_1) - \alpha(p)\left(-\dot{L}_1 + |\dot{T}_1|\right), p\right] + \mu_L$$
$$> \alpha(p)u'\left[Y(p, L_1, T_1), p\right] + \mu_L$$
$$\geq 0,$$

another contradiction.

*Lemma 7.4* Along any solution path, and for $\{X, Z\} = \{L, T\}$, $X \neq Z$,

(a) if $|\mu_X| > |\mu_Z|$ and $|\mu_X| > \alpha(p)u'[Y(p, L_1, T_1), p]$, then, uniquely,

$$\dot{X}_1 = \begin{cases} \left[Y(p, L_1, T_1) - \beta\left(\mu_X / \alpha(p), p\right)\right] / \alpha(p) & \text{if } \mu_X > 0 \\ -\left[Y(p, L_1, T_1) + \beta\left(\mu_X / \alpha(p), p\right)\right] / \alpha(p) & \text{if } \mu_X < 0 \end{cases}$$

(b) if $|\mu_X| = |\mu_Z|$ then

$$|\dot{X}_1| + |\dot{Z}_1| = \left[Y(p, L_1, T_1) - \beta\left(|\mu_X| / \alpha(p), p\right)\right] / \alpha(p),$$

where $\beta(\cdot)$ is the inverse function of $u'(\cdot)$ with respect to household income.

*Proof* This proposition follows readily from the first-order conditions, the two suppositions, Lemma 7.2, the concavity of u and the Inada conditions.

Turning to the co-state variables, we recall that, along any optimal trajectory,

$$\dot{\mu}_X = \rho\mu_X - u'Y_X \tag{A2}$$

From the proof of Lemma 7.1, if $|\dot{X}_1| > 0$ then $\alpha(p)u' = |\mu_X|$. Substituting in (A2),

$$\dot{\mu}_X = \begin{cases} \mu_X\left[\rho - \dfrac{1}{\alpha(p)}\dfrac{\partial Y}{\partial X_1}\right] & \text{if } \mu_X > 0 \\ \mu_X\left[\rho + \dfrac{1}{\alpha(p)}\dfrac{\partial Y}{\partial X_1}\right] & \text{if } \mu_X < 0 \end{cases}$$

That concludes our discussion of the necessary conditions of optimality. We next note that, in view of (A1), the maximized Hamiltonian is strictly concave with respect to the state and co-state variables $X$ and $\mu_X$, $X = L, T$. Hence we may rely on Arrow's sufficiency condition. Any trajectory that satisfies the foregoing necessary conditions, as well as the transversality and initial conditions, is an optimal trajectory; moreover, it is the unique solution for those initial conditions.

We can now return to Figure 7.2 and begin the construction of the phase diagram of our dynamic system. Let us first examine the case in which the initial point $(L_1(0), T_1(0))$ lies below the side $HJ$ of the diamond, in the shaded region $aHJb$ of Figure 7.4. The relevant system of differential equations is

$$\dot{T}_1 = \frac{1}{\alpha(p)}\left[Y\left(p, L_1(0), T_1\right) - \beta\left(\mu_T / \alpha(p), p\right)\right], \tag{A3a}$$

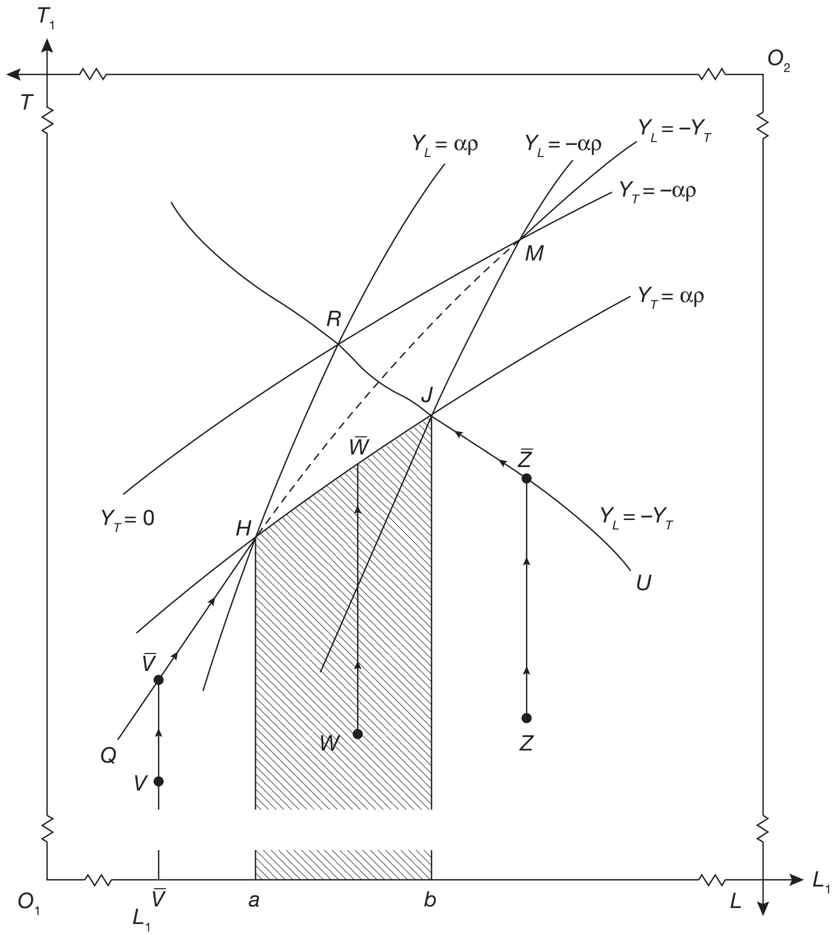


*Figure 7.4*

$$\dot{\mu}_T = \mu_T \left[ \rho - \frac{1}{\alpha(p)} Y_T \left( L_1(0), T_1 \right) \right], \tag{A3b}$$

$$\dot{\mu}_L = \rho\mu_L - \frac{1}{\alpha(p)} \mu_T Y_L \left( L_1(0), T_1 \right). \tag{A3c}$$

By routine calculations, the system can be shown to possess a unique and saddlepoint stable stationary state. Indeed inspection of (A3) reveals that the first two members determine the movement of $\mu_T$ and $T_1$ and that the third member then determines the movement of $\mu_L$. Thus, the U-shaped curve in Figure 7.5 is the graph of

$$Y(p, L_1(0), T_1) - \beta(\mu_T / \alpha(p), p) = 0$$

and the vertical line is the graph of

$$\rho\alpha(p) - Y_T(L_1(0), T_1) = 0.$$

The arrows indicate the stable arm. Thus, if the initial allocation is represented by point *W* in Figure 7.4, then there is an appropriate $(\mu_L(0), \mu_T(0))$

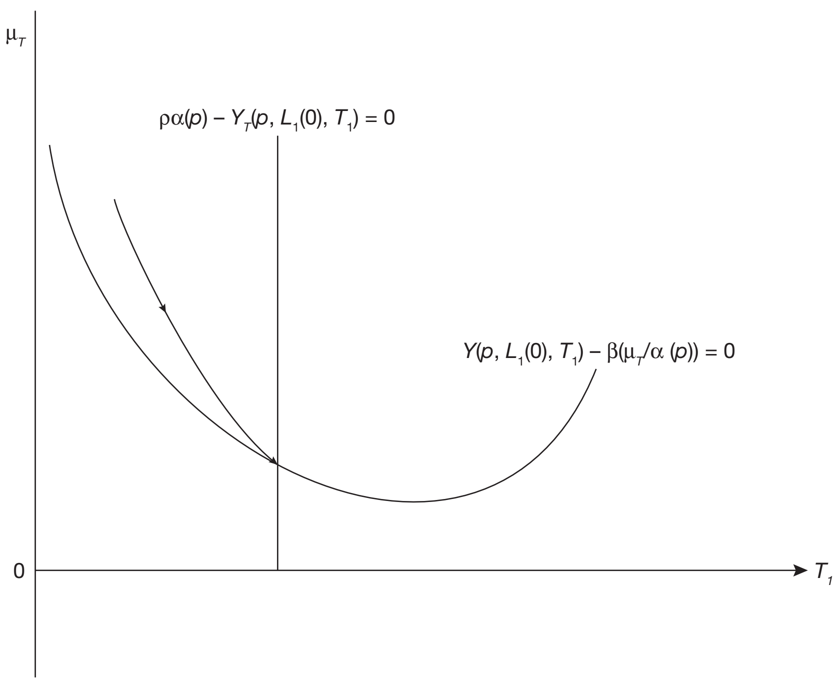


*Figure 7.5*

such that the system asymptotically converges to the stationary state ($L_1(0)$, $\bar{T}_1$, $\bar{\mu}_T$, $\bar{\mu}_L$), represented by $\overline{W}$.

To establish the optimality of the convergent path, it now suffices to show that, everywhere along that path, $|\mu_L| < |\mu_T|$. First, from (A3b) and (A3c), at the stationary state $\bar{\mu}_L/\bar{\mu}_T = Y_L/Y_T$. On the other hand, from Figure 7.2 it is clear that, on the segment *HJ* (but not at the endpoints), $0 < |Y_L| < \rho\alpha(p)$ and $Y_T = \rho\alpha(p)$. Hence $|\bar{\mu}_L| < |\bar{\mu}_T|$. Second, suppose that, somewhere on the trajectory, $|\bar{\mu}_L|$ happens to be equal to $|\bar{\mu}_T|$. Now consider the expression

$$Y_T - (\mu_L / \mu_T)Y_L, \tag{A4}$$

and recall that, along the trajectory, $\mu_T > 0$. If $\mu_L = \mu_T$ then expression (A4) reduces to $Y_T - Y_L$ which, again from Figure 7.2, is positive; and if $\mu_L < 0$ then (A4) reduces to $Y_T + Y_L$ which, again from Figure 7.2, is positive. It then follows from (A3b) and (A3c) that

$$\frac{\dot{\mu}_L}{\mu_L} - \frac{\dot{\mu}_T}{\mu_T} = Y_T - \frac{\mu_L}{\mu_T} Y_L > 0, \tag{A5}$$

implying that, thereafter, $|\mu_L| > |\mu_T|$, a contradiction. It follows that, everywhere along the trajectory, $|\mu_L| < |\mu_T|$ , and we may be sure that, if the initial point lies in the region *aHJb* of Figure 7.4, then the stable arm is the optimal trajectory from the initial point.

Let us next examine the case in which the initial allocation lies in the region *QHa* of Figure 7.4. Suppose that the system of differential equations is

$$\dot{T}_1 = \left[1/\alpha(p)\right]\left[Y\left(p, L_1^V, T_1\right) - \beta\left(\mu_T / \alpha(p), p\right)\right], \tag{A6a}$$

$$\dot{\mu}_T = \mu_T\left[\rho - \left(1/\alpha(p)\right)Y_L\left(p, L_1^V, T_1\right)\right], \tag{A6b}$$

$$\dot{u}_L = \rho\mu_L - \left(1/\alpha(p)\right)\mu_T Y_L\left(p, L_1^V, T_1\right). \tag{A6c}$$

Evidently, the two systems (A3) and (A6) differ only in the initial allocation. The arrowed solution path beginning at $V$ eventually reaches *QH* at ($L_1^V = L_1^{\bar{V}}, T_1$), represented by point $\bar{V}$, at time $\bar{t}$. Evidently $\bar{V}$ is saddlepoint stable. By familiar reasoning, $|\mu_L| < |\mu_T|$ everywhere along the trajectory $V\bar{V}$. Now let us switch our attention to the system

$$Y_L\left(p, L_1, T_1\right) = Y_T\left(p, L_1, T_1\right), \tag{A7a}$$

$$\dot{L}_1 + \dot{T}_1 = \left[1/\alpha(p)\right]\left[Y\left(p, L_1, T_1\right) - \beta\left(\mu / \alpha(p), p\right)\right], \tag{A7b}$$

$$\dot{\mu} = \mu\left[\rho - Y_L\left(p, L_1, T_1\right)\right], \tag{A7c}$$

$$L_1(\bar{t}) = L_1^V = L_1^{\bar{V}}, \qquad T_1(\bar{t}) = T_1^{\bar{V}}. \tag{A7d}$$

The stationary state of this system is represented by point $H$ in Figure 7.4. In view of the positive slope of $QH$, the system is saddlepoint stable and the arrowed path $V\bar{V}H$ is the stable arm of the saddle. Thus, the kinked trajectory $V\bar{V}H$ is the optimal trajectory.

A parallel argument applies when the initial allocation is represented by a point in the region $bJU$, say $Z$. The optimal trajectory is then the kinked locus $Z\bar{Z}J$. Notice, however, that, along $JU$, the relevant system of differential equations is not (A7) but

$$Y_L(p, L_1, T_1) + Y_T(p, L_1, T_1) = 0 \tag{A8a}$$

$$-\dot{L}_1 + \dot{T}_1 = [1/\alpha(p)][Y(p, L_1, T_1) - \beta(\mu/\alpha(p), p)] \tag{A8b}$$

$$\dot{\mu} = \mu[\rho + Y_L(p, L_1, T_1)] \tag{A8c}$$

$$L_1(\bar{t}) = L_1^Z = L_1^{\bar{Z}}, \qquad T_1(\bar{t}) = T_1^{\bar{Z}}. \tag{A8d}$$

Other regions can be examined in a similar manner. Thus, we eventually arrive at the complete phase diagram, Figure 7.3.

## Appendix 7.2 Proof of revised Stolper-Samuelson proposition

In Figure 7.6, the contract curve of Figure 7.1 is reproduced as $O_1E'EO_2$, with $E$ and $E'$ static equilibrium points compatible with the international prices $p$ and $p + \Delta p$, respectively, where $\Delta p > 0$. $E$ and $E'$ lie in the interior of the diamonds $D(p)$ and $D(p + \Delta p)$ associated with $p$ and $p + \Delta p$.

Our first objective is to impose restrictions on $\Delta p$ and the rate of time preference $\rho$ such that each point in $D(p)$ lies north-east of every point in $D(p + \Delta p)$. In achieving that intermediate objective, the following lemma will be useful:

*Lemma 7.5* The loci $Y_L|_p = 0$ and $Y_L|_{p+\Delta p} = 0$ fail to intersect in Figure 7.6, and the same is true of the loci $Y_T|_p = 0$ and $Y_T|_{p+\Delta p} = 0$.

*Proof* Consider the locus

$$Y_L = Y_L^1(L_1, T_1) - pY_L^2(L - L_1, T - T_1) = 0.$$

Totally differentiating with respect to $T_1$ and $p$, with $L_1$ held constant, we obtain

$$\left(Y_{LT}^1 + pY_{LT}^2\right)\mathrm{d}T_1 - Y_L^2\mathrm{d}p = 0$$

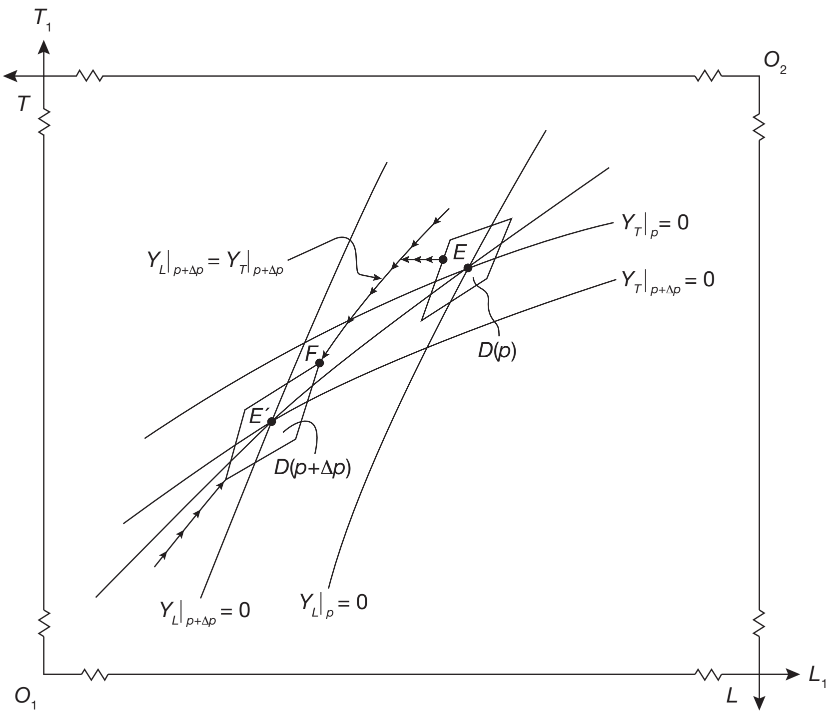


*Figure 7.6*

so that

$$\frac{\mathrm{d}T_1}{\mathrm{d}p} = \frac{Y_L^2}{Y_{LT}^1 + pY_{LT}^2} > 0.$$

Since $\Delta p$ is positive, and since the locus $Y_L = 0$ is positively sloped, the locus $Y_L|_{p+\Delta p} = 0$ lies everywhere to the left of the locus $Y_L|_p = 0$. By a parallel argument, the locus $Y_T|_p = 0$ lies everywhere to the left of the locus $Y_T|_{p+\Delta p} = 0$.

The four loci of Lemma 7.5 ($Y_L|_p = 0$, $Y_L|_{p+\Delta p} = 0$, $Y_T| = 0$ and $Y_T|_{p+\Delta p} = 0$) are independent of the rate of time preference $\rho$. However, this is not true of the loci ($Y_L|_p = \rho\alpha(p)$, $Y_L|_p = -\rho\alpha(p)$, $Y_T|_p = \rho\alpha(p)$ and $Y_T|_p = -\rho\alpha(p)$) that define the diamond $D(p)$; neither is it true of the loci ($Y_L|_{p+\Delta p} = \rho\alpha(p + \Delta p)$, $Y_L|_{p+\Delta p} = -\rho\alpha(p + \Delta p)$, $Y_T|_{p+\Delta p} = \rho\alpha(p + \Delta p)$ and $Y_T|_{p+\Delta p} = -\rho\alpha(p + \Delta p)$) that define the diamond $D(p + \Delta p)$. It follows that, for any positive $\Delta p$ and for sufficiently small $\rho$, (i) the diamonds $D(p)$ and $D(p + \Delta p)$ have no points in common; (ii) the diamond $D(p)$ lies between the loci $Y_L|_{p+\Delta p} = 0$ and $Y_T|_{p+\Delta p} = 0$; and (iii) each point in $D(p)$ lies north-east of every point in $D(p + \Delta p)$. It further follows that, along any optimal trajectory that begins in $D(p)$, the point $(L_1(t), T_1(t))$ initially travels due west until it reaches the locus $Y_L|_{p+\Delta p} = Y_T|_{p+\Delta p}$ and thereafter converges to point $F$ along that locus; see the arrowed trajectory in Figure 7.6.

Point $F$ marks the intersection of the loci $Y_L|_{p+\Delta p} = -\rho\alpha(p + \Delta p)$ and $Y_T|_{p+\Delta p} = -\rho\alpha(p + \Delta p)$. At that point,

$$w \equiv Y_L^1 = \left(p + \Delta p\right) Y_L^2 - \rho\alpha\left(p + \Delta p\right)$$

$$r \equiv Y_T^1 = \left(p + \Delta p\right) Y_T^2 - \rho\alpha\left(p + \Delta p\right)$$

so that

$$1 = c^1(w, r) \tag{B1}$$

$$p + \Delta p = c^2(w + \rho\alpha(p + \Delta p), r + \rho\alpha(p + \Delta p)) \tag{B2}$$

where $c^i$ is the average cost of producing commodity $i$. Returning to the initial equilibrium point in $D(p)$, let us denote by $(\bar{w}(\rho), \bar{r}(\rho))$ the solution to the equations

$$1 = c^1(w, r), \tag{B1}$$

$$p = c^2(w + \rho\alpha(p), r + \rho\alpha(p)). \tag{B3}$$

Then (B1) and (B2) can be rewritten as

$$1 = c^1\left(\bar{w}\left(\rho\right) + \Delta w, \bar{r}\left(\rho\right) + \Delta r\right), \tag{B1$'$}$$

$$p + \Delta p = c^2\left(\bar{w}\left(\rho\right) + \Delta w + \rho\alpha\left(p + \Delta p\right), \bar{r}\left(\rho\right) + \Delta r \rho\alpha\left(p + \Delta p\right)\right). \tag{B2$'$}$$

The unknowns, $\Delta w$ and $\Delta r$, depend on $\rho$ and $\Delta p$: $\Delta w(\rho, \Delta p)$ and $\Delta r(\rho, \Delta p)$. Substituting Taylor's expansions for the right-hand sides of (B1′) and (B2′), we obtain

$$\begin{aligned}
&c^1\left(\bar{w}\left(\rho\right) + \Delta w, \bar{r}\left(\rho\right) + \Delta r\right) \\
&\quad = c^1\left(\bar{w}\left(\rho\right), \bar{r}\left(\rho\right)\right) + \bar{c}_w^1\left(\rho, \Delta p\right)\Delta w\left(\rho, \Delta p\right) + \bar{c}_r^1\left(\rho, \Delta p\right)\Delta r\left(\rho, \Delta p\right) \\
&c^2\left(\bar{w}\left(\rho\right) + \Delta w + \rho\alpha\left(\rho + \Delta p\right), \bar{r}\left(\rho\right) + \Delta r \rho\alpha\left(\rho + \Delta p\right)\right) \\
&\quad = c^2\left(\bar{w}\left(\rho\right) + \rho\alpha\left(\rho\right), \bar{r}\left(\rho\right) + \Delta r \rho\alpha\left(\rho\right)\right) \\
&\quad + \bar{c}_w^2\left(\rho, \Delta p\right)\left[\Delta w\left(\rho, \Delta p\right) + \rho\alpha'\left(\rho + \theta_2 \Delta p\right)\Delta p\right] \\
&\quad + \bar{c}_r^2\left(\rho, \Delta p\right)\left[\Delta r\left(\rho, \Delta p\right) + \rho\alpha'\left(\rho + \theta_2 \Delta p\right)\Delta p\right],
\end{aligned}$$

where

$$\bar{c}_w^1(\rho,\Delta p) \equiv c_w^1\left(\bar{w}(\rho)+\theta_1\Delta w(\rho,\Delta p),\bar{r}(\rho)+\theta_1\Delta r(\rho,\Delta p)\right)$$
$$\bar{c}_r^1(\rho,\Delta p) \equiv c_r^1\left(\bar{w}(\rho)+\theta_1\Delta w(\rho,\Delta p),\bar{r}(\rho)+\theta_1\Delta r(\rho,\Delta p)\right)$$
$$\bar{c}_w^2(\rho,\Delta p) \equiv c_w^2\left(\bar{w}(\rho)+\theta_2\Delta w(\rho,\Delta p)+\rho\alpha(\rho+\theta_2\Delta p),\bar{r}(\rho)+\theta_2\Delta r(\rho,\Delta p)+\rho\alpha(\rho+\theta_2\Delta p)\right)$$
$$\bar{c}_r^2(\rho,\Delta p) \equiv c_r^2\left(\bar{w}(\rho)+\theta_2\Delta w(\rho,\Delta p)+\rho\alpha(\rho+\theta_2\Delta p),\bar{r}(\rho)+\theta_2\Delta r(\rho,\Delta p)+\rho\alpha(\rho+\theta_2\Delta p)\right)$$

and where $0 \le \theta_i \le 1$, $i = 1, 2$. Note that, like $\Delta w$ and $\Delta r$, $\theta_i$ depends on $\rho$ and $\Delta p$.

From (B1′) and (B2′), recalling the definitions of $\bar{w}(\rho)$ and $\bar{r}(\rho)$,

$$\bar{c}_w^1(\rho,\Delta p)\Delta w(\rho,\Delta p)+\bar{c}_r^1(\rho,\Delta p)\Delta r(\rho,\Delta p)=0$$
$$\left[1-\rho\alpha'(p+\theta_2\Delta p)\left\{\bar{c}_w^2(\rho,\Delta p)+\bar{c}_r^2(\rho,\Delta p)\right\}\right]\Delta p$$
$$=\bar{c}_w^2(\rho,\Delta p)\Delta w(\rho,\Delta p)+\bar{c}_r^2(\rho,\Delta p)\Delta r(\rho,\Delta p)$$

Solving for $\Delta w(\rho, \Delta p)$ and $\Delta r(\rho, \Delta p)$, we obtain

$$\frac{p}{\Delta p}\;\frac{\Delta w(\rho,\Delta p)}{\bar{w}(\rho)}=\frac{-\left[1-\rho\alpha'\cdot\left\{\bar{c}_w^2+\bar{c}_r^2\right\}\right]\bar{c}_r^1 p}{\left[\bar{c}_w^1\bar{c}_r^2-\bar{c}_w^2\bar{c}_r^1\right]\bar{w}(\rho)}$$
$$=\frac{-\left[1-(\rho\alpha'/\alpha)\left\{(\rho\alpha/\bar{w})(\overline{wc}_w^2/p)+(\rho\alpha/\bar{r})(\overline{rc}_r^2/p)\right\}\right](\overline{rc}_r^1/1)}{\left[(\bar{c}_w^1\bar{w}/1)(\overline{rc}_r^2/p)-(\overline{rc}_w^2/p)(\overline{wc}_r^1/1)\right]} \tag{B4}$$

$$\frac{p}{\Delta p}\frac{\Delta r(\rho,\Delta p)}{\bar{r}(\rho)}=\frac{-\left[1-\rho\alpha'\cdot\left\{\bar{c}_w^2+\bar{c}_r^2\right\}\right]\bar{c}_w^1 p}{\left[\bar{c}_w^1\bar{c}_r^2-\bar{c}_w^2\bar{c}_r^1\right]\bar{r}(\rho)}$$
$$=\frac{-\left[1-(\rho\alpha'/\alpha)\left\{(\rho\alpha/\bar{w})(\overline{wc}_w^2/p)+(\rho\alpha/\bar{r})(\overline{rc}_r^2/p)\right\}\right](\overline{wc}_w^1)}{\left[(\bar{c}_w^1\bar{w}/1)(\overline{rc}_r^2/p)-(\overline{rc}_w^2/p)(\overline{wc}_r^1/1)\right]} \tag{B5}$$

Now as $(\rho, \Delta p) \to (0, 0)$,

$$\bar{c}_w^1(\rho,\Delta p)\bar{w}(\rho)/1,\quad \bar{c}_r^2(\rho,\Delta p)\bar{r}(\rho)/p,$$
$$\bar{c}_w^2(\rho,\Delta p)\bar{w}(\rho)/p,\quad \bar{c}_r^1(\rho,\Delta p)\bar{r}(\rho)/1,$$

converge to

$$\begin{aligned}&\bar{c}_w^1(\bar{w}(0),\bar{r}(0))\bar{w}(0)/1,\quad \bar{c}_r^2(\bar{w}(0),\bar{r}(0))\bar{r}(0)/p,\\ &\bar{c}_w^2(\bar{w}(0),\bar{r}(0))\bar{w}(0)/p,\quad \bar{c}_r^1(\bar{w}(0),\bar{r}(0))\bar{r}(0)/1,\end{aligned} \tag{B6}$$

respectively. Since (B6) is the set of distributive shares at the static equilibrium point *E* in Figure 7.6, and since the square-bracketed terms in the numerators of (B4) and (B5) converge to 1 as $(\rho, \Delta p) \to (0, 0)$, (B4) and (B5) imply that

$$\lim_{(\rho,\Delta p)\to(0,0)} \frac{p}{\Delta p}\frac{\Delta w(\rho,\Delta p)}{\bar{w}(\rho)} \quad \text{and} \quad \lim_{(\rho,\Delta p)\to(0,0)} \frac{p}{\Delta p}\frac{\Delta r(\rho,\Delta p)}{\bar{r}(\rho)}$$

follow the Stolper-Samuelson patterns. For example, if the second good is relatively labour-intensive, then

$$1 < \lim_{(\rho,\Delta p)\to(0,0)} \frac{p}{\Delta p}\frac{\Delta w(\rho,\Delta p)}{\bar{w}(\rho)} \quad \text{and} \quad 0 > \lim_{(\rho,\Delta p)\to(0,0)} \frac{p}{\Delta p}\frac{\Delta r(\rho,\Delta p)}{\bar{r}(\rho)}$$

It then follows from the continuity properties of (B4) and (B5) that there is a small neighbourhood of the origin (0, 0) in $(\rho, \Delta p)$-space, say $V(0, 0)$, such that, for any $(\rho, \Delta p)$ in $V(0, 0)$,

$$1 < \frac{p}{\Delta p}\frac{\Delta w(\rho,\Delta p)}{\bar{w}(\rho)} \quad \text{and} \quad 0 > \frac{p}{\Delta p}\frac{\Delta r(\rho,\Delta p)}{\bar{r}(\rho)}$$

By way of illustration, consider the parameter space depicted by Figure 7.7. The dashed curve is the boundary of the neighbourhood $V(0, 0)$; and below the locus *OA* conditions (i) to (iii) are satisfied. Thus, the Stolper-Samuelson patterns prevail for any parameter pair $(\rho, \Delta p)$ in the shaded region.

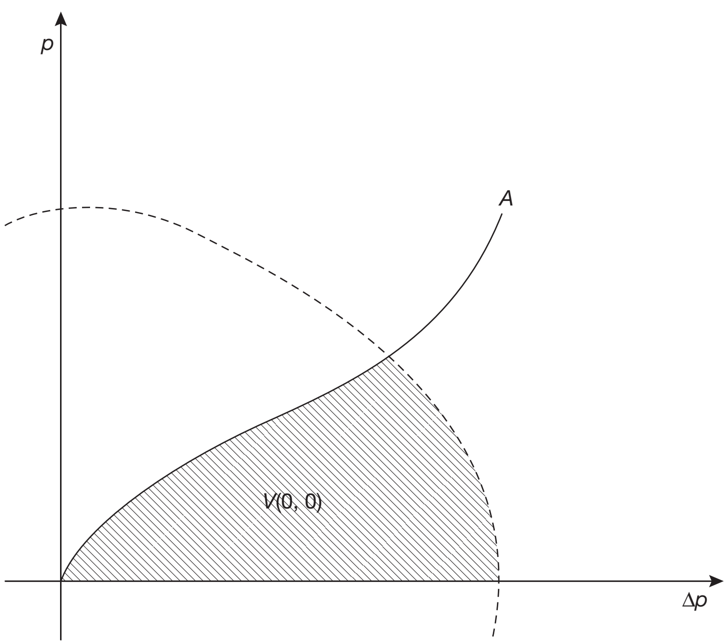


*Figure 7.7*

# 8 A second correspondence principle

Paul Samuelson's Correspondence Principle (henceforth CP) first appeared in a pair of wartime articles; see Samuelson (1941: 97–120 and 1942: 1–25). The CP asserts that economic dynamics and comparative statics stand in a two-way relationship of mutual support and dependence. In particular, the precision of comparative statics may be enhanced by imposing the assumption of dynamic stability.

Since its appearance sixty years ago, the CP has been widely accepted, especially by economists who work primarily with the Heckscher-Ohlin or other two-by-two models. However, the CP has been subjected to criticism, principally on two grounds:

1 For systems only slightly larger than two-by-two, the assumption of dynamic stability typically adds little to the precision of comparative statics.
2 The dynamic models in terms of which Samuelson illustrated the potential usefulness of the CP were of the *tâtonnement* variety; and nearly all later applications of the CP have been in the context of a *tâtonnement* process. In such models, costless adjustment takes place at a finite pace. However, if adjustment is costless, then, given any finite speed of adjustment, there is a profit or utility incentive to increase the speed. In short, *tâtonnement* models are internally inconsistent. That this was not recognized long ago is a bit of a puzzle, best explained perhaps in terms of our failure to ask whose maximizing behaviour is being modelled.

The first of these two criticisms has long been recognized; but we have learned to live with it. The second criticism is more fundamental. What can be done about it? Recently, Kemp and Shimomura (2003a) have examined a possible escape route. Specifically, they have proposed that *tâtonnement* models be abandoned in favour of models in which markets always clear and in which the dynamics are both resource using and rooted in the choices of resource owners.

In Appendix 8.1, we present and examine a modified version of Kemp and Shimomura (2003a), in which the effect of learning associated with the movement of factors is explicitly recognized. Naturally, the modified version relaxes but cannot extinguish the destructiveness to Samuelson's CP, which the modelling changes introduced by Kemp and Shimomura (2003a) brought about. Let us explain.

Consider an arbitrary initial allocation of resources to industries. The rewards earned by any particular factor may differ from industry to industry. Indeed they may differ by more than the marginal cost of moving the factor. Hence there may be incentives for the owners of the factor to move it from one industry to another. Each owner of a factor must consider the control problem of choosing the optimal employment path for each factor owned, given the paths of expected factor rewards in each industry and given the (positive) marginal costs of moving factors. Over time, the inter-industrial earnings disparities may increase or decrease; but, whatever happens, the disparities will remain. In fact, the phase diagram (Figure 8.3) shows that any optimal trajectory ceases its motion at a certain point on the corridor $H'J'M'R'$ surrounding the point $E$ where the disparities vanish or at one of the points $A$, $B$, $C$, and $D$.

On the other hand, all of the well-known general equilibrium comparative static propositions (for example, the Stolper-Samuelson, Rybczynski, Heckscher-Ohlin, Factor Price Equalization and Hicks-Ikema propositions) rely on each factor earning the same reward in all industries, both before and after the relevant disturbance. Hence none of those propositions survives the replacement of *tâtonnement* by market-clearing, resource-using dynamic processes. This is the second CP, announced in our title. Perhaps it would be as appropriately called the non-CP.

Whereas Samuelson's CP was put forward in a spirit of constructive optimism about the possibility of deriving broad comparative-static results from purely theoretical considerations, the second CP is compatible only with a profound pessimism. The second CP does not signal the end of general equilibrium comparative statics. Indeed it never denies the idea, immanent in Samuelson's CP, that the thorough analysis of dynamic or transitional processes can help establish meaningful comparative statics. However, it does signal the end of traditional general equilibrium comparative statics, which rest on theoretical specifications of the Heckscher-Ohlin type.

Having admitted the second CP, we must be satisfied with fairly attenuated versions of the traditional comparative-static propositions. For example, with the aid of the observation that the corridor (or diamond) $H'J'M'R'$ is narrower than the Kemp-Shimomura corridor (or diamond) $HJMR$ obtained by putting $\gamma = 0$ (see Figure 8.3), the revised Stolper-Samuelson Proposition of Kemp and Shimomura (2003a) is slightly relaxed to read as follows:

*Re-revised Stolper-Samuelson Proposition* Given any constant-returns production functions and any initial and final values of the commodity price ratio $p$, say $p'$ and $p''$, and a positive rate $\lambda$ of technical progress in the technology associated with the movement of factors, there exists a sufficiently small but positive rate of time preference, say $\rho(p', p'', \gamma)$ such that, for all $\rho \leq \rho(p', p'', \gamma)$ and comparing the initial and final steady states, the Stolper-Samuelson conclusions remain intact. Moreover, the larger the value of $\gamma$, the larger the value of $\rho(p', p'', \gamma)$.

The remaining comparative static propositions listed above may be similarly attenuated.

We bring this short chapter to an end by arguing the robustness of the second CP. A close scrutiny of the analysis of the appendices reveals that, if and only if (a) the cost of moving each factor can be extinguished by learning and (b) the process of learning lasts long enough to achieve that end, the destructive conclusions of the second CP would need to be abandoned. However, assumption (a) is completely implausible; and in many plausible situations assumption (b) would not be satisfied. Thus the second CP is intrinsically robust.

## Appendix 8.1

In this appendix we present a modified version of Kemp and Shimomura (2003a), which is henceforth abbreviated as the K-S paper, and construct the phase diagram, making use of the lemmas stated and proved in Appendix 8.2.

### *The modified K-S model*

Consider a small and fully employed open economy, which potentially produces two final goods (labelled 1 and 2) with the aid of two factors of production (labour and land, available in amounts $L$ and $T$). The short-run gross national product (GNP) function for the economy is defined as

$$Y(p, L_1, T_1) = Y^1(L_1, T_1) + pY^2(L-L_1, T-T_1), \tag{8.1}$$

where $p$ is the given and constant relative price of the second commodity and $Y^i$ is the output of industry $i$, $i = 1, 2$. It is assumed that each production function is increasing, strictly quasi-concave, homogeneous of degree one in the two factor inputs, and satisfies the Inada conditions.

To move a factor of production from one industry to the other requires inputs of commodities 1 and 2. The production functions are defined by

$$\left|\dot{X}_1\right| = G\left(y_{1X}, y_{2X}, \tilde{I}(t)\right), \quad X \equiv L, T,$$

where $\dot{X}_1$ is the (positive or negative) rate of re-allocation of factor $X$ from industry 2 to industry 1, $y_{iX}$ is the amount of commodity $i$ ($i = 1, 2$) employed in reallocating factor $X$ and production function $G(.)$ is supposed to satisfy the same conditions as the production function $Y^i(.)$ with respect to $y_{1X}$ and $y_{2X}$ and to be increasing in the indicator $\tilde{I}(t)$ of learning by watching.

A rational owner of a factor of production would naturally take into account the observations cumulated from the outset of moving the factor. $\tilde{I}(t)$ is, so to speak, a measure of the cumulated observations. In what follows, we assume that

$$\tilde{I}(t) \equiv \exp\left[\gamma \int_0^t \left[\left|\dot{L}_1(s)\right| + \left|\dot{T}_1(s)\right|\right] ds\right],$$

where $\gamma$ is a given positive constant. We further assume that the cumulated observations work like Hicksian neutral technical progress. Thus, the production functions related to the movement of factors take the form

$$\left|\dot{X}_1\right| = \tilde{I}(t) G\left(y_{1X}, y_{2X}\right), \quad X = L, T. \tag{8.2}$$

Given (8.2), the minimum cost, in terms of the first commodity, of a pair of inter-industrial factor movements ($\dot{L}_1$, $\dot{T}_1$) is

$$\alpha(p) I(t)\left[\left|\dot{L}_1\right| + \left|\dot{T}_1\right|\right], \tag{8.3}$$

where $I(t)$ denotes the reciprocal of $\tilde{I}(t)$.

All households are identical, that is, at all times they have the same homothetic preferences, own the same amounts of the two factors of production, and allocate those amounts in the same way. Bearing in mind these assumptions, the competitive market may be viewed as solving the control problem

$$\max_{L_1, T_1} \int_0^\infty \frac{\exp[-\rho t]}{e(p)} u\left[Y\left(p, L_1, T_1\right) - \alpha(p) I(t)\left(|\ell| + |h|\right)\right] dt \tag{8.4}$$

subject to the constraints

$$\dot{L}_1 = l, \ \dot{T}_1 = h,$$
$$0 \leq T_1 \leq T, \ 0 \leq L_1 \leq L,$$
$$L_1(0), \ T_1(0) \text{ given,}$$

where $e(p)$ is the unit expenditure function, $u(.)/e(p)$ is the indirect utility function and $\rho$ is the given, constant and positive rate of time preference.[1] It is assumed that $u(.)$ is increasing and strictly concave and that it satisfies the Inada conditions.

Normalizing $e(p)$ to be unity for simplicity, associated with (8.4) is the Hamiltonian

$$H \equiv u\left[Y\left(p, L_1, T_1\right) - I\alpha(p)\left(|\ell| + |h|\right)\right] + \mu_L \ell + \mu_T h. \tag{8.5}$$

***The optimal trajectories***

Consider Figure 8.1, which is the same as Figure 2 in the K-S paper. As Figure 8.1 shows, we can divide the Edgeworth box diagram into four sub-regions. Let us concentrate on the case in which the initial condition $(L_1(0), T_1(0))$ is in Region I. The other three cases can be treated in a similar manner.

Before actually deriving the optimal trajectory, let us explain the principle involved in the derivation. First, considering the assumed properties of the functions $Y(\cdot)$ and $u(\cdot)$ and the definition of $|\,|$, it is clear that the Hamiltonian (8.5) is *strictly* concave in state variables $(L_1, T_1)$ and instrument variables $(l, h)$. Therefore, by virtue of Mangasarian's sufficiency theorem for the *unique* existence of an optimal trajectory, we can be sure that any trajectory $(L_1(t), T_1(t))$ that (i) starts from the initial condition, (ii) satisfies the first-order conditions for maximizing the Hamiltonian, (iii) involves the co-state variables each of which satisfies the solution of the differential equation

$$\dot{\mu}_{X_1} = \rho\mu_{X_1} - \frac{\partial H}{\partial X_1}, \;\; X = L, T,$$

and (iv) satisfies the transversality condition, is optimal. If the state variables converge to constant values and the growth rate of each co-state variable is less than $\rho$ then the transversality condition must be satisfied. Hence, our aim is to obtain such a trajectory.

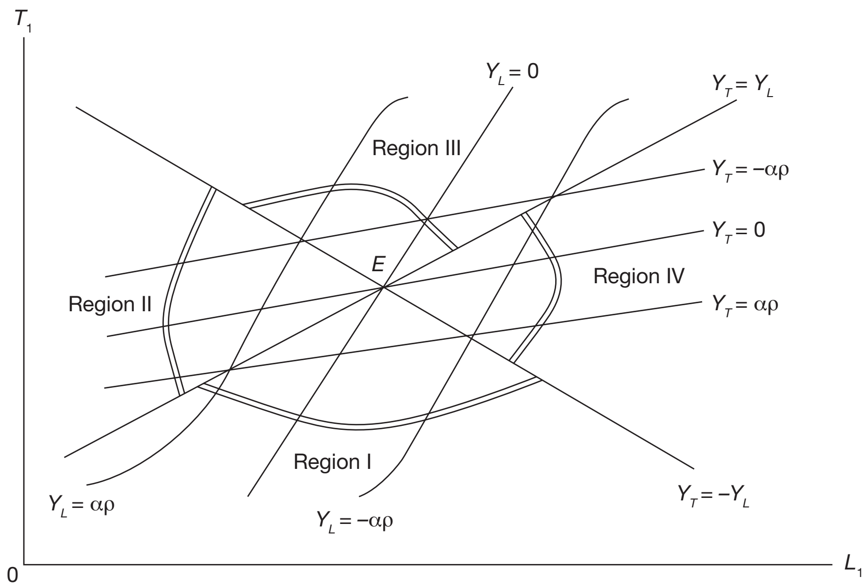


*Figure 8.1*

Now let us consider the case in which the initial condition is not only in Region I but also below the locus $Y_T = \alpha\rho$. Choose $\mu_X(0)$, $X = L$, $T$, such that

$$\mu_T(0) > \alpha(p)u'\left(Y\left(L_1(0), T(0)\right)\right) > \mu_L(0) > 0.$$

Then, the pair $(l(t), h(t))$, which satisfies

$$\mu_T(0) = u'\left(Y\left(L_1(0), T_1(0)\right) - \alpha(p)h(0)\right) \text{ and } \ell(0) = 0,$$

maximizes the Hamiltonian at $t = 0$, where $h(0) > 0$, due to the strict concavity of the utility function. Even for $t > 0$, as far as

$$\mu_T(t) > \alpha(p)I(t)u'\left(Y\left(L_1(t), T_1(t)\right)\right) > \mu_L(t) > 0,$$

the pair $(l(t), h(t))$ satisfying

$$\begin{aligned} &\mu_T(t) = \alpha(p)I(t)u'\left(Y\left(L_1(t), T_1(t)\right) - \alpha(p)I(t)h(t)\right) \\ &\ell(t) = 0 \end{aligned} \tag{8.6}$$

maximizes the Hamiltonian at $t > 0$. It follows that $L_1(t) = L_1(0)$. Note also that the co-state variables are the solutions of the differential equations

$$\begin{aligned} \dot{\mu}_T(t) &= \rho\mu_T(t) - u'\left[Y\left(L_1(t), T_1(t)\right) - \alpha(p)I(t)h(t)\right]Y_T\left(L_1(0), T_1(t)\right) \\ &= \mu_T(t)\left[\rho - \frac{Y_T\left(L_1(0), T_1(t)\right)}{\alpha(p)I(t)}\right] \end{aligned} \tag{8.7}$$

$$\begin{aligned} \dot{\mu}_L(t) &= \rho\mu_L(t) - u'\left[Y\left(L_1(0), T_1(t)\right) - \alpha(p)I(t)h(t)\right]Y_L\left(L_1(0), T_1(t)\right) \\ &= \mu_L(t)\left[\rho - \frac{\mu_T(t)Y_t\left(L_1(0), T_1(t)\right)}{\mu_L(t)\alpha(p)I(t)}\right]. \end{aligned} \tag{8.8}$$

Note finally that $Y_T > \alpha I(t)\,\rho$ below the locus $Y_T = \alpha I(t)\,\rho$. Hence, it follows from (8.7) that below the locus $\dot{\mu}_T(t)/\mu_T(t) < 0$. The trajectory can be depicted like the vertical arrow $ll'$ in Figure 8.2.

The present analysis differs from the K-S paper in that the locus $Y_T = \alpha I(t)\rho$ itself gradually shifts toward the locus $Y_T = 0$ since, from the definition of $I(t)$,

$$I(t) = \exp\left[-\gamma\int_0^t \dot{T}(s)\,ds\right] = \exp\left[-\gamma\left\{T_1(t) - T_1(0)\right\}\right] \tag{8.9}$$

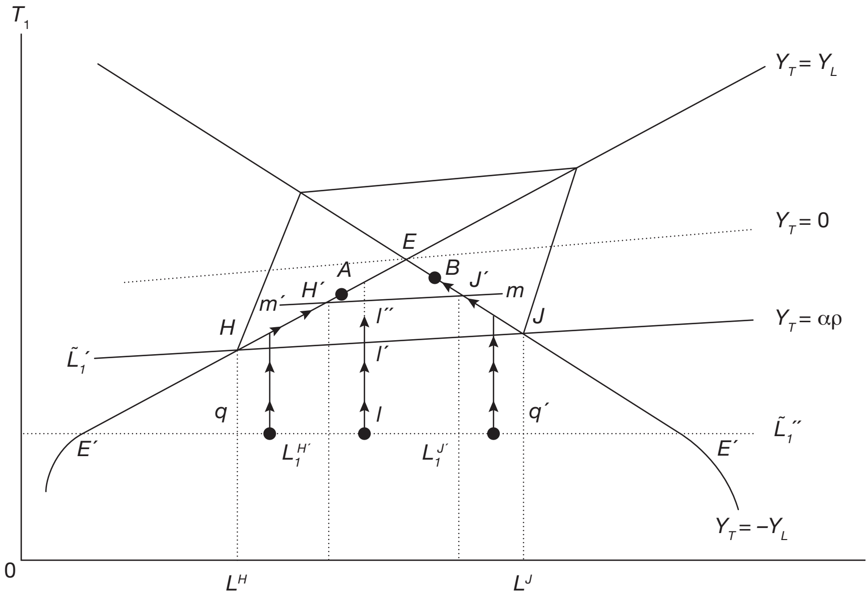


*Figure 8.2*

Substituting (8.9) into (8.6), (8.7), and (8.8) we find that

$$\dot{T}_1 = \frac{\exp\left[\gamma\left(T_1 - T_1(0)\right)\right]}{\alpha}\left[Y\left(T_1, L_1(0)\right) - \beta\left(\frac{\mu_T \exp\left[\gamma\left(T_1 - T_1(0)\right)\right]}{\alpha}\right)\right] \tag{8.10}$$

$$\dot{\mu}_T = \mu_T\left[\rho - \frac{Y_T\left(T_1, L_1(0)\right)\exp\left[\gamma\left(T_1 - T_1(0)\right)\right]}{\alpha}\right] \tag{8.11}$$

$$\dot{\mu}_L = \rho\mu_L - \frac{\mu_T Y_L\left(T_1, L_1(0)\right)\exp\left[\gamma\left(T_1 - T_1(0)\right)\right]}{\alpha} \tag{8.12}$$

where $\beta(\cdot) \equiv (\mu')^{-1}$. The differential equations (8.10) through (8.12) constitute the system on which our argument is based.

Notice that for any given $L_1(0)$ between $L_1^{H'}$ and $L_1^{J'}$, there exist $\bar{T}_1$ and $\tilde{T}_1$ which satisfy

$$\alpha\rho = Y_T\left(L_1(0), \bar{T}_1\right)$$
$$0 = Y_T\left(L_1(0), \tilde{T}_1\right).$$

Evidently, $\tilde{T}_1 > \bar{T}_1$ (since $Y_{TT} < 0$). Further define $\Gamma(L_1(0), T_1)$ as

$$\rho - \frac{1}{\alpha} Y_T\left(L_1(0), T_1\right) \exp\left[\gamma\left(T_1 - T_1(0)\right)\right].$$

Then, it is immediate that for $T_1(0)$, which is smaller than $\bar{T}_1$,

$$\begin{aligned} &\Gamma\left(L_1(0), \bar{T}_1\right) = 1 - \exp\left[\gamma\left(\bar{T}_1 - T_1(0)\right)\right] < 0 \\ &\Gamma\left(L_1(0), \bar{T}_1\right) = \rho > 0. \end{aligned} \tag{8.13}$$

Hence, the continuity of $\Gamma(\cdot)$ ensures that the equation

$$\Gamma(L_1(0), T_1) = 0 \tag{8.14}$$

has a solution $T_1^S$. Since $\Gamma(\cdot)$ is not necessarily increasing in $T_1$, $T_1^S$ may be multiple. In such a case, it suffices to define $T_1^S$ as the minimum of the solutions to (8.14). Since $Y_{TT} < 0$ and $Y_{TL} > 0$, both $\bar{T}_1$ and $\tilde{T}_1$ are increasing in $L_1(0)$. Moreover, a direct computation, coupled with the facts that $Y_{TL} > 0$ and that $\dot{T}_1 > 0$ (or $T_1 > T_1(0)$), yields

$$\Gamma\left(L_1(0), T_1\right) - \Gamma\left(L_1'(0), T_1\right) = \frac{\exp\left[\gamma\left(T_1 - T_1(0)\right)\right]}{\alpha}\left[Y_T\left(L_1'(0), T_1\right) - Y_T\left(L_1(0), T_1\right)\right] > 0,$$

provided that $L_1'(0) > L_1(0)$. Therefore, the locus $(L_1(0), T_1^S)$, or the line $m'H'J'm$ of Figure 8.2 is positively sloped or, algebraically, $Y_{TT} + Y_T < 0$ along the locus.

Once we derive the locus to which the optimal trajectory converges, the rest of our argument basically follows the K-S paper.

(i) First, suppose that the initial condition $(L_1(0), T_1(0))$ is on the segment $L_1^{H'} L_1^{J'}$. Lemmas 8.3 and 8.4 (see Appendix 8.2) imply that, if we choose positive $\mu_T(0)$ and $\mu_L(0)$ such that $\mu_T(0) > \mu_L(0)$, the optimal trajectory must follow the system (8.10) through (8.12). The steady state of the system (8.10) through (8.12), which corresponds to point $l''$ in Figure 8.2, is the solution to the system of equations

$$\dot{T}_1 = \frac{\exp\left[\gamma\left(T_1 - T_1(0)\right)\right]}{\alpha}\left[Y\left(T_1, L_1(0)\right) - \beta\left(\frac{\mu_T \exp\left[\gamma\left(T_1 - T_1(0)\right)\right]}{\alpha}\right)\right] = 0 \tag*{(8.10)$'$}$$

$$\dot{\mu}_T = \mu_T\left[\rho - \frac{Y_T\left(T_1, L_1(0)\right)\exp\left[\gamma\left(T_1 - T_1(0)\right)\right]}{\alpha}\right] = 0 \tag*{(8.11)$'$}$$

$$\dot{\mu}_L = \rho\mu_L - \frac{\mu_T Y_L\left(T_1, L_1(0)\right)\exp\left[\gamma\left(T_1 - T_1(0)\right)\right]}{\alpha} = 0. \tag*{(8.12)$'$}$$

As discussed above, $T_1^S$ satisfies (8.11)′. Substituting $T_1^S$ into (8.10)′, $\mu_T$ is uniquely determined. Then, from (8.12)′, so is $\mu_L$. Note that $Y_T > Y_L$ on the segment $H'J'$ except at point $H'$, where $Y_T = Y_L$. It follows from (8.11)′ and (8.12)′ that $\mu_T > \mu_L$ on the segment $H'J'$ except for point $H'$, where $\mu_T = \mu_L$.

Let $f(x)$ be the characteristic equation of the Jacobian matrix evaluated at the steady state of the system (8.10) through (8.12). Then,

$$f(x) \equiv (x-\rho)\left[x^2 - \left(Y_T - \frac{\gamma\beta' e^{\Delta}}{\alpha}\right)\frac{e^{\Delta}}{\alpha}x - \frac{e^{2\Delta}\beta'\mu_T\left(Y_{TT}+\gamma Y_T\right)}{\alpha^2}\right] = 0 \tag{8.15}$$

where $\Delta \equiv \gamma(T_1 - T_1(0))$. From (8.14),

$$-\frac{e^{2\Delta}\beta'\mu_T\left(Y_{TT}+\gamma Y_T\right)}{\alpha^2} < 0.$$

Hence, the characteristic equation (8.15) has two positive and one negative real roots, which means that there exists a one-dimensional stable arm crossing the steady state. Since the number of state variables is one, the foregoing result means that the steady state is saddlepoint stable.

Next, let us consider the case in which the initial condition $(L_1(0), T_1(0))$ is either (ii) on $\tilde{L}_1' L_1^{H'}$ or (iii) on $\tilde{L}_1'' L_1^{J'}$ in Figure 8.2.

(ii) If the initial condition is on the segment $\tilde{L}_1' L_1^{H'}$, the optimal trajectory follows the system (8.10) through (8.12) until it reaches the locus $Y_T = Y_L$ at some finite time $\bar{t}$ at which $\mu_T(\bar{t}) = \mu_L(\bar{t})$, and then converges to a point along the locus $Y_T = Y_L$ by following the system of differential equations (from Lemma 8.3 of Appendix 8.2)

$$Y_T(T_1, L_1) = Y_L(T_1, L_1) \tag{8.16}$$

$$\dot{L}_1 + \dot{T}_1 = \frac{1}{\alpha I}\left[Y\left(T_1, L_1\right) - \beta\left(\frac{\mu}{\alpha I}\right)\right] \tag{8.17}$$

$$\dot{\mu} = \mu\left[\rho - \frac{Y_T\left(T_1, L_1\right)}{\alpha I}\right] \tag{8.18}$$

Differentiating (8.16) with respect to time, we have

$$(Y_{TL} - Y_{LL})\dot{L}_1 = (Y_{LT} - Y_{TT})\dot{T}_1 \tag{8.19}$$

or

$$\dot{L}_1 = \frac{\left(Y_{LT} - Y_{TT}\right)}{\left(Y_{TL} - Y_{LL}\right)}\dot{T}_1. \tag{8.20}$$

Let us denote by $L_1 = \eta(T_1)$ the relation between $L_1$ and $T_1$ that is defined by (8.16). Equation (8.20) means that

$$\eta' = \frac{Y_{LT} - Y_{TT}}{Y_{TL} - Y_{LL}} > 0.$$

Therefore, along the locus $Y_T = Y_L$, both $L_1$ and $T_1$ are increasing, which implies that

$$\begin{aligned} I(t) &= \exp\left[-\gamma\left\{\left(T_1(t) - T_1(0)\right) + \left(L_1(t) - L_1(\bar{t})\right)\right\}\right] \\ &= \exp\left[-\gamma\left\{T_1(t) - T_1(0) + \eta\left(T_1(t)\right) - L_1(\bar{t})\right\}\right]. \end{aligned} \tag{8.21}$$

Note that $L_1(\bar{t}) = L_1(0)$. Substituting (8.21) into (8.17) and (8.18), we see that

$$\dot{T}_1 = \frac{Y\left(T_1, \eta(T_1)\right) - \beta\left(\dfrac{\mu}{\alpha}\right)\exp\left[\gamma\left\{\left(T_1 - T_1(0)\right) + \left(\eta(T_1) - L_1(\bar{t})\right)\right\}\right]}{\alpha(1+\eta')\exp\left[-\gamma\left\{\left(T_1 - T_1(0)\right) + \left(\eta(T_1) - L_1(\bar{t})\right)\right\}\right]} \tag{8.22}$$

$$\dot{\mu} = \mu\left[\rho - \frac{Y_T\left(T_1, \eta(T_1)\right)\exp\left[\gamma\left\{\left(T_1 - T_1(0)\right) + \left(\eta(T_1) - L_1(\bar{t})\right)\right\}\right]}{\alpha}\right]. \tag{8.23}$$

To consider whether the system (8.22) and (8.23) has a steady state or not, we verify that at point $H'$

$$\begin{aligned} &\rho - \frac{Y_T\left(T_1, \eta(T_1)\right)\exp\left[\gamma\left\{\left(T_1 - T_1(0)\right) + \left(\eta(T_1) - L_1(\bar{t})\right)\right\}\right]}{\alpha} \\ &\qquad < \rho - \frac{Y_T\left(T_1, \eta(T_1)\right)\exp\left[\gamma\left(T_1 - T_1(0)\right)\right]}{\alpha} = 0. \end{aligned} \tag{8.24}$$

On the other hand, since $Y_T = Y_L = 0$ at point $E$, we see that

$$\rho - \frac{Y_T\left(T_1, \eta(T_1)\right)\exp\left[\gamma\left\{\left(T_1 - T_1(0)\right) + \left(\eta(T_1) - L_1(\bar{t})\right)\right\}\right]}{\alpha} = \rho > 0. \tag{8.25}$$

Equations (8.24) and (8.25) imply that there exists a point on $E'E$ such that

$$\rho - \frac{Y_T\left(T_1, \eta(T_1)\right)\exp\left[\gamma\left\{\left(T_1 - T_1(0)\right) + \left(\eta(T_1) - L_1(\bar{t})\right)\right\}\right]}{\alpha} = 0. \tag{8.26}$$

Denote by $T_1^{SS}$ the minimum value of $T_1$ that satisfies (8.26). Substituting it into

$$0 = Y\left(T_1, \eta(T_1)\right) - \beta\left(\frac{\mu}{\alpha}\right)\exp\left[-\gamma\left\{\left(T_1(t) - T_1(0)\right) + \left(\eta\left(T_1(t)\right) - L_1(\bar{t})\right)\right\}\right], \tag{8.27}$$

we can determine the steady-state value of $\mu$ uniquely. Differentiation of the left side of (8.26) with respect to $T_1$ yields

$$-\frac{(1+\eta')}{\alpha}\left[Y_{TT} + \gamma Y_T\right]\exp\left[\gamma\left\{\left(T_1(t) - T_1(0)\right) + \left(\eta\left(T_1(t)\right) - L_1(\bar{t})\right)\right\}\right], \tag{8.28}$$

which is positive.

Once we verify the existence of the steady state of the system (8.22) and (8.23), we can proceed to check the saddlepoint stability of it. We see that at the steady state

$$\partial \dot{T}_1 / \partial \mu = \frac{-\frac{1}{\alpha}\beta' \exp\left[\gamma\left\{\left(T_1 - T_1(0)\right) + \eta(T_1) - \left(L_1(\bar{t})\right)\right\}\right]}{\alpha(1+\eta')\exp\left[-\gamma\left\{\left(T_1 - T_1(0)\right) + \left(\eta(T_1) - L_1(\bar{t})\right)\right\}\right]} > 0 \tag{8.29}$$

$$\partial \dot{\mu} / \partial T_1 = \mu \times (8.25) > 0. \tag{8.30}$$

Thus, the determinant of the Jacobian matrix of the system (8.22) and (8.23) evaluated at the steady state, which is – (8.29) × (8.30), is negative. It follows that the characteristic equation has one positive and one negative real root. Therefore, the steady state is saddlepoint stable.

(iii) Third, suppose the initial condition is on the segment $L_1^{J'}\tilde{L}_1''$. Then the optimal trajectory follows the system (8.10) through (8.12) until it reaches the locus $-Y_T = Y_L$ at some finite time $\tilde{t}$ at which $\mu_T(\tilde{t}) = \mu_L(\tilde{t})$, and finally converges to a point along the locus $-Y_T = Y_L$ by following the system of differential equations (from Lemma 8.3 of Appendix 8.2):

$$-Y_T\left(T_1, L_1\right) = Y_L\left(T_1, L_1\right) \tag{8.31}$$

$$\left|\dot{L}_1\right| + \left|\dot{T}_1\right| = \frac{1}{\alpha I}\left[Y\left(T_1, L_1\right) - \beta\left(\frac{\mu}{\alpha I}\right)\right] \tag{8.32}$$

$$\dot{\mu} = \mu\left[\rho - \frac{Y_T\left(T_1, L_1\right)}{\alpha I}\right]. \tag{8.33}$$

Here we focus on the case in which the locus $-Y_T(T_1, L_1) = Y_L(T_1, L_1)$ is negatively sloped as in Figure 8.2. Then, if the trajectory proceeds towards point $E$ along the locus, $L_1$ decreases and $T_1$ increases. Thus (8.32) can be rewritten as

$$-\dot{L}_1 + \dot{T}_1 = \frac{1}{\alpha I}\left[Y\left(T_1, L_1\right) - \beta\left(\frac{\mu}{\alpha I}\right)\right]. \tag{8.34}$$

Moreover, in this case we see that

$$I = \exp\left[-\gamma\left\{\left(L_1\left(\tilde{t}\right) - L_1\left(t\right)\right) + \left(T_1\left(t\right) - T_1\left(0\right)\right)\right\}\right]. \tag{8.35}$$

Note that $L_1\ (\tilde{t}) = L_1\ (0)$. Totally differentiating (8.31) with respect to time, we obtain

$$\Delta_L \dot{L}_1 + \Delta_T \dot{T}_1 = 0, \tag{8.36}$$

where $\Delta_L \equiv Y_{LL} + Y_{TL}$ and $\Delta_T \equiv Y_{TL} + Y_{TT}$. A direct calculation yields

$$\Delta \equiv \Delta_L + \Delta_T < 0,$$

and (8.36), together with the fact that $\dot{L}_1 < 0$ and $\dot{T}_1 > 0$, implies that $\Delta_L \Delta_T > 0$. Hence, it is certain that

$$\Delta_L < 0 \text{ and } \Delta_T < 0. \tag{8.37}$$

Solving (8.34) and (8.36), we obtain

$$\dot{L}_1 = \frac{-\Delta_T}{\Delta \alpha I}\left[Y\left(T_1, L_1\right) - \beta\left(\frac{\mu}{\alpha}\right)\exp\left[\gamma\left\{\left(L_1\left(\tilde{t}\right) - L_1\left(t\right)\right) + \left(T_1\left(t\right) - T_1\left(0\right)\right)\right\}\right]\right] \tag{8.38}$$

$$\dot{T}_1 = \frac{-\Delta_L}{\Delta \alpha I}\left[Y\left(T_1, L_1\right) - \beta\left(\frac{\mu}{\alpha}\right)\exp\left[\gamma\left\{\left(L_1\left(\tilde{t}\right) - L_1\left(t\right)\right) + \left(T_1\left(t\right) - T_1\left(0\right)\right)\right\}\right]\right] \tag{8.39}$$

$$\dot{\mu} = \mu\left[\rho - \frac{Y_T\left(T_1, Y_1\right)}{\alpha}\exp\left[\gamma\left\{\left(L_1\left(\tilde{t}\right) - L_1\left(t\right)\right) + \left(T_1\left(t\right) - T_1\left(0\right)\right)\right\}\right]\right]. \tag{8.40}$$

The steady state is determined by

$$-Y_T\left(T_1,L_1\right)+Y_L\left(T_1,L_1\right)=0 \tag{8.41}$$

$$Y\left(T_1,L_1\right)-\beta\left(\frac{\mu}{\alpha}\right)\exp\left[\gamma\left\{\left(L_1\left(\tilde{t}\right)-L_1\left(t\right)\right)+\left(T_1\left(t\right)-T_1\left(0\right)\right)\right\}\right]=0 \tag{8.42}$$

$$\rho-\frac{Y_T\left(T_1,L_1\right)}{\alpha}\exp\left[\gamma\left\{\left(L_1\left(\tilde{t}\right)-L_1\left(t\right)\right)+\left(T_1\left(t\right)-T_1\left(0\right)\right)\right\}\right]=0, \tag{8.43}$$

First, at point $J'$ in Figure 8.2,

$$0=\rho-\frac{Y_T\left(T_1,L_1\right)}{\alpha}\exp\left\{\gamma\left(T_1\left(t\right)-T_1\left(0\right)\right)\right\}.$$

Thus,

$$0>\rho-\frac{Y_T\left(T_1,L_1\right)}{\alpha}\exp\left[\gamma\left\{\left(L_1\left(\tilde{t}\right)-L_1\left(t\right)\right)+\left(T_1\left(t\right)-T_1\left(0\right)\right)\right\}\right]. \tag{8.44}$$

Second, at $E$,

$$\rho-\frac{Y_T\left(T_1,L_1\right)}{\alpha}\exp\left[\gamma\left\{\left(L_1\left(\tilde{t}\right)-L_1\left(t\right)\right)+\left(T_1\left(t\right)-T_1\left(0\right)\right)\right\}\right]=\rho>0. \tag{8.45}$$

Therefore, there exists a point on the locus $Y_T(T_1, L_1) + Y_L(T_1, L_1) = 0$ such that (8.43) holds. Let $(T_1^S, L_1^S)$ be such that that $T_1^S$ is the smallest among such points. Totally differentiating the right-hand side of (8.43) with respect to time, we have

$$0<-\left[\left(\gamma Y_T+Y_{TT}\right)\dot{T}_1+\left(-\gamma Y_T+Y_{TL}\right)\dot{L}_1\right]. \tag{8.46}$$

The sign of (8.46) follows from (8.44) and (8.45). Since $\dot{T}_1 > 0$, it follows from (8.36) and (8.37) that at the steady state

$$0>\left(\gamma Y_T+Y_{TT}\right)\Delta_L+\left(\gamma Y_T-Y_{TL}\right)\Delta_T. \tag{8.47}$$

Using (8.47), we can prove that the steady state is saddlepoint stable. The characteristic equation corresponding to the Jacobian matrix evaluated at the steady state of the system (8.38) through (8.40) is

$$g\left(x\right)\equiv x\left[x^2-\frac{2\Delta_T\beta'}{\alpha\beta I\Delta}\left(Y_L+\frac{\gamma\mu\beta'}{\alpha I}\right)x+\frac{\beta'}{aI\Delta}\left\{\Delta_T\left(Y_{YL}-\gamma Y_T\right)+\Delta_L\left(Y_{TT}+\gamma Y_T\right)\right\}\right]=0.$$

From (8.47) and since $\beta' < 0$, $\Delta < 0$ and $\mu < 0$,

$$\frac{\beta'}{\alpha I \Delta}\left\{\Delta_T\left(Y_{YL} - \gamma Y_T\right) + \Delta_L\left(Y_{TT} + \gamma Y_T\right)\right\} < 0.$$

Therefore, the characteristic equation has a zero root, a positive real root and a negative real root. This ensures that the steady state is saddlepoint stable.

We have just considered the case in which the initial condition is in Region I. The case in which the initial condition is in one of the other three regions can be treated in a similar way. Thus, the desired phase diagram is shown in Figure 8.3.

## Appendix 8.2

This appendix states and proves four lemmas that clarify the properties of the optimal trajectories of the control problem treated in Appendix 8.1.

As preliminary observations, we draw attention to: (i) the convention that $X$ denotes any one member of the set $\{L, T\}$, the remaining member of the set being denoted by $Z$, and (ii) the fact that, if the Hamiltonian is differentiable,

$$\left(\partial H / \partial \dot{X}_1\right) = -\left(\operatorname{sgn} \dot{X}_1\right)\alpha\left(p\right)Iu'\left(Y\left(p, L_1, T_1\right) - I\alpha\left(p\right)\left\{\left|\dot{L}_1\right| + \left|\dot{T}_1\right|\right\}\right) + \mu_X,\ X = L, T, \tag{8.48}$$

where sgn $x$ means the sign of variable $x$.

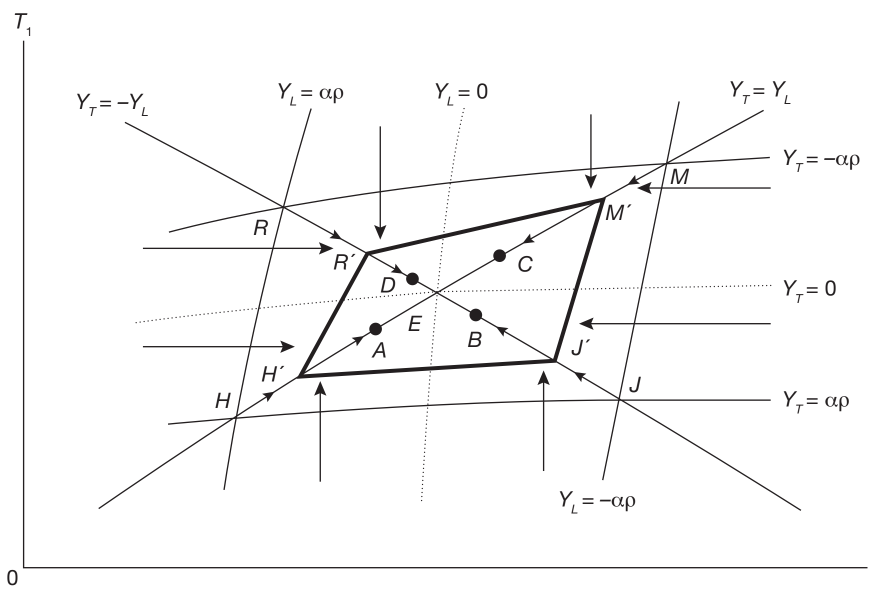


*Figure 8.3*

*Lemma 8.1* For any optimal plan, if $\mu_X \geq 0$ ($\mu_X \leq 0$) then $\dot{X}_1 \geq 0$ ($\dot{X}_1 \leq 0$), $X = L, T$. Consequently, $\mu_X = 0$ implies that $\dot{X}_1 = 0$.

*Proof* Consider the case $\mu_X \geq 0$ and suppose that $\dot{X}_1 < 0$. Then the assumed optimality and (8.48) imply that

$$0 = \left(\partial H / \partial \dot{X}_1\right) = \alpha(p) Iu'\left(Y\left(p, L_1, T_1\right) - I\alpha(p)\left\{\left|\dot{L}_1\right| + \left|\dot{T}_1\right|\right\}\right) + \mu_X > 0,$$

a self-contradiction. Similarly the assumption that $\mu_X \leq 0$ is incompatible with $\dot{X}_1 > 0$. Noting that $\mu_X = 0$ if and only if $\mu_X \geq 0$ and $\mu_X \leq 0$, the remaining assertion follows at once.

In view of Lemma 8.1 and the identity $\mu_X = (\operatorname{sgn} \mu_X)|\mu_X|$, (8.48) reduces to

$$\left(\operatorname{sgn} \mu_X\right)\left(\partial H / \partial \dot{X}_1\right) = \left|\mu_X\right| - I\alpha(p) u'\left(Y\left(p, L_1, T_1\right) - I\alpha(p)\left\{\left|\dot{L}_1\right| + \left|\dot{T}_1\right|\right\}\right),\ X = L, T \tag{8.49}$$

if $\dot{X}_1 \neq 0$.

*Lemma 8.2* Under the assumption that $\dot{X}_1$ is an optimal control ($X = L, T$), (i) if $\dot{X}_1 \neq 0$ then

$$\left|\mu_X\right| > I\alpha(p) u'\left(Y\left(p, L_1, T_1\right)\right), \tag{8.50}$$

and (ii) if $\dot{Z}_1 = 0$ and if (8.50) is satisfied then $\dot{X}_1 \neq 0$.

*Proof* (i) Arguing by contradiction, assume that $\dot{X}_1 \neq 0$ and that

$$\left|\mu_X\right| \leq I\alpha(p) u'\left(Y\left(p, L_1, T_1\right)\right) \tag{8.51}$$

Then, the assumed optimality, (8.51) and the strict concavity of $u(\cdot)$ imply that

$$\begin{aligned} 0 = \left(\operatorname{sgn} \mu_X\right)\left(\partial H / \partial \dot{X}_1\right) &= \left|\mu_X\right| - I\alpha(p) u'\left(Y\left(p, L_1, T_1\right) - I\alpha(p)\left\{\left|\dot{X}_1\right| + \left|\dot{Z}_1\right|\right\}\right) \\ &< \left|\mu_X\right| - I\alpha(p) u'\left(Y\left(p, L_1, T_1\right)\right) \leq 0, \end{aligned}$$

a self-contradiction.

We verify that the assumed conditions are incompatible with $\dot{X}_1 = 0$. Suppose that $\mu_X \geq 0$. Then the assumed optimality requires that there exists a positive number $d$ such that the Hamiltonian viewed as a function of $\dot{X}_1$ is monotonically non-increasing in the open interval $(0, d)$. Letting $\dot{X}_1 \in (0, d)$ tend to zero, the differentiability of the Hamiltonian in this interval, the assumed optimality and the assumption that $\dot{Z}_1 = 0$ imply that

$$\mu_X - I\alpha(p) u'\left(Y\left(p, L_1, T_1\right)\right) \leq 0.$$

This violates (8.50).

When $\mu_X$ is non-positive, it suffices to note that there can be found an open interval $(-d, 0)$ in which the Hamiltonian is differentiable as well as non-decreasing and to proceed as above, *mutatis mutandis*.

*Lemma 8.3* Along the optimal path, the following assertions hold true:

(i) if $|\mu_X| > |\mu_Z|$ then $\dot{Z}_1 = 0$,

(ii) if $|\mu_X| > \max\left\{|\mu_X|, I\alpha(p)u'\left(Y(p, L_1, T_1)\right)\right\}$ then $\dot{X}_1 \neq 0$ and $\dot{Z}_1 = 0$,

(iii) if $\dot{X}_1 \neq 0$ then $|\dot{X}_1| + |\dot{Z}_1| = \left(Y(p, L_1, T_1) - \beta\left(|\mu_X| / I\alpha(p)\right)\right) / I\alpha(p)$,

and if, in addition, $|\mu_X| = |\mu_Z|$ then

$$|\dot{X}_1| + |\dot{Z}_1| = \left(Y(p, L_1, T_1) - \beta\left(|\mu_Z| / I\alpha(p)\right)\right) / I\alpha(p),$$

and, finally,

(iv) if $\dot{X}_1 \neq 0$ and $\dot{Z}_1 = 0$ then

$$\dot{X}_1 = (\operatorname{sgn} \mu_X)\left\{Y(p, L_1, T_1) - \beta\left(|\mu_X| / I\alpha(p)\right)\right\} / I\alpha(p).$$

*Proof* (i) Suppose the contrary. Then $\dot{Z}_1 \neq 0$. If $\dot{X}_1 \neq 0$, we obtain

$$\begin{aligned} 0 &= (\operatorname{sgn} \mu_X)\left(\partial H / \partial \dot{X}_1\right) \\ &= |\mu_X| - I\alpha(p)u'\left(Y(p, L_1, T_1) - I\alpha(p)\left\{|\dot{X}_1| + |\dot{Z}_1|\right\}\right) \\ &> |\mu_Z| - I\alpha(p)u'\left(Y(p, L_1, T_1) - I\alpha(p)\left\{|\dot{X}_1| + |\dot{Z}_1|\right\}\right) \\ &= (\operatorname{sgn} \mu_Z)\left(\partial H / \partial \dot{Z}_1\right), \end{aligned}$$

because $\dot{Z}_1 \neq 0$ and from (8.49). Hence $(\partial H / \partial \dot{Z}_1) \neq 0$, which contradicts the assumed optimality.

Now consider the case $\dot{X}_1 = 0$. Since $\dot{Z}_1 \neq 0$, the assumed optimality, with the aid of (8.49), yields

$$\begin{aligned} 0 &= \left(\partial H / \partial \dot{Z}_1\right)(\operatorname{sgn} \mu_Z) \\ &= |\mu_Z| - I\alpha(p)u'\left(Y(p, L_1, T_1) - I\alpha(p)|\dot{Z}_1|\right) \\ &< |\mu_X| - I\alpha(p)u'\left(Y(p, L_1, T_1) - I\alpha(p)|\dot{Z}_1|\right). \end{aligned}$$

Furthermore, $\mu_X$ is either positive or negative for $|\mu_X| > |\mu_Z| \geq 0$. Since the Hamiltonian viewed as a function of $\dot{X}_1$ attains a local maximum at $\dot{X}_1 = 0$, there exists a positive number $d_1$ such that the Hamiltonian is non-increasing in the open interval $(0, d_1)$ and non-decreasing in $(-d_1, 0)$. Therefore, if $\mu_X > 0$, by letting $X_1 \in (0, d_1)$ tend to zero, we reach the self-contradiction

$$0 < \mu_X - I\alpha(p)u'\left(Y(p, L_1, T_1) - I\alpha(p)|\dot{Z}_1|\right) \le 0.$$

And, if $\mu_X < 0$, the same reasoning applied to $(-d_1, 0)$ leads to the self-contradiction

$$0 > \mu_X + I\alpha(p)u'\left(Y(p, L_1, T_1) - I\alpha(p)|\dot{Z}_1|\right) \ge 0.$$

(ii) The implication follows directly from Lemma 8.1(ii) and Lemma 8.3(i).

(iii) Noticing that $u'(\cdot)$ is monotonically decreasing, the assertion follows from (8.49) and the Inada conditions. The remaining assertion is rather trivial.

(iv) The conclusion is derived directly from (iii) above and the fact that $|\dot{X}_1| = (\text{sgn}\ \dot{X}_1)\dot{X}_1$.

*Lemma 8.4* Along the optimal path, we can assert that

(i) if $\dot{X}_1 \neq 0$ then $\dot{\mu}_X = \mu_X(\rho - ((\text{sgn}\ \mu_X)Y_X)/I\alpha(p))$,

and that

(ii) if $\dot{X}_1 \neq 0$ and $\dot{Z}_1 = 0$ then

$$\dot{\mu}_Z = \rho\mu_Z - \left((\text{sgn}\,\mu_X)\mu_X Y_Z\right)/I\alpha(p),$$

where $Y_X = (\partial Y/\partial X_1)$, $X = L, T$.

*Proof* (i) The assumed optimality and (8.49) imply that

$$|\mu_X| - I\alpha(p)u'\left(Y(p, L_1, T_1) - I\alpha(p)\left\{|\dot{X}_1| + |\dot{Z}_1|\right\}\right) = 0.$$

Hence,

$$u'\left(Y(p, L_1, T_1) - I\alpha(p)\left\{|\dot{X}_1| + |\dot{Z}_1|\right\}\right) = (\text{sgn}\,\mu_X)\mu_X/I\alpha(p). \quad (8.52)$$

On the other hand, the optimal principle requires that

$$\dot{\mu}_X = \rho\mu_X - (\partial H/\partial X_1) = \rho\mu_X - u'\left(Y(p, L_1, T_1) - I\alpha(p)\left\{|\dot{X}_1| + |\dot{Z}_1|\right\}\right)Y_X.$$

Substituting (8.52) into the above equation, the assertion is immediate.

(ii) Using (8.52) with $\dot{Z}_1 = 0$, eliminate $u'(Y(p, L_1, T_1) - I\alpha(p)|\dot{X}_1|)$ from

$$\dot{\mu}_Z = \rho\mu_Z - u'\left(Y(p, L_1, T_1) - I\alpha(p)|\dot{X}_1|\right)Y_Z.$$

Then, the assertion is seen to hold trivially.

# 9 A theory of involuntary unrequited international transfers[1]

## 9.1 Introduction

The theory of involuntary and unrequited transfers (war indemnities) has been constructed on the assumption that the donor and recipient countries are completely indifferent to each other's well-being. The assumption is hard to justify since usually the transfers closely follow periods during which the countries have been dropping bombs on each other. In the present paper, therefore, we rework the theory on the more plausible assumption that the well-being of each country is influenced (negatively) by the well-being of the other country; that is, we introduce an international externality.

This small analytical innovation has quite startling consequences. Whereas in conventional Samuelsonian theory (Samuelson 1947: 29n), which incorporates the assumption of local Walrasian stability, the recipient necessarily benefits at the expense of the donor, it now emerges that the donor might benefit at the expense of the recipient, even when local Walrasian stability is imposed. The traditional stability condition, that the determinant of the Jacobian matrix of the static system be negative, no longer ensures stability in all circumstances.

The finding is counter-intuitive. One might have expected that, under our revised assumptions, the donor would incur the additional burden of improving the lot of a nation it scorns and that the recipient would reap the additional satisfaction of putting down a people it cordially dislikes. Thus, we have unearthed a new transfer paradox that, unlike the well-known paradox described by Gale (1974), does not rely on the presence of third or bystander countries.[2]

## 9.2 Analysis

Two countries, $\alpha$ and $\beta$, produce and trade in two commodities, 1 and 2. Let $p$ denote the relative price of the first commodity, $u^j$ the per capita utility or well-being of the $j$th country, $e^j(p, u^\alpha, u^\beta)$ the expenditure function of the $j$th country, $r^j(p)$ the national income function of the $j$th country, and $E^j(p, u^\alpha, u^\beta) \equiv e^j(p, u^\alpha, u^\beta) - r^j(p)$ the excess expenditure function of the

$j$th country, $j = \alpha, \beta$. The national income functions are derived from conventional convex technologies and from primary factor endowments. The technologies may differ from country to country. In each country, there are at least two primary factors, and they are in completely inelastic supply; accordingly, they are not explicit in the national income functions. Let $\alpha$ be the defeated nation, required to pay to $\beta$ an indemnity $T \geq 0$. Of course, $e^j$, $r^j$, $E^j$, and $T$ are, in terms of the numeraire, commodity 2. Finally, let subscripts indicate partial or total derivatives; for example, $E^j_p \equiv \partial E^j/\partial p$, $E^j_k \equiv \partial E^j/\partial u^k$ and $r^j_p \equiv dr^j/dp$ $(j, k = \alpha, \beta)$. The 'direct' utility derivatives of $e^j$, and therefore of $E^j$, are positive.

*Assumption 1* $\partial E^j/\partial u^j > 0$ for $j = \alpha, \beta$ and for all feasible $(u^\alpha, u^\beta)$.

The lingering ill feeling between the former adversaries ensures that the 'indirect' utility derivatives also are positive.

*Assumption 2* $\partial E^j/\partial u^k > 0$ for $j, k = \alpha, \beta, j \neq k$, and for all feasible $(u^\alpha, u^\beta)$.

In terms of the notation introduced in this section, we have two aggregate budget constraints, one for each country,

$$E^\alpha(p, u^\alpha, u^\beta) = -T \tag{1}$$

and

$$E^\beta(p, u^\alpha, u^\beta) = T \tag{2}$$

and the market-clearing condition for the first commodity,

$$E^\alpha_p(p, u^\alpha, u) + E^\beta_p(p, u^\alpha, u^\beta) = 0. \tag{3}$$

Equations (1)–(3) can be viewed as determining $u^\alpha$, $u^\beta$, and $p$ in terms of feasible $T$. Let us assume that a solution exists and is unique. By varying $T$ within the bounds of feasibility, we can trace the locus of equilibrium pairs $(u^\alpha(T), u^\beta(T))$. Given the assumption of uniqueness, each positive value of $T$ is associated with a different point on the locus, all south-east or all north-west of the initial point, at which $T = 0$; moreover, as the associated point moves farther from the initial point, $T$ grows in magnitude.

Differentiating the system (1)–(3) totally, we obtain

$$\begin{bmatrix} E^\alpha_p & E^\alpha_\alpha & E^\alpha_\beta \\ E^\beta_p & E^\beta_\alpha & E^\beta_\beta \\ E^\alpha_{pp} + E^\beta_{pp} & E^\alpha_{p\alpha} + E^\beta_{p\alpha} & E^\alpha_{p\beta} + E^\beta_{p\beta} \end{bmatrix} \begin{bmatrix} dp \\ du^\alpha \\ du^\beta \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} dT \tag{4}$$

and, solving,

$$\Delta\left(\frac{dp}{dT}\right) = -\left(E_\alpha^\alpha + E_\alpha^\beta\right)\left(E_{p\beta}^\alpha + E_{p\beta}^\beta\right) + \left(E_\beta^\alpha + E_\beta^\beta\right)\left(E_{p\alpha}^\alpha + E_{p\alpha}^\beta\right), \tag{5a}$$

$$\Delta\left(\frac{du^\alpha}{dT}\right) = -\left(E_{pp}^\alpha + E_{pp}^\beta\right)\left(E_\beta^\alpha + E_\beta^\beta\right) > 0, \tag{5b}$$

$$\Delta\left(\frac{du^\beta}{dT}\right) = \left(E_{pp}^\alpha + E_{pp}^\beta\right)\left(E_\alpha^\alpha + E_\alpha^\beta\right) < 0, \tag{5c}$$

where

$$\begin{aligned}\Delta \equiv &\left(E_{pp}^\alpha + E_{pp}^\beta\right)\left(E_\alpha^\alpha E_\beta^\beta - E_\beta^\alpha E_\alpha^\beta\right)\\ &- E_p^\beta\left[\left(E_\alpha^\alpha + E_\alpha^\beta\right)\left(E_{p\beta}^\alpha + E_{p\beta}^\beta\right) - \left(E_\beta^\alpha + E_\beta^\beta\right)\left(E_{p\alpha}^\alpha + E_{p\alpha}^\beta\right)\right]\end{aligned} \tag{6}$$

is the determinant of the Jacobian matrix of equations (1)–(3), evaluated at the equilibrium associated with the chosen value of $T$ and assumed to be non-zero. Hence the slope of the locus at the equilibrium point is

$$\frac{du^\alpha}{du^\beta} = -\frac{E_\beta^\alpha + E_\beta^\beta}{E_\alpha^\alpha + E_\alpha^\beta} < 0. \tag{7}$$

(Here and in equation (5), the inequalities follow from assumptions 1 and 2.)

Figures 9.1 and 9.2 display the downward-sloping utility locus, with the initial equilibrium, associated with a zero value of $T$, marked by point $F$. In view of (5), *either* any point associated with a positive and feasible value of $T$ must lie north-west of $F$ *or* any such point must lie south-east of $F$.

It does not seem possible to say more without resorting to additional restrictions on the excess expenditure functions. Even the assumption that all points on the locus are locally stable in the Walrasian sense fails to establish the priority of one figure over the other. This claim is confirmed in Appendix 9.1. It is there shown also that if and only if the direct utility terms dominate the cross utility terms, in the sense that

$$\delta \equiv E_\alpha^\alpha E_\beta^\beta - E_\beta^\alpha E_\alpha^\beta > 0, \tag{8}$$

does local Walrasian stability ensure that $\Delta$ is negative, so that conventional Samuelsonian conclusions continue to hold. If $\delta < 0$, then stability merely ensures that $\Delta$ is positive and the outcomes paradoxical.

Thus, in summary, the victorious $\beta$, unsure of the sign of $\Delta$, cannot confidently choose to impose an indemnity on the vanquished $\alpha$. It might be in $\beta$'s interest to offer aid to $\alpha$. This finding is in sharp contrast to the

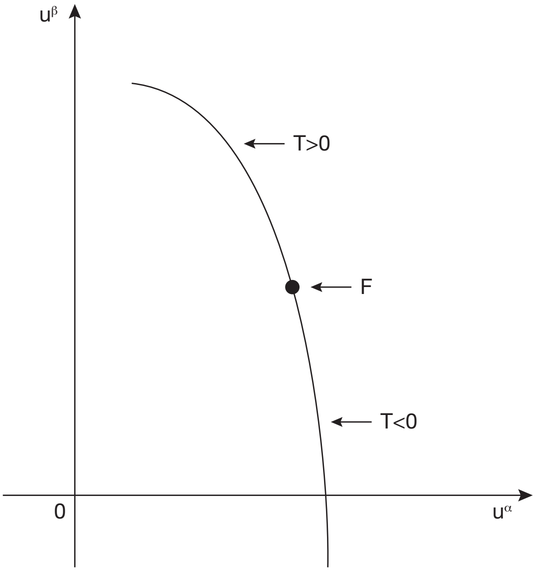


*Figure 9.1* Stability with $\Delta < 0$ yields conventional outcomes

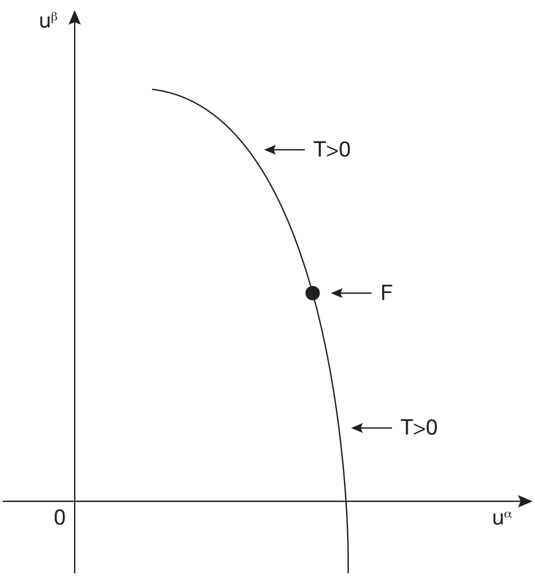


*Figure 9.2* Stability with $\Delta < 0$ yields paradoxical outcomes

central result of conventional theory. Perhaps we may refer to the 'victor's dilemma'.

## 9.3 An alternative perspective

With the heavy work behind us, we now offer, in conclusion, a brief alternative perspective that allows us to focus on the key role of $\delta$ and thereby extract the common sense of our argument. The new approach rests on our earlier stability analysis but is otherwise self-contained.

From (1), (2), and (3),

$$\begin{aligned} E_{\alpha}^{\alpha}\left(\frac{du^{\alpha}}{dT}\right)+E_{\beta}^{\alpha}\left(\frac{du^{\beta}}{dT}\right)&=-\left[1-E_{p}^{\beta}\left(\frac{dp}{dT}\right)\right], \\ E_{\alpha}^{\beta}\left(\frac{du^{\alpha}}{dT}\right)+E_{\beta}^{\beta}\left(\frac{du^{\beta}}{dT}\right)&=1-E_{p}^{\beta}\left(\frac{dp}{dT}\right), \end{aligned} \tag{9}$$

where, in view of (5a),

$$1-E_{p}^{\beta}\left(\frac{dp}{dT}\right)=\left(\frac{\delta}{\Delta}\right)\left(E_{pp}^{\alpha}+E_{pp}^{\beta}\right). \tag{10}$$

On the left-hand side of (10), the first term records the direct effect on $\beta$'s income of the transfer by $\alpha$ of one unit of the numeraire and the second term represents the indirect or terms-of-trade effect of the transfer. Thus (10) records the net effect of the transfer on $\beta$'s income. Since the pure substitution term $E_{pp}^{\alpha}+E_{pp}^{\beta}$ is negative and since $\delta/\Delta$ is assumed to be negative as a sufficient condition of local Walrasian stability, the net effect of the transfer is to raise $\beta$'s income.

Inspection of (9) reveals that (a) if there are no externalities (so that $E_{\beta}^{\alpha}=0=E_{\alpha}^{\beta}$) or if the externalities are positive but sufficiently small that $\delta$, the determinant of the coefficient matrix in (9), is positive, then the conventional conclusions remain intact; and (b) if the externalities are sufficiently large that $\delta$ is negative, then the transfer has paradoxical implications. Thus, given stability, a negative $\delta$ is essential for a paradoxical outcome.

## Appendix 9.1 Local Walrasian stability

Let us replace the static market-clearing condition (3) with the dynamic adjustment equation

$$\dot{p}\equiv\frac{dp}{dt}=\psi\left(E_{p}^{\alpha}\left(p,u^{\alpha},u^{\beta}\right)+E_{p}^{\beta}\left(p,u^{\alpha},u^{\beta}\right)\right), \tag{A1}$$

where $\psi(\cdot)$ is an increasing function with $\psi(0) = 0$. Linearizing about the initial equilibrium, with $T = 0$, we find that

$$\dot{p} = \psi'(0)\left[E^{\alpha}_{pp} + E^{\beta}_{pp} + \left(E^{\alpha}_{p\alpha} + E^{\beta}_{p\alpha}\right)\frac{du^{\alpha}}{dp} + \left(E^{\alpha}_{p\beta} + E^{\beta}_{p\beta}\right)\frac{du^{\beta}}{dp}\right]\left(p - p^{*}\right), \tag{A2}$$

where $\psi'(0) \equiv (d/dX)\psi(X)|_{X=0} > 0$ and all the terms in brackets are evaluated at the initial equilibrium values of $p$, $u^{\alpha}$ and $u^{\beta}$. The term $p^{*}$ denotes the initial equilibrium value of $p$. From (1), (2), and (3), treating $T$ as a constant and $p$ as a parameter, we obtain

$$\begin{aligned} \frac{du^{\alpha}}{dp} &= \left(\frac{1}{\delta}\right)E^{\alpha}_{p}\left(E^{\alpha}_{\beta} + E^{\beta}_{\beta}\right), \\ \frac{du^{\beta}}{dp} &= -\left(\frac{1}{\delta}\right)E^{\beta}_{p}\left(E^{\alpha}_{\alpha} + E^{\beta}_{\alpha}\right), \end{aligned} \tag{A3}$$

where $\delta$, defined by equation (8), is assumed to be finite and non-zero. Hence, substituting from (A3) into (A2) and recalling (6), we get

$$\dot{p} = \psi'(0)\left(\frac{\Delta}{\delta}\right)\left(p - p^{*}\right). \tag{A4}$$

The linear system (A4) is stable if and only if $\Delta/\delta$ is negative. However, we have assumed only that $\Delta$ and $\delta$ are not equal to zero and that $\delta$ is finite. Hence, the assumption that the linear system is stable does not pin down the sign of $\Delta$. Only if the mutual dislike of $\alpha$ and $\beta$ is sufficiently mild to ensure that $\delta$ is positive does negative $\Delta/\delta$ imply that $\Delta$ is negative.[3] Finally, we note that, given the restrictions imposed on $\Delta$ and $\delta$, the non-linear system (A1) is locally stable if and only if the linear system is stable.

# 10 A theory of involuntary unrequited international transfers

## A reply to Carlos da Costa[1]

### 10.1 Introduction

In Kemp and Shimomura (2003b) we noted that, after a war between two countries, there may be a lingering dislike of each country for the other. We then argued that, in those circumstances, an indemnity paid by the defeated country to the victorious country might improve the well-being of the defeated country and reduce that of the victorious country, even when there are no third or bystander countries and the world equilibrium is locally stable in the sense of Walras. Necessary and sufficient conditions for the paradoxical outcome were provided and have been the subject of comment by Carlos da Costa (2005).

Da Costa does not question the logical validity of our conclusions; but he does suggest that the utility functions implicit in our analysis are unrealistic in implying that if consumption externalities were somehow internalized, then, in any post-transfer world equilibrium, each country would be satiated by its own consumption. However, in establishing that possibility, he focuses on a special case of our model. In this reply we show that his is not a general result.[2]

### 10.2 Da Costa's special case

In our paper we relied on the assumption that the well-being of each family (whatever its country of residence) depends on the well-being of all other families (whatever their countries of residence). We relied also on the assumption that all families in a country are identical in all respects but unaware that they are identical. Given the latter assumption, we felt free to further assume that each household takes commodity prices and the well-being of other families (whatever their countries of residence) as given, beyond its control; that is, we assumed that each household behaves non-cooperatively in all of its decision-making.[3]

Our entire analysis was conducted in terms of household expenditure functions. Da Costa, on the other hand, employs also the utility functions that, is his view, lie behind our expenditure functions; and it is at this point that our difference of opinion arises. Implicit in our analysis are the utility functions

$$u^z = U^z(x^z, \bar{u}^z, \bar{u}^y) \;\; \{z, y\} = \{\alpha, \beta\},\; z \neq y \tag{1}$$

where $\alpha$ and $\beta$ are the two countries, $u^z$ is the utility of a representative household in country $z$, $x^z$ is the consumption vector of that household and $\bar{u}^z$ is the average utility level of country $z$. The expenditure function of a representative household in country $z$ is then defined as the solution to the minimization problem

$$\bar{e}^z\left(p, u^z; \bar{u}^z, \bar{u}^y\right) \equiv \min_{x^z} px^z \quad \text{s.t.}\;\; U^z(x^z, \bar{u}^z, \bar{u}^y) \geq u^z \tag{2}$$

where $p$ is the commodity price vector. Bearing in mind that all households in country $z$ are identical, $\bar{u}^z$ must be equal to $u^z$, allowing us to write the expenditure function

$$e^z(p, u^z, u^y) \equiv \bar{e}^z(p, u^z; u^z, u^y) \tag{3}$$

and the excess expenditure function

$$E^z(p, u^z, u^y) \equiv e^z(p, u^z, u^y) - r^z(p)\,, \tag{4}$$

as in Kemp and Shimomura (2003b). It then follows that

$$\begin{aligned} E^z_z &\equiv \partial E^z(p, u^z, u^y) / \partial u^z \\ &\equiv [\partial \bar{e}^z(p, u^z, \bar{u}^z, \bar{u}^y) / \partial u^z + \partial \bar{e}^z(p, u^z, \bar{u}^z, \bar{u}^y) / \partial \bar{u}^z]_{u^z = \bar{u}^z} \\ &\equiv \tilde{E}^z_z + \tilde{E}^z_{\bar{z}}. \end{aligned} \tag{5}$$

$\tilde{E}^z_z$ is naturally positive but $\tilde{E}^z_{\bar{z}}$ may be of either sign and, if negative, may outweigh $\tilde{E}^z_z$. Hence the role of our Assumption 9.1.

In our exposition of 2003 (Kemp and Shimomura 2003b), we allowed $\tilde{Z}^z_{\bar{z}}$ to be negative but not to outweigh $\tilde{E}^z_z$. In da Costa (2005), on the other hand, no allowance is made for local externalities; that is, $\bar{u}^z$ is omitted as an argument of the function $U^z$. Given that omission, $E^z_z$ reduces to positive $\tilde{E}^z_z$. At the other extreme, in our further calculations, it will be assumed that $\tilde{E}^z_{\bar{z}}$ always outweighs $\tilde{E}^z_z$.

## 10.3 Unrealistic implications?

It can now be shown that, even if all externalities are somehow internalized, paradoxical outcomes of the Kemp-Shimomura type are possible without satiety in own consumption. In the course of the demonstrations we shall have occasion to modify our earlier Assumptions 9.1 and 9.2.

For simplicity, we re-specify utility function (1) as

$$u^z = U^z(g^z(x^z), \bar{u}^z, \bar{u}^y) \quad \{z, y\} = \{\alpha, \beta\},\ z \neq y \tag{1'}$$

with $U^z$ strictly increasing in $g^z$ and $g^z$ strictly increasing in each component of $x^z$. Resorting again to the envelope theorem, and denoting by $\lambda^z$ the Lagrangean multiplier associated with problem (2), we find that

$$\tilde{E}^z_z \equiv \partial\tilde{E}^z / \partial u^z = \lambda^z \tag{6a}$$

$$\tilde{E}^z_{\bar{z}} \equiv \partial\tilde{E}^z / \partial\bar{u}^z = -\lambda^z(\partial U^z / \partial\bar{u}^z) \equiv -\lambda^z U^z_{\bar{z}} = -\tilde{E}^z_z U^z_{\bar{z}} \tag{6b}$$

$$\tilde{E}^z_{\bar{y}} \equiv \partial\tilde{E}^z / \partial\bar{u}^y = -\lambda^z(\partial U^z / \partial\bar{u}^y) \equiv -\tilde{E}^z_z U^z_{\bar{y}}. \tag{6c}$$

In view of (5) and (6), and since $E^z_{\bar{y}} = \tilde{E}^z_y$, we can now re-introduce $\delta$ from Kemp and Shimomura (2003b: equation (8)) as

$$\begin{aligned}\delta &\equiv E^z_z E^y_y - E^z_y E^y_z \\ &\equiv \partial E^z(p, u^z, u^y) / \partial u^z \\ &= \tilde{E}^z_z \tilde{E}^y_y[(1 - U^z_{\bar{z}})(1 - U^y_{\bar{y}}) - U^z_{\bar{y}} U^y_{\bar{z}}].\end{aligned} \tag{7}$$

Since $\tilde{E}^z_z > 0$ and $\tilde{E}^y_y > 0$,

$$\delta < 0 \text{ iff } \quad \Gamma \equiv (1 - U^z_{\bar{z}})(1 - U^y_{\bar{y}}) - U^z_{\bar{y}} U^y_{\bar{z}} < 0. \tag{8}$$

Next, we recall (from Kemp and Shimomura 2003b: equation (5)) that, for the Kemp-Shimomura paradox, it is necessary that $E^z_y + E^y_y > 0$ or, in view of (5) and (6), that

$$\tilde{E}^z_{\bar{y}} + (\tilde{E}^y_y + \tilde{E}^y_{\bar{y}}) = -\tilde{E}^z_z U^z_{\bar{y}} + \tilde{E}^y_y(1 - U^y_{\bar{y}}) > 0. \tag{9}$$

Finally, setting $u^z = \bar{u}^z$ and totally differentiating (1′),

$$(1 - U^z_{\bar{z}})du^z - U^z_{\bar{y}} du^y = U^z_g dg^z(x^z) \quad \{z, y\} = \{\alpha, \beta\},\ z \neq y \tag{10}$$

where $U^z_g \equiv \partial U^z / \partial g^z$. Solving (10) for $du^z$ and $du^y$, we obtain

$$\begin{aligned}&du^z = (1/\Gamma)[(1 - U^y_{\bar{y}})U^z_g dg^z(x^z) + U^z_{\bar{y}} U^y_g dg^y(x^y)] \\ &\{x, y\} = \{\alpha, \beta\},\ z \neq y.\end{aligned} \tag{11}$$

Recalling (9) and (10), we see that if

$$1 - U^y_{\bar{y}} < 0 \text{ and } U^z_{\bar{y}} < 0 \tag{12}$$

then

$$[(1/U_g^z)du^z/dg^z(x^z)]_{dx^y=0} = (1/\Gamma)(1-U_{\bar{y}}^y) > 0 \tag{13a}$$

$$[(1/U_g^y)du^z/dg^y(x^y)]_{dx^z=0} = (1/\Gamma)(1-U_{\bar{y}}^z) > 0. \tag{13b}$$

Evidently (11) is compatible with $\underset{\cdot}{\delta} < 0$ if $1 - U_{\bar{z}}^z$ is sufficiently small in absolute value and if $U_{\bar{y}}^z$ is sufficiently large in absolute value, where $\{(z, y)\} = \{\alpha, \beta\}, z \neq y$. Thus, given (8), (9) and (12), the Kemp-Shimomura transfer paradox is compatible with utility increasing in own consumption in any world equilibrium in which all externalities have been internalized. Assumptions 9.1 and 9.2 can be replaced by

*Assumption 9.1′* $\quad E_\alpha^\alpha < 0,\ E_\beta^\beta < 0,\ E_\alpha^\alpha + E_\alpha^\beta > 0,\ E_\beta^\alpha + E_\beta^\beta > 0.$

# 11 A theory of voluntary unrequited international transfers

## 11.1 Introduction

The present theory of voluntary and unrequited international transfers (foreign aid), most of it developed during the post-Second World War period, rests on two incompatible assumptions:

1 The well-being of each country depends only on the consumption vector of that country, implying that each country is indifferent to the well-being of its trading partners and therefore has no incentive to offer aid.
2 Voluntary unrequited international transfers do take place.

One may wonder how such a serious flaw could have been so long overlooked. Perhaps the explanation lies in the fact that the present theory has long served in the analysis of involuntary unrequited transfers (war indemnities). However that may be, we now seek to remove the inconsistency by allowing for the possibility that the well-being of each country is influenced by the well-being of other countries; that is, we introduce an international externality. Given the externality, it no longer makes sense to treat the extent of foreign aid as arbitrary. Instead, it must be chosen optimally by the donor; that is, it must maximize the donor's well-being, with full allowance for feedbacks from the recipient and other countries.

For the most part, we confine attention to a simple world of just two countries, each of which produces two commodities with the aid of two or more primary factors of production. Our central proposition states that either neither country extends aid to the other or one country extends aid and both countries benefit from the aid, and that there exist some acceptable (Arrow-Debreu) economies such that neither country extends aid to the other and other acceptable economies such that one country extends aid to the other. This differs substantially from the conventional conclusion, which we owe to Paul Samuelson (1947: 29n.), that, given dynamic stability of the Walrasian kind, the recipient necessarily benefits from arbitrary aid while the donor is necessarily harmed.

## 11.2 General analysis

Two countries, $\alpha$ and $\beta$, produce and trade in two commodities, 1 and 2. Let $p$ denote the relative price of the first commodity, $u^j$ the per capita utility or well-being of the $j$th country, $e^j(p, u^\alpha, u^\beta)$ the expenditure function of the $j$th country, $r^j(p)$ the national income function of the $j$th country and $E^j(p, u^\alpha, u^\beta) \equiv e^j(p, u^\alpha, u^\beta) - r^j(p)$ the excess expenditure function of the $j$th country, $j = \alpha$ and $\beta$. The national income functions are derived from conventional convex technologies and from primary factor endowments. The technologies may differ from country to country. In each country the primary factors are at least two in number and in completely inelastic supply; accordingly, they are not explicit in the national income functions.

Continuing with the notation, let $T^{\alpha\beta}$ (positive or negative) denote the amount transferred from $\alpha$ to $\beta$. Of course, $e^j$, $r^j$, $E^j$ and $T^{\alpha\beta}$ are in terms of the numeraire, commodity 2. Finally, let subscripts indicate partial or total derivatives; for example, $E^j_p \equiv \partial E^j/\partial p$ and $r^j_p \equiv dr^j/dp$. The 'direct' utility derivatives of $e^j$, and therefore of $E^j$, are positive:

$$\partial E^j/\partial u^j > 0 \text{ for } j = \alpha, \beta \text{ and for all feasible values of } u^\alpha \text{ and } u^\beta. \tag{A1}$$

The signs of the 'cross' derivatives, on the other hand, are assumed to depend on the relative values of $u^\alpha$ and $u^\beta$:

$$\begin{aligned} &\partial E^\alpha/\partial u^\beta \lesseqgtr 0 \text{ if and only if } u^\alpha \lesseqgtr k^\alpha u^\beta \\ &\partial E^\beta/\partial u^\alpha \lesseqgtr 0 \text{ if and only if } u^\beta \lesseqgtr k^\beta u^\alpha \end{aligned} \tag{A2}$$

where $k^\alpha$ and $k^\beta$ are positive and summarize the notions of distributive justice that prevail in $\alpha$ and $\beta$. In an extreme case, $k$ is so large (small) that $u^\alpha <$ $(>)$ $ku^\beta$ for all feasible $(u^\alpha, u^\beta)$. Further assumptions will be introduced later.

In terms of the notation introduced in this section, we have two aggregate budget constraints, one for each country,

$$E^\alpha(p, u^\alpha, u^\beta) = -T^{\alpha\beta}, \tag{1}$$

$$E^\beta(p, u^\beta, u^\alpha) = T^{\alpha\beta}, \tag{2}$$

and the market-clearing condition for the first commodity,

$$E^\alpha_p(p, u^\alpha, u^\beta) + E^\beta_p(p, u^\alpha, u^\beta) = 0. \tag{3}$$

Equations (1)–(3) can be viewed as determining $u^\alpha$, $u^\beta$ and $p$ in terms of feasible $T^{\alpha\beta}$. Let us assume that a solution exists and is unique. By varying

$T^{\alpha\beta}$ within the bounds of feasibility, we can trace the locus of equilibrium pairs $(u^{\alpha}(T^{\alpha\beta}), u^{\beta}(T^{\alpha\beta}))$. Given the assumption of uniqueness, each point on the locus is associated with a different feasible value of $T^{\alpha\beta}$; moreover, as the associated point moves farther from the initial (pre-aid) equilibrium, $T^{\alpha\beta}$ grows in magnitude.

Let us focus on a particular feasible $T^{\alpha\beta}$, possibly but not necessarily equal to zero, and let us examine the slope of the locus at the associated point. For present purposes we can view (3) as determining $p$ in terms of $u^{\alpha}$ and $u^{\beta}$: $p = \tilde{p}(u^{\alpha}, u^{\beta})$. Differentiating (3) totally, we obtain

$$\left(E^{\alpha}_{pp} + E^{\beta}_{pp}\right)dp + \left(E^{\alpha}_{pu^{\alpha}} + E^{\beta}_{pu^{\alpha}}\right)du^{\alpha} + \left(E^{\alpha}_{pu^{\beta}} + E^{\beta}_{pu^{\beta}}\right)du^{\beta} = 0$$

whence

$$\begin{aligned} \partial\tilde{p} / \partial u^{\alpha} &= -\left(E^{\alpha}_{pu^{\alpha}} + E^{\beta}_{pu^{\alpha}}\right) / \left(E^{\alpha}_{pp} + E^{\beta}_{pp}\right) \\ \partial\tilde{p} / \partial u^{\beta} &= -\left(E^{\alpha}_{pu^{\beta}} + E^{\beta}_{pu^{\beta}}\right) / \left(E^{\alpha}_{pp} + E^{\beta}_{pp}\right). \end{aligned} \tag{4}$$

Turning to (1) and (2), differentiating totally with respect to $T^{\alpha\beta}$, recalling (4) and solving for $du^{\alpha}/dT^{\alpha\beta}$ and $du^{\beta}/dT^{\alpha\beta}$, we find that

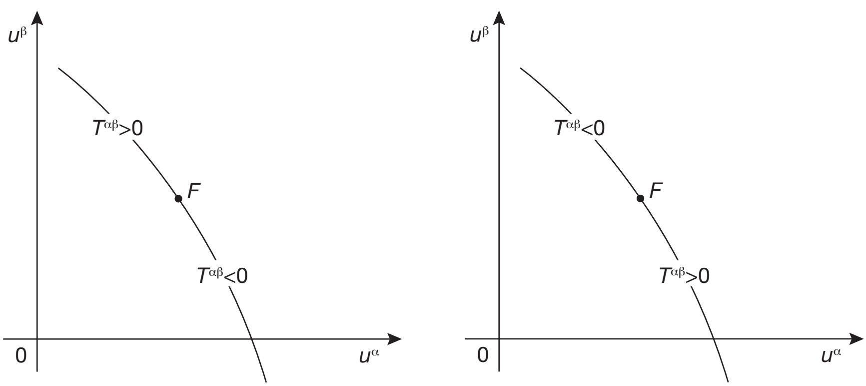


*Figure 11.1(a) & (b)*

$$\begin{aligned} \Delta\left(du^{\alpha} / dT^{\alpha\beta}\right) &= -\left(E^{\alpha}_{u^{\beta}} + E^{\beta}_{u^{\beta}}\right) \\ \Delta\left(du^{\beta} / dT^{\alpha\beta}\right) &= \left(E^{\alpha}_{u^{\alpha}} + E^{\beta}_{u^{\alpha}}\right) \end{aligned} \quad , \tag{5}$$

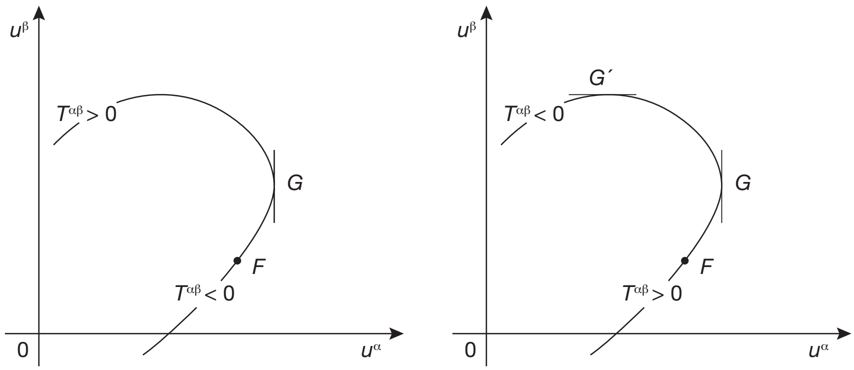


*Figure 11.2(a) & (b)*

where

$$\begin{aligned}\Delta \equiv &\left(E^{\alpha}_{u^\alpha}E^{\beta}_{u^\beta} - E^{\alpha}_{u^\beta}E^{\beta}_{u^\alpha}\right)\\ &+\Big\{E^{\alpha}_{p}\left[E^{\beta}_{u^\alpha}\left(E^{\alpha}_{pu^\beta}+E^{\beta}_{pu^\beta}\right)-E^{\beta}_{u^\beta}\left(E^{\alpha}_{pu^\alpha}+E^{\beta}_{pu^\alpha}\right)\right]\\ &-E^{\beta}_{p}\left[E^{\alpha}_{u^\alpha}\left(E^{\alpha}_{pu^\beta}+E^{\beta}_{pu^\beta}\right)-E^{\alpha}_{u^\beta}\left(E^{\alpha}_{pu^\alpha}+E^{\beta}_{pu^\alpha}\right)\right]/\left(E^{\alpha}_{pp}+E^{\beta}_{pp}\right)\Big\}/\left(E^{\alpha}_{pp}+E^{\beta}_{pp}\right)\end{aligned} \tag{6}$$

is the determinant of the coefficient matrix, evaluated at the equilibrium associated with the chosen value of $T^{\alpha\beta}$ and, from the assumption of uniqueness, is non-zero and therefore either always positive or always negative. Hence the slope of the locus is

$$du^\alpha / du^\beta = -\left(E^{\alpha}_{u^\beta}+E^{\beta}_{u^\beta}\right)/\left(E^{\alpha}_{u^\alpha}+E^{\beta}_{u^\alpha}\right). \tag{7}$$

It is possible, but perhaps implausible, that the locus is everywhere negatively sloped, as in Figure 11.1. Let $F$ be the initial equilibrium point with a zero transfer ($T^{\alpha\beta} = 0$). If, as in Figure 11.2(a), points on the locus north-west (south-east) of $F$ are associated with positive (negative) values of $T^{\alpha\beta}$, then neither country would choose to aid the other: any feasible transfer would benefit the recipient but harm the donor; and if, as in Figure 11.2(b), points on the locus north-west (south-east) of $F$ are associated with negative (positive) values of $T^{\alpha\beta}$, then each country would choose to offer aid but neither country would accept it. Thus, in each case, the equilibrium would stay at $F$.

Alternatively, the locus might be positively sloped, either near one or both of its endpoints (as in Figure 11.2) or elsewhere.[1] In particular, from (7), the locus is positively sloped near its endpoints if

$$\begin{aligned} E^{\alpha}_{u^{\beta}} &\to -\infty \text{ as } u^{\beta} \to \min u^{\beta} \\ E^{\beta}_{u^{\alpha}} &\to -\infty \text{ as } u^{\alpha} \to \min u^{\alpha} \end{aligned} \tag{A3}$$

and

$$E^{\alpha}_{u^{\alpha}} \text{ and } E^{\beta}_{u^{\beta}} \text{ are finite for all feasible } (u^{\alpha}, u^{\beta}). \tag{A4}$$

(However, it will shortly be shown by example that (A4) is not a necessary assumption.) If the initial equilibrium (with $T^{\alpha\beta} = 0$) can be represented by a point on one of the positively sloped segments of the locus, one of the countries has an incentive to offer aid and the other has an incentive to accept all or part of the aid offered. Thus if, as in Figure 11.2(a), points on the locus counter-clockwise (clockwise) from $F$ are associated with positive (negative) values of $T^{\alpha\beta}$, then country $\alpha$ has an incentive to offer whatever aid will shift the equilibrium to point $G$, and $\beta$ has an incentive to accept the offer in full; and if, as in Figure 11.2(b), points on the locus counter-clockwise (clockwise) from $F$ are associated with negative (positive) values of $T^{\alpha\beta}$, then country $\beta$ has an incentive to offer whatever aid will shift the equilibrium to point $G'$, and $\alpha$ has an incentive to accept the smaller amount associated with point $G$. In each case, the equilibrium shifts to the north-east of $F$ and both countries benefit. If, on the other hand, the initial equilibrium can be represented by a point on $GG'$, the negatively sloped segment of the locus, then the equilibrium stays there.

The foregoing analysis suggests that our general proposition is valid. However, it is not decisive. For it rests on the assumption that, given $T^{\alpha\beta}$, equations (1)–(3) possess a unique solution. Moreover, to establish that the utility locus may contain increasing and decreasing segments is not to establish that it may contain an increasing segment containing the initial equilibrium point (with $T^{\alpha\beta} = 0$) or that it may contain a decreasing segment containing the initial equilibrium point.

## 11.3 An example

In the present section we show by example that there exist excess expenditure functions that satisfy (A1)–(A3) and ensure the existence of a unique initial equilibrium (with $T^{\alpha\beta} = 0$), which can be represented by a point chosen either on a positively sloped segment or on a negatively sloped segment of the utility locus. Moreover, we show how to allocate a positive or negative value of $T^{\alpha\beta}$ to each point on the locus other than the initial equilibrium point.

Let us begin by specifying our example, focusing for the time being on country $\alpha$. The direct utility function for $\alpha$ takes the form

$$u^{\alpha} = f\left(g\left(c_1^{\alpha}, c_2^{\alpha}\right), u^{\beta}\right), \quad c_1^{\alpha}, c_2^{\alpha} \geq 0, \quad u^{\beta} > 0 \tag{8}$$

where $c_i^{\alpha}$ is the per capita consumption of the $i$th commodity, $g(c_1^{\alpha}, c_2^{\alpha})$ is linearly homogeneous and strictly quasi-concave in $c_1^{\alpha}$ and $c_2^{\alpha}$,

$$f\left(g,u^{\beta}\right)=\begin{cases} u^{\beta}\exp\left[1+\dfrac{u^{\beta}}{ln\left(1-g/e\right)}\right] \text{ if } 0\leq g<e \\ u^{\beta}g \quad \text{if } e\leq g \end{cases} \tag{9}$$

and $e \sim 2.78$ is the base of the natural logarithms.

We pause to show that $f$ is strictly increasing in $g$, implying that the direct utility function is homothetic in $c_1^{\alpha}$ and $c_2^{\alpha}$. We begin by noting that, when positive $g$ converges to zero, $u^{\beta}/ln(1-g/e)$ diverges to $-\infty$, implying that $\exp[1+u^{\beta}/ln(1-g/e)]$ converges to zero. That is, for any positive $u^{\beta}$, $u^{\alpha} = f(0, u^{\beta}) = 0$. Thus, zero is the minimum level of $\alpha$'s utility. Next, for any given and positive $u^{\beta}$ and for $g \in [0, e)$, $ln(1-g/e)$ is monotonically decreasing in $g$; and, for $g \geq e$, $u^{\beta}g$ also is monotonically increasing in $g$. Finally,

$$\lim_{g\uparrow e} u^{\beta}\exp\left[1+u^{\beta}/ln\left(1-g/e\right)\right]=u^{\beta}e.$$

Thus, for any positive $u^{\beta}$, $f(g, u^{\beta})$ is strictly increasing in $g$.

Given the utility function (8), the expenditure function for $\alpha$ is

$$e^{\alpha}\left(p,u^{\alpha},u^{\beta}\right)=h\left(p\right)U^{\alpha}\left(u^{a},u^{\beta}\right), \tag{10}$$

where

$$U^{\alpha}\left(u^{\alpha},u^{\beta}\right)\equiv\begin{cases} e\left[1-\exp\left(-\dfrac{u^{\beta}}{1-ln\left(u^{\alpha}/u^{\beta}\right)}\right)\right] & \text{if } eu^{\beta}>u^{\alpha}\geq 0 \\ u^{\alpha}/u^{\beta} \quad \text{if } u^{\alpha}\geq eu^{\beta}>0 \end{cases} \tag{11}$$

and $h(p)$ is the unit expenditure function defined as

$$h\left(p\right)\equiv \min pc_1^{\alpha}+c_2^{\alpha}, \text{ s.t. } 1\leq g\left(c_1^{\alpha},c_2^{\alpha}\right). \tag{12}$$

That completes our discussion of the direct utility and expenditure functions of $\alpha$. It is assumed that $\beta$'s direct utility and expenditure functions are of the same form as $\alpha$'s; one set of functions may be obtained from the other by interchanging the superscripts $\alpha$ and $\beta$. This assumption implies that the two countries have populations of the same size. Proceeding, we add (1) and (2) then substitute the specific expenditure functions (10) to obtain

$$\sum_{j=\alpha,\beta} U^{j}\left(u^{\alpha},u^{\beta}\right)=\frac{1}{h\left(p\right)}\sum_{j=\alpha,\beta} r^{j}\left(p\right). \tag{13}$$

Again inserting the specific expenditure functions, this time in the market-clearing equation (3), we find that

$$\sum_{j=\alpha,\beta} U^j\left(u^\alpha, u^\beta\right) = \frac{1}{h_p\left(p\right)} \sum_{j=\alpha,\beta} r_p^j\left(p\right). \tag{14}$$

It then follows from (13) and (14) that

$$\frac{1}{h\left(p\right)} \sum_{j=\alpha,\beta} r^j\left(p\right) = \frac{1}{h_p\left(p\right)} \sum_{j=\alpha,\beta} r_p^j\left(p\right) \tag{15}$$

Given the homotheticity of preferences and the conventional assumptions imposed on technologies and factor endowments, and therefore on $r^j(p)$, $j = \alpha, \beta$, the terms of trade $p$ are uniquely determined by (15). Let us denote the equilibrium terms of trade by $p^*$. Recalling (13), the locus of utilities can now be obtained from

$$\sum_{j=\alpha,\beta} U^j\left(u^\alpha, u^\beta\right) = \frac{1}{h\left(p^*\right)} \sum_{j=\alpha,\beta} r\left(p^*\right) \equiv \Gamma, \tag{16}$$

where $\Gamma$ is a positive constant. We already know that min. $u^\alpha = 0 =$ min. $u^\beta$. Thus, the utility locus begins on one axis and ends on the other.

We now turn our attention to the slope of the locus. Consider the function $U^\alpha(u^\alpha, u^\beta)$. Its level or indifference curves are displayed in Figure 11.3. The non-linear curves are described by the equation

$$u^\alpha = u^\beta \exp\left(1 - u^\beta / v\right) \equiv \theta_v\left(u^\beta\right), \quad v \geq 0, u^\beta > 0,$$

with higher curves associated with larger values of a parameter $v$. It is easy to verify that, for finite $v$, $\theta_v(0) = 0 = \theta_v(\infty)$, and that $d\theta_v(u^\beta)/du^\beta = (1 - u^\beta/v)\exp(1 - u^\beta/v)$. However, as $v \to \infty$ the non-linear curves converge to the straight line $Ob_0$ with slope $e$. Above $Ob_0$, the indifference curves are straight lines with slopes greater than $e$. Recalling (11), however, $U(u^\alpha, u^\beta)$ is not defined for $u^\beta = 0$. Hence the indifference curves do not reach the origin.

With the aid of Figure 11.3, we can verify that the excess expenditure functions satisfy (A1)–(A3). That (A1) is satisfied is immediately apparent from the figure. Next, noting the values taken by $U^\alpha$ along the dashed line $la_2l'$ of the figure, we can see that it reaches a minimum at $a_2$, implying that (A2) is satisfied. Finally, since the utility value of each straight level curve is given by its slope, the utility value of the vertical straight line is infinity, implying that (A3) is satisfied. Thus, the function $U^\alpha(u^\alpha, u^\beta)$ satisfies all of conditions (A1)–(A3). By similar reasoning, $U^\beta(u^\alpha, u^\beta)$ also satisfies (A1)–(A3). Notice that in the present example, $k^\alpha = 1 = k^\beta$.

We can now demonstrate that the utility locus associated with our example contains a positively sloped portion. In the light of the foregoing analysis, the locus can be represented in the following way.

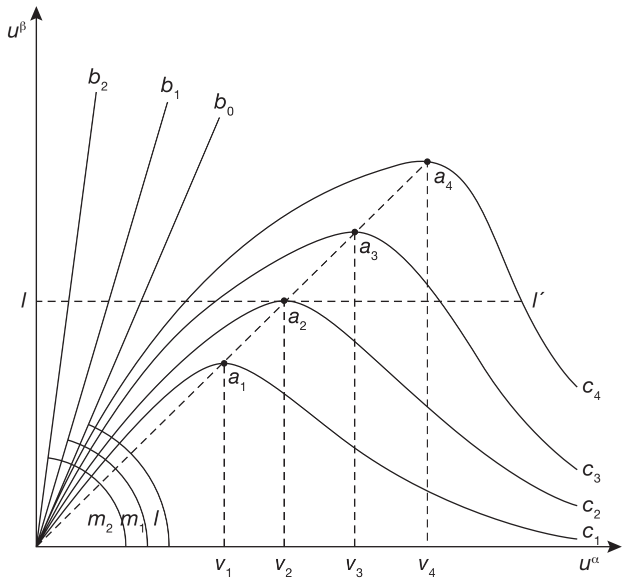


*Figure 11.3*

(i) For $u^\beta/u^\alpha \geq e$,

$$U^\alpha\left(u^\alpha,u^\beta\right)=e\left[1-\exp\left(-\frac{u^\beta}{1-ln\left(u^\alpha / u^\beta\right)}\right)\right],$$

$$U^\beta\left(u^\alpha,u^\beta\right)=u^\beta / u^\alpha,$$

and the locus consists of those pairs $(u^\alpha, u^\beta)$ that satisfy

$$U^\alpha+U^\beta=\frac{u^\beta}{u^\alpha}+e\left[1-\exp\left(-\frac{u^\beta}{1-ln\left(u^\alpha / u^\beta\right)}\right)\right]=\Gamma. \tag{17}$$

(ii) For $e > u^\beta/u^\alpha > 1/e$.

$$U^\alpha\left(u^\alpha,u^\beta\right)=e\left[1-\exp\left(-\frac{u^\alpha}{1-ln\left(u^\beta / u^\alpha\right)}\right)\right],$$

$$U^\beta\left(u^\alpha,u^\beta\right)=e\left[1-\exp\left(-\frac{u^\beta}{1-ln\left(u^\alpha / u^\beta\right)}\right)\right],$$

and the locus consists of those pairs ($u^\alpha$, $u^\beta$) that satisfy

$$U^\alpha + U^\beta = e\left[1-\exp\left(-\frac{u^\alpha}{1-ln\left(u^\beta / u^\alpha\right)}\right)\right] + e\left[1-\exp\left(-\frac{u^\beta}{1-ln\left(u^\alpha / u^\beta\right)}\right)\right] = \Gamma$$

(iii) For $1/e \geq u^\beta/u^\alpha > 0$,

$$U^\alpha\left(u^\alpha, u^\beta\right) = u^\alpha / u^\beta,$$

$$U^\beta\left(u^\alpha, u^\beta\right) = e\left[1-\exp\left(-\frac{u^\alpha}{1-ln\left(u^\beta / u^\alpha\right)}\right)\right],$$

and the locus consists of those pairs ($u^\alpha$, $u^\beta$) that satisfy

$$U^\alpha + U^\beta = u^\alpha / u^\beta + e\left[1-\exp\left(-\frac{u^\alpha}{1-ln\left(u^\beta / u^\alpha\right)}\right)\right] = \Gamma. \tag{18}$$

Let us focus for the time being on case (i). Rewriting (14), we obtain

$$\Gamma - u^\beta / u^\alpha = e\left[1-\exp\left(-\frac{u^\alpha\left(u^\beta / u^\alpha\right)}{1+ln\left(u^\beta / u^\alpha\right)}\right)\right]. \tag{19}$$

If $u^\beta/u^\alpha = e$, this equation reduces to

$$\Gamma = e\left[2-\exp\left(-\frac{eu^\alpha}{2}\right)\right],$$

or, solving for $u^\alpha$, to

$$u^\alpha = -\frac{2}{e}ln\left(2-\frac{\Gamma}{e}\right).$$

Hence $u^\alpha$ is positive if $e < \Gamma < 2e$. On the other hand, as $u^\alpha / u^\beta$ approaches $\Gamma$ from below, the left-hand side of (19) goes to zero; and, recalling that $\Gamma > e > 1$, so that $1 + ln\ \Gamma > 1$, the right-hand side of (19) goes to zero if and only if $u^\alpha$ goes to zero. Thus, we arrive at the following result.

*Lemma 11.1* If $2e > \Gamma > e$, then, along the utility locus,

$$u^\alpha \begin{cases} = -\left(2/e\right)ln\left(2-\Gamma/e\right) & \text{if } u^\beta / u^\alpha = e \\ \to 0 \text{ as } u^\beta / u^\alpha \to \Gamma & \end{cases}.$$

It follows from the lemma that there exist $(u^\beta/u^\alpha)^*$ and $(u^\beta/u^\alpha)^{**}$, $e \leq (u^\beta/u^\alpha)^* < (u^\beta/u^\alpha)^{**} \leq \Gamma$, such that, for all $u^\beta/u^\alpha$, i.e., $(u^\beta/u^\alpha)^* < u^\beta/u^\alpha < (u^\beta/u^\alpha)^{**}$, the locus is positively sloped.

Let us now turn our attention to case (iii), rewriting (18) as

$$e\left[1-\exp\left(-\frac{u^\alpha}{1-ln\left(u^\beta/u^\alpha\right)}\right)\right]=\Gamma-u^\alpha/u^\beta. \tag{20}$$

If $u^\beta/u^\alpha = 1/e$, (20) reduces to

$$e\left[2-\exp\left(-u^\alpha/2\right)\right]=\Gamma,$$

whence, solving,

$$u^\alpha=-2\ ln\left(2-\Gamma/e\right)>0.$$

On the other hand, as $u^\alpha/u^\beta$ approaches $\Gamma$ from above, the right-hand side of (20) goes to zero and the left-hand side goes to zero if and only if $u^\alpha$ goes to zero.

*Lemma 11.2* If $2e > \Gamma > e$, then, along the utility locus,

$$u^\alpha\begin{cases}=-2\ ln\left(2-\Gamma/e\right)>0 & \text{if } u^\beta/u^\alpha=1/e\\ \to 0 \text{ as } u^\beta/u^\alpha\to 1/\Gamma & \end{cases}.$$

It then follows from Lemma 11.2 that there exist $(u^\beta/u^\alpha)^\dagger$ and $(u^\beta/u^\alpha)^{\dagger\dagger}$, $1/e \geq (u^\beta/u^\alpha)^\dagger > (u^\beta/u^\alpha)^{\dagger\dagger} \geq 1/\Gamma$, such that, for all $u^\beta/u^\alpha$, $(u^\beta/u^\alpha)^\dagger > u^\beta/u^\alpha > (u^\beta/u^\alpha)^{\dagger\dagger}$, the locus is positively sloped.

Figure 11.4 is compatible with the information so far gleaned about the utility locus. However, we have not yet allocated positive and negative $T^{\alpha\beta}$ values to the points on the locus. That is our next task. Since $h(p)U^\alpha - r^\alpha = -T^{\alpha\beta}$ and $h(p)U^\beta - r^\beta = T^{\alpha\beta}$, $T^{\alpha\beta}$ is negative if $U^\beta = 0$ and positive if $U^\alpha = 0$. On the other hand, we have shown that $U^\alpha \to \Gamma$ as $u^\beta/u^\alpha \to \Gamma$. Hence, recalling that $U^\alpha + U^\beta = \Gamma$, $U^\beta$ must converge to zero as $u^\beta/u^\alpha \to \Gamma$. Thus, approaching the origin along the utility locus with a counter-clockwise motion, $T^{\alpha\beta} = h(p)U^\beta - r^\beta \to -r^\beta < 0$. By the same logic, approaching the origin along the locus in a clockwise way, $-T^{\alpha\beta} = h(p)U^\alpha - r^\alpha \to -r^\alpha < 0$. It follows that, if the initial equilibrium point (with $T^{\alpha\beta} = 0$) is unique, then $T^{\alpha\beta}$ is negative or positive as we move in a counter-clockwise or clockwise direction from that point.

Our work is still not quite done. Suppose that the initial equilibrium (with $T^{\alpha\beta} = 0$) can be represented by an interior point $F$ in the lower segment $OG$ of Figure 11.5. To the left of $F$ in that segment, $T^{\alpha\beta} > 0$; and at any

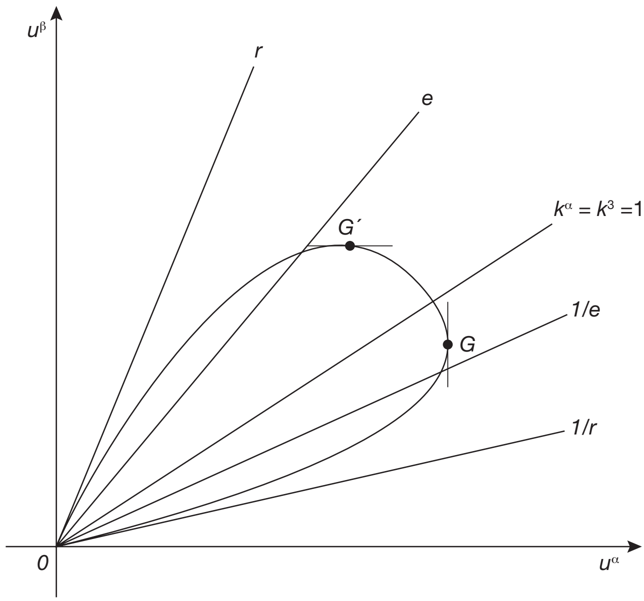


*Figure 11.4*

other point in the locus except $F$ itself, $T^{\alpha\beta} < 0$. Hence country $\beta$ has an incentive to aid $\alpha$. Indeed, *only* if $F$ lies in the lower segment $OG$ does $\beta$ have that incentive. Similarly, if and only if $F$ lies in the upper segment $OG'$ does country $\alpha$ have an incentive to aid $\beta$.

These observations suggest a vital question: can our example be tailored to ensure that the initial equilibrium is represented by any arbitrarily chosen point of the utility locus? In particular, can the example be tailored to ensure that the initial equilibrium is represented by a point in an upward-sloping segment (or in the downward-sloping segment)?

Indeed it can. We have noted that $p^*$, the equilibrium price ratio, is independent of $T^{\alpha\beta}$. It follows that we can ensure that the initial equilibrium is represented by any point in the locus simply by changing the national income functions to $\hat{r}^\alpha(p)$ and $\hat{r}^\beta(p)$ subject to the condition that $\hat{r}^\alpha(p^*) + \hat{r}^\beta(p^*) = r^\alpha(p^*) + r^\beta(p^*)$.

Now, at last, we can return to our proposition, stated in Section 11.1. In the case represented by Figure 11.5, country $\beta$ offers to $\alpha$ whatever aid is associated with point $G'$ but $\alpha$ accepts only the smaller amount associated with point $G$. If, on the other hand, the initial point $F$ were in the upper segment $OG'$ of the locus, country $\alpha$ would offer to $\beta$ whatever aid is associated with point $G$, but $\beta$ would accept only the smaller amount associated with point $G'$. And finally, if the initial point $F$ were in the downward-sloping

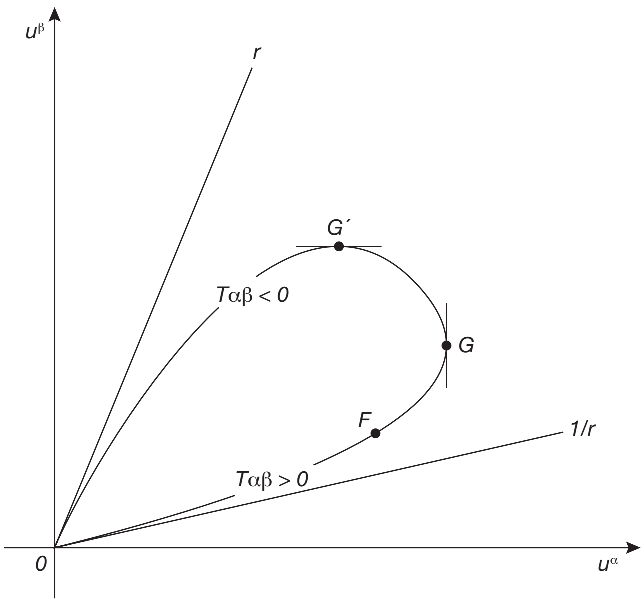


*Figure 11.5*

segment, between $G'$ and $G$, each country would offer aid, but neither country would accept it; the pre-and post-aid equilibria would coincide.

## 11.4 A final remark

In the conventional theory of voluntary unrequited transfers, the addition of a third country (a 'bystander') throws up paradoxical possibilities. Thus, David Gale (1974) showed that, if there is a bystander, the donor might be left better off and the recipient might be left worse off by a transfer; and it was later added that these possibilities would remain even if the world economy was stable in the sense of Walras. However, in Gale's model as in Samuelson's, the utility functions imply that the donor and recipient are quite indifferent to each other's well-being. When that inadequacy is repaired, the possibility of paradoxes disappears. Thus, suppose that a third country $\gamma$, indifferent to the well-being of $\alpha$ and $\beta$, is added to the model. A budget constraint for $\gamma$ must be written in, and the market-clearing condition (3) must accommodate the excess demand of $\gamma$ for the first commodity. However, a (new) locus in $(u^\beta, u^\alpha)$-space can be derived, and it can contain both positively and negatively sloped segments. Thus, additional countries do not generate paradoxes if the transfers are voluntary and if utility functions accommodate international caring.

Again in the conventional theory, but this time with just two countries, the imposition of local Walrasian stability is quite decisive in ruling out paradoxical outcomes. In our own analysis, on the other hand, there has been no mention of it. The reason is simple: the assumption of local Walrasian stability does not significantly curtail the variety of possible outcomes. Both Figure 11.2(a) and Figure 11.2(b) are compatible with local stability for each feasible value of $T^{\alpha\beta}$. In fact, our model enjoys global Walrasian stability if, as we implicitly assume, each commodity is indispensable in consumption, so that the excess demand for a commodity is positive for any positive but sufficiently low relative price. Figure 11.6 illustrates.

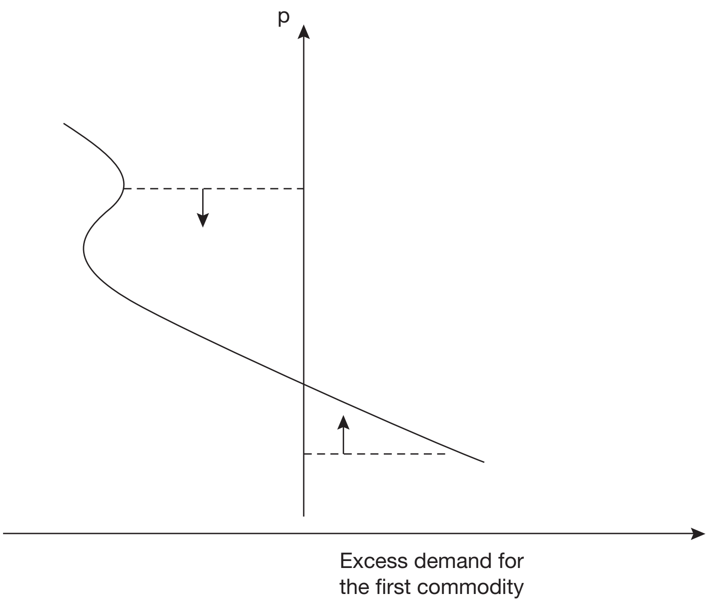


*Figure 11.6*

# 12 Aid tied to the donor's exports

## 12.1 Introduction

Until recently, foreign aid was, for the most part, government to government; and much of inter-governmental aid was tied by the requirement that at least a specified proportion of the aid be spent by the recipient on the exports of the donor.

Aid tied in that way attracted the attention of theorists. However, the theory that emerged focused on the particular case in which only private consumption goods are produced, consumed and traded; see Ohyama (1974), Kemp and Kojima (1985) and Schweinberger (1990). In those papers it was assumed, in effect, that the recipient receives the aid, spends the required proportion $m$ on the donor's exports, the balance on other goods, then distributes to its households the basket of goods obtained in this way. Evidently, the recipient's households have an incentive to untie the aid by disposing of unwanted goods on world markets. Hence, the existing theory makes sense only if the recipient's households are somehow prevented from re-selling on world markets. This seems to imply direct and unwelcome intervention by the recipient in the decision-making of its households. Much the same objection applies to an alternative assumption, that the recipient subsidizes its households' consumption of the donor's exports to whatever extent is required by the donor.

The root of the difficulty is that in the emerging theory only private consumption goods are recognized. In a context of foreign aid the assumption is unrealistic, for much of inter-governmental aid is now in terms of dams, bridges and highways. In the present paper, therefore, I admit the possibility that consumption goods have a dual function: they can be privately consumed or they can be transformed into public consumption goods. On the basis of that assumption, I put forward a model of trade and aid that allows us to explore the implications of tying but that is free of the weaknesses of existing theory. In the model proposed, the recipient converts the aid basket into public consumption goods that are then made available without cost to its households. Since the public goods cannot be traded by households, the model does not rely on direct intervention by government in household decision-making.

To keep complications to a minimum, it will be assumed that the conversion of traded goods into public goods is costless. However, this strict assumption could be relaxed without destroying the conclusions. It will also be assumed, conventionally, that, in each trading country, households are identical in all respects and that, overlooking the tying constraint, each country is a free trader, imposing neither taxes on foreign trade nor internal taxes on consumption or production. These last assumptions are mutually compatible for, as Kemp and Shimomura (1995) have argued, identical households that know that they are identical will 'see through' commodity taxes and conduct their affairs as though the taxes do not exist.

Only minor formal change needs be made to the earlier analysis of Kemp and Kojima (1985). However, substantial changes in interpretation are required. Nevertheless, one of the principal conclusions of Kemp and Kojima remains intact. Thus, it will be shown that, even in a world of just two trading countries and two traded commodities, tied aid might benefit the donor and harm the recipient; that is, paradoxes might recur without the intervention of third or 'bystander' countries.

## 12.2 Analysis

There are two countries, $\alpha$ and $\beta$, and there are two traded commodities, 1 and 2. Each of the traded commodities is a private consumption good. In an initial world trading equilibrium, country $\alpha$ exports commodity 1 and country $\beta$ exports commodity 2. The initial equilibrium is disturbed when $\alpha$ extends aid to $\beta$.

The following notation will be employed:

$T$ the amount of aid, in terms of commodity 2, from country $\alpha$ (the donor) to country $\beta$ (the recipient); initially, $T=0$;

$p$ the price of commodity 1 in terms of commodity 2;

$u^j$ the utility derived from privately budgeted consumption in country $j$ ($j = \alpha, \beta$);

$v^\beta$ the utility derived in country $\beta$ from consumption not privately budgeted, that is, from the consumption of public goods; initially, $v^\beta = 0$;

$w^\beta \equiv u^\beta + v^\beta$ the welfare of country $\beta$;

$e^j(p, u^j)$ the private expenditure function of country $j$, expenditure in terms of commodity 2 ($j = \alpha, \beta$);

$r^j(p)$ the private revenue function of country $j$, revenue in terms of commodity 2 ($j = \alpha, \beta$);

$x^{ji}(p)$ the private output of commodity $i$ in country $j$ ($i = 1, 2; j = \alpha, \beta$);

$c^{ji}(p, u^j)$ the compensated private demand for commodity $i$ by country $j$ ($i = 1, 2; j = \alpha, \beta$);

$z^{ji}(p, u^j) \equiv c^{ji}(p, u^j) - x^{ji}(p)$ the excess private demand for commodity $i$ by country $j$ ($i = 1, 2; j = \alpha, \beta$).

The aid is government to government. In $\alpha$ there is already an absolute sufficiency of public consumption goods; hence the aid is financed by a lump sum tax. The private budget constraint in $\alpha$ is therefore

$$e^{\alpha}(p, u^{\alpha}) = r^{\alpha}(p) - T. \tag{1}$$

The aid is spent by the government of $\beta$, $mT$ on the first commodity, $(1-m)T$ on the second commodity; the basket of goods obtained in this way is therefore $[mT/p, (1-m)T]$. The basket of goods is passed on to the households of $\beta$, the same share to each household. Simply by passing through the hands of government officials, the commodities (without cost) change their characteristics, turning into desirable public goods evaluated by the second of the two household utility functions $v$, with $v(0, 0) = 0$. The public goods are not tradable on international markets. Moreover, given the separability of $w^{\beta}$ in $u^{\beta}$ and $v^{\beta}$, the public goods do not enter the private budget constraint:

$$e^{\beta}(p, u^{\beta}) = r^{\beta}(p). \tag{2}$$

The description of world equilibrium is completed by adding to equations (1) and (2) the market-clearing condition

$$z^{\alpha 1}(p, u^{\alpha}) + z^{\beta 1}(p, u^{\beta}) + mT/p = 0. \tag{3}$$

Equations (1)–(3) contain the three variables $u^{\alpha}$, $u^{\beta}$ and $p$, as well as the parameter $T$. The system is assumed to possess a unique solution $(p^*, u^{\alpha *}, u^{\beta *})$, with $p^*$ positive and finite.

We wish to know how each of the three variables responds to a small change in $T$. Differentiating (1)–(3) with respect to $T$ we find that

$$\begin{pmatrix} e_p^{\alpha} - r_p^{\alpha} & 1 & 0 \\ e_p^{\beta} - r_p^{\beta} & 0 & 1 \\ z_p^{\alpha 1} + z_p^{\beta 1} & z_u^{\alpha 1} & z_u^{\beta 1} \end{pmatrix} \begin{pmatrix} dp \\ du^{\alpha} \\ du^{\beta} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -m/p \end{pmatrix} dT \tag{4}$$

where subscripts indicate differentiation ($e_p^{\alpha} \equiv \partial e^{\alpha}/\partial p$, $r_p^{\alpha} \equiv dr^{\alpha}/dp$, etc.). Recalling the envelope result that $e_p^j - r_p^j = z^{j1}$ ($j = \alpha, \beta$), choosing units of utility so that $e_u^j = 1$, and recalling that $T = 0$, (4) reduces to

$$\begin{pmatrix} z^{\alpha 1} & 1 & 0 \\ z^{\beta 1} & 0 & 1 \\ z_p^{\alpha 1} + z_p^{\beta 1} & z_u^{\alpha 1} & z_u^{\beta 1} \end{pmatrix} \begin{pmatrix} dp \\ du^{\alpha} \\ du^{\beta} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -m/p \end{pmatrix} dT \tag{5}$$

Solving,

$$\Delta\frac{dp}{dT}=-\frac{m-pz_u^{\alpha 1}}{p}\qquad \Delta\frac{du^\alpha}{dT}=-(z_p^{\alpha 1}+z_p^{\beta 1})-\frac{z^{\beta 1}(m-pz_u^{\beta 1})}{p} \tag{6}$$

$$\Delta\frac{du^\beta}{dT}=\frac{z^{\beta 1}(m-pz_u^{\beta 1}}{p}$$

where

$$\Delta \equiv z_p^{\alpha 1}+z_p^{\beta 1}+z^{\beta 1}(z_u^{\alpha 1}-z_u^{\beta 1}) \tag{7a}$$

is the determinant of the matrix of coefficients in (5).

We seek to attach a sign to $\Delta$. To this end, consider the dynamic system consisting of (1), (2) and

$$\dot{p}=z^{\alpha 1}(p,u^\alpha)+z^{\beta 1}(p,u^\beta). \tag{8}$$

Linearizing the system at the equilibrium values of the variables, we obtain

$$\begin{aligned}
&z^{\alpha 1}\cdot(p-p^*)+(u^\alpha-u^{\alpha}*)=0\\
&z^{\beta 1}\cdot(p-p^*)+(u^\beta-u^{\beta}*)=0\\
&\dot{p}=z_p^{\alpha 1}\cdot(p-p^*)+z_u^{\alpha 1}\cdot(u^\alpha-u^{\alpha}*)+z_p^{\beta 1}\cdot(p-p^*)+z_u^{\beta 1}\cdot(u^\beta-u^*)
\end{aligned} \tag{9}$$

where the functions $z^{ji}$, $z_p^{ji}$ and $z_u^{ji}$ are evaluated at ($p^*$, $u^{\alpha *}$, $u^{\beta *}$). Eliminating ($u^\alpha - u^{\alpha *}$) and ($u^\beta - u^{\beta *}$), and defining $\pi \equiv p - p^*$, (9) reduces to

$$\dot{\pi} = \Delta\pi \tag{10}$$

For local stability of the linear system it is necessary and sufficient that

$$\Delta < 0 \tag{7b}$$

Returning to (6), we see immediately that if the system (9) is stable and if the tying proportion $m$ exceeds the donor households' marginal propensity to consume the donor's exported commodity, that is, if $m > pz_u^{\alpha 1}$, then the terms of trade must turn in favour of the donor. Can they turn so far in favour of the donor as to cancel the welfare loss directly associated with aid? Bearing in mind that the pure substitution terms $z_p^{\alpha 1}$ *and* $z_p^{\beta 1}$ are negative and that, by assumption, $z^{\beta 1} > 0$, it follows from (6) and (7) that for both stability and $du^\alpha/dT > 0$ it is sufficient and almost[1] necessary that

$$-\frac{z^{\beta 1}}{p}(m-pz_u^{\beta 1})<z_p^{\alpha 1}+z_p^{\beta 1}<-z^{\beta 1}(z_u^{\alpha 1}-z_u^{\beta 1}) \tag{11}$$

Evidently, this condition can be satisfied without inferiority. However, it does require that the recipient's offer curve be inelastic at the initial equilibrium point. Consider the first inequality of (11). Making use of the well-known relationship between substitution terms

$$pz_p^{\beta 1} + z_p^{\beta 2} = 0$$

and of the identity between marginal propensities to consume

$$pz_u^{\beta 1} + z_u^{\beta 2} = 1,$$

that inequality can be rewritten as

$$z_p^{\beta 2} - z^{\beta 1} z_u^{\beta 2} < pz_p^{\alpha 1} - (1-m) z^{\beta 1} < 0. \tag{12}$$

However the left-hand expression in (12) is the total derivative $dz^{\beta 2}/dp$; hence, $-dz^{\beta 2}/d(1/p) < 0$. Thus, the recipient's offer of its export commodity decreases when its terms of trade improve, implying that the recipient's offer curve is inelastic.

The fate of the recipient is easily determined. Since the initial equilibrium is Pareto-efficient, a small change in the real income of $\alpha$ must be accompanied by an opposite change in the real income of $\beta$. Indeed, given the normalization $e_u^j = 1$,

$$\frac{dw^{\beta}}{dT} = -\frac{du^{\alpha}}{dT}.$$

Thus we arrive at the following proposition:

*Proposition 12.1* Suppose that aid is wholly tied in the recipient country, wholly untied in the donor. Then, if and only if condition (11) is satisfied, the world economy is stable, the donor benefits from aid and the recipient suffers.

## 12.3 Final remarks

In Section 12.2 the focus was on a common defect of available theories of tied aid – they contain only freely tradable private consumption goods. However, all theories of voluntary foreign aid, whether the aid is tied or untied, share a second weakness. They are based on the assumption that the well-being of each country depends solely on its consumption of goods and is quite independent of the well-being of its trading partners. This is a very convenient assumption; but it leaves unexplained why any country would ever offer aid to another. Kemp and Shimomura (2002a) have recently shown that this defect can be corrected by introducing international externalities of well-being. However, in the corrected model there is an incentive to offer aid, but there is no incentive to tie the aid offered. For in that (two-by-two) model a donor can improve the well-being of a recipient efficiently (that is, at least cost in terms of its own well-being) only in the absence of all distortions, including those related to tying. On the other hand, in a world of three countries, the donor might be able to achieve its objective (of raising the recipient's well-being) at least cost to itself if its aid is tied to its own exports and if the third country (to the well-being of which both the donor and recipient are indifferent) exports the same commodity as the donor; for in those circumstances the tying of aid might enable the donor and recipient to jointly exploit the third country. Of course, in such a multi-country case, the exploitation would be more efficiently achieved by discriminatory tariffs; but, in contrast to tied aid, discriminatory tariffs are ruled out by GATT.

## Addendum (2007): The welfare implications of reciprocally untying foreign aid

Chapter 12 was drafted several years ago as part of an international discussion in which Ohyama (1974), Kemp and Kojima (1985), Schweinberger (1990) and Kemp (2005) all participated. Throughout that discussion, the analytical focus was on a world that produces only consumption goods, either private consumption goods (as in the above papers) or public consumption goods (as in the main body of Chapter 12). That focus was not inappropriate, given the customs of donor governments in the 1960s and 1970s. Since that period, however, donor governments have increasingly focused on aid in the form of public goods, both public consumer goods (such as medical facilities) and public producer goods or infrastructure (such as educational facilities, highways and railroads, dockyards and reservoirs). Each donor's choice of aid package is normally made after consultation with the recipient government and with other potential donors. This trend is understandable in view of the long-run needs of most less-developed countries.

While donor governments finance such projects, the actual provision of public goods is usually left to private firms, which bid or tender for the privilege. Access to the tendering process is usually restricted to firms that, in some sense, are 'from' the donor or recipient countries; the aid is 'tied' in this novel sense. However, there is now a substantial measure of agreement among the leading donor governments that, without loss of control and with some gain in efficiency, firms from any country might be allowed to bid for any aid-financed contract. Indeed, this is already the operating policy of the United Kingdom; and, very recently, the rest of the European Union (EU) has moved in the same direction by passing legislation untying its aid to the 'least developed countries' and by untying all of its aid on a reciprocal basis with other donor nations: If you allow our firms to bid for your aid-financed contracts, we will allow your firms to bid for our aid-financed contracts.

Evidently, if all donor countries were to accept this principle, then each donor government would be freed of the need to determine the 'home' of a firm seeking to bid for that government's aid-financed contracts.

The EU legislation has been welcomed by other donor nations. Indeed the untying of all aid has been described in an Australian government white paper (Australian Government 2006: 23) as 'international best practice'. However, that generous assessment may need qualification.

It is clear that, in the absence of distorting tariffs and other taxes and under conventional assumptions of the Walras-Arrow-Debreu-McKenzie type, the reciprocal untying of any given pattern of aid in terms of durable public goods unambiguously expands the set of world production possibilities. However, even under those assumptions, individual countries (whether donor or recipient) might be harmed by the untying of aid. If, in fact, not all countries are free-trading and without domestic commodity taxes, we cannot even be sure that the reciprocal untying of a given pattern of aid will enhance world efficiency; the principle of second best rules out that generalization.[2]

Since we do still inhabit a tax-ridden world economy and since, even if that were not so, some households would be harmed by the untying, the white paper's assessment of untying can be accepted only with caution. Certainly the assessment cannot be based solely on expected improvements in national efficiency.

# 13 Variable returns to scale and factor price equalization[1]

## 13.1 Introduction

In their classical expositions of the Factor Price Equalization (FPE) theorem, Heckscher (1919), Lerner (1952), Samuelson (1948, 1949) and McKenzie (1955) provided sufficient conditions for the equality of equilibrium factor rewards in two or more countries. Those conditions invariably included the specification of perfectly competitive markets supported by convex production sets and freedom of entry. The focus on convexity continues in modern textbook presentations of the theory, suggesting a widespread belief that FPE is 'less likely' in a context of non-convexities. In contrast, we shall argue that if the non-convexities flow from external economies associated with changes in worldwide industry outputs then the existing theory of FPE is already sufficiently general to accommodate the non-convexities. In particular, it will be shown that, leaving aside singular cases in which the input vectors of industries are linearly dependent, the set of international factor assignments compatible with FPE is of full rank if and only if there are at least as many tradable commodities as primary factors.

The assumption that externalities are associated with changes in *worldwide* outputs is essential to our conclusions; for, without it, the efficiency of each country depends on its size. Moreover, the assumption is still unconventional even though it is nearly seventy years since Allyn Young's classical contribution (1928) and a decade and a half since the championing of the assumption by Wilfred Ethier (1979). However, the globalization of the economy continues apace and, sooner or later, the assumption will appear to be the natural one.

## 13.2 Analysis

Let there be just two countries ($\alpha$ and $\beta$), just two industries, each producing a single tradable good, and just two primary factors of production, labour and land. The same technology prevails in each country. Everywhere, individual *firms* perceive that they operate under constant returns to scale. However, external economies and diseconomies of production are admitted and give rise to increasing and decreasing returns to the *industries*. As already emphasized, these economies and diseconomies are associated with changes in the industries' worldwide outputs.

Consider the Edgeworth box of Figure 13.1. The dimensions of the box are determined by the world endowments of labour and land. Point $E$ represents an integrated competitive world equilibrium, attainable when both factors of production are costlessly mobile between $\alpha$ and $\beta$. In that equilibrium, the labour:land ratio of the first industry is indicated by the slope of $O_\alpha E$ and that of the second industry is indicated by the slope of $O_\beta E$. In the absence of externalities and non-constant returns, the parallelogram $O_\alpha E O_\beta E'$ is the Lancaster (1957)-Travis (1964) FPE region.[2] Evidently the region is convex, is symmetrical about the diagonal $O_\alpha O_\beta$ and has full dimensionality of two. However, it is clear that, even in a context of externalities and non-constant returns, the parallelogram has the same properties and admits of the same interpretation.

Thus we have shown that, in the familiar 2×2×2 case, *the dimensionality of the set of endowments compatible with FPE is independent of scale returns.* For that conclusion, it suffices that an integrated world equilibrium exist (it need not be unique) and that the externalities be linked to worldwide outputs.

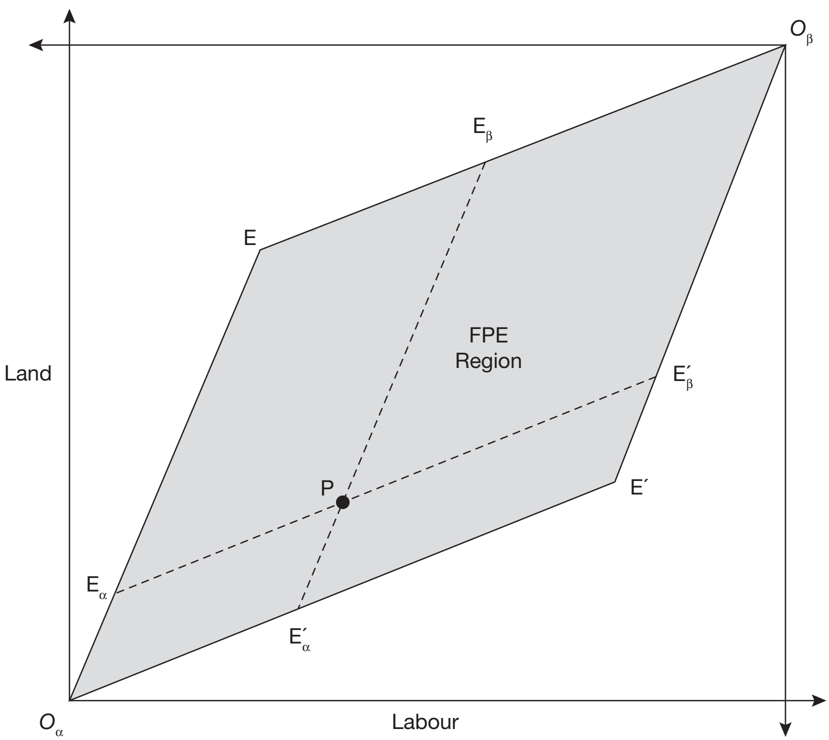


*Figure 13.1* The FPE region in a 2 × 2 world economy

The italicized conclusion is valid in more general contexts, with $m$ traded products, $n$ internationally immobile factors and $q$ countries ($m, n, q \geq 2$). In the more general setting, the convex FPE region is of full dimensionality $n$ if in the integrated world equilibrium there are $n$ linearly independent input vectors, each associated with a particular traded good. This condition can be satisfied only if the number of produced and traded goods is not less than the number of primary factors.

Moreover, the conclusion is valid in capitalistic economies with produced means of production. Let us complicate the $2\times2\times2$ case by allowing one or both of the two products to be means of production in one or both of the two industries, without ruling out the possibility that a product may be both a consumption and an intermediate good. For simplicity only, let us suppose that intermediate goods are used up in one period of time. Then, in any country, the choice of technique depends not only on conventional primary factor rewards but also on product prices and the rate of interest. Nevertheless, if there exists an integrated world equilibrium then it can be depicted as a point in the Edgeworth box, with all inputs now taken to be total (direct *plus* indirect) inputs; and, based on that point, we can construct a two-dimensional FPE region such that, if the world endowment point lies in that region and if there are international markets for both products and for credit, then, in the unintegrated world equilibrium, factor rewards are everywhere the same.

## 13.3 International public goods

That concludes our analysis. However, it may be noted that, by essentially the same reasoning as in Section 13.2, it can be shown that the dimensionality of the set of endowments consistent with FPE is independent of the publicness or privateness of the goods produced and traded, and that this is so whether they are intermediate or final consumption goods. All that matters is that, if they be public goods, they are *international* public goods and that they are *privately* supplied.[3]

# 14 Market structure and factor price equalization[1]

## 14.1 Introduction

In their classical expositions of the Factor Price Equalization (FPE) theorem, Heckscher (1919), Lerner (1952), Samuelson (1948, 1949) and McKenzie (1955) provided sufficient conditions for the equality of equilibrium factor rewards in two or more countries. Those conditions invariably included the specification of perfectly competitive markets supported by convex production sets and freedom of entry. The focus on perfect competition continues in modern textbook presentations of the theory.[2] Moreover, Blackorby *et al.* (1993) have recently proposed a set of conditions that, they claim, are *necessary and sufficient* for FPE, implicitly including in the set the requirement that all markets be competitive.

The continuing emphasis on perfect competition in product markets suggests a widespread belief that FPE is 'less likely' if some industries are imperfectly competitive. However that may be,[3] we shall argue that, if one or more product markets are imperfectly competitive (oligopolistic, to be precise) and if the oligopolists are treated as primary factors of production, on the same footing as various categories of labour and land, then the existing theory of FPE is already sufficiently general to cover any mixture of perfectly and imperfectly competitive industries. In particular, it will be shown that, leaving aside singular cases in which the input vectors of industries are linearly dependent, the set of international factor assignments compatible with FPE is of full rank if and only if there are at least as many tradable commodities as primary factors. Thus, the findings of Blackorby *et al.* (1993) must be interpreted as *conditional* on the presence of perfect competition in all markets.

## 14.2 Simple cases

Suppose that there are just two countries ($\alpha$ and $\beta$) and just two industries, each producing a tradable good under constant returns to scale. Suppose further that the first industry is oligopolistic, the second competitive. Suppose finally that there are two primary factors of production, labour and oligopolists, each in fixed supply. Labour is employed in both industries; oligopolists

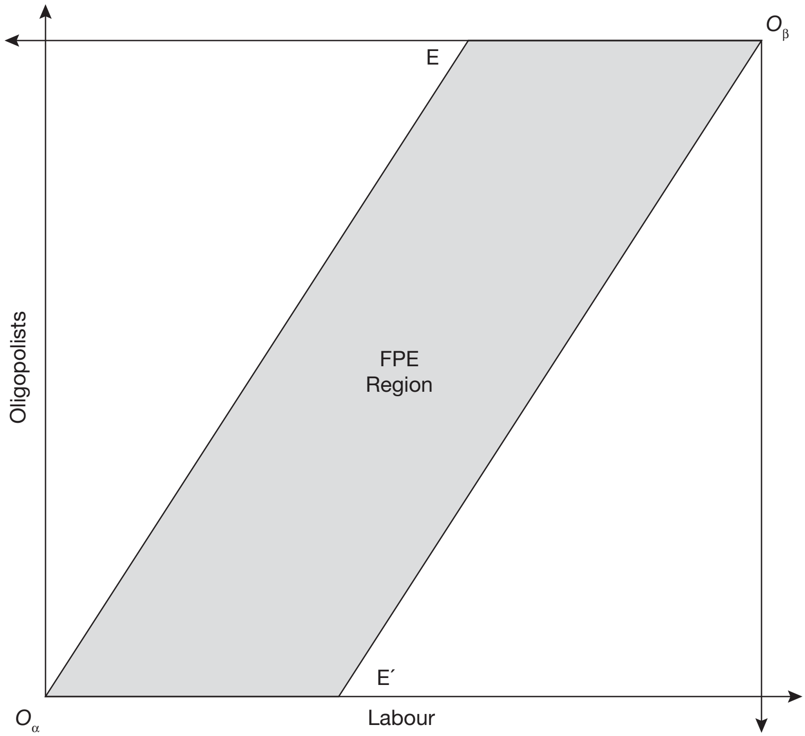


*Figure 14.1*

are found only in the first industry. Labourers earn wages; oligopolists earn profits.

Consider the modified Edgeworth box of Figure 14.1. The dimensions of the box are determined by the world endowments of labour and oligopolists. Points $E$ and $E'$ represent the integrated world equilibrium, and $O_\alpha E O_\beta E'$ is the Lancaster (1957)-Travis (1964) *FPE* region. Evidently that region is convex, is symmetrical about the diagonal $O_\alpha O_\beta$, and has full dimensionality of 2.[4]

Suppose now that both industries are oligopolistic. If the oligopolists are homogeneous and, like labour, can move freely from industry to industry, then the integrated world equilibrium is represented by points $E$ and $E'$ of Figure 14.2, and the convex FPE region again has full dimensionality of 2.

Alternatively, we can introduce two types of industry-specific oligopolist. To remain within the $2\times2$ framework, we must then assume that there are no other inputs. In this case, the integrated world equilibrium is represented by points $E$ and $E'$ of Figure 14.3, and the convex *FPE* region coincides with the box $O_\alpha E O_\beta E'$. Again, the *FPE* region has full dimensionality.

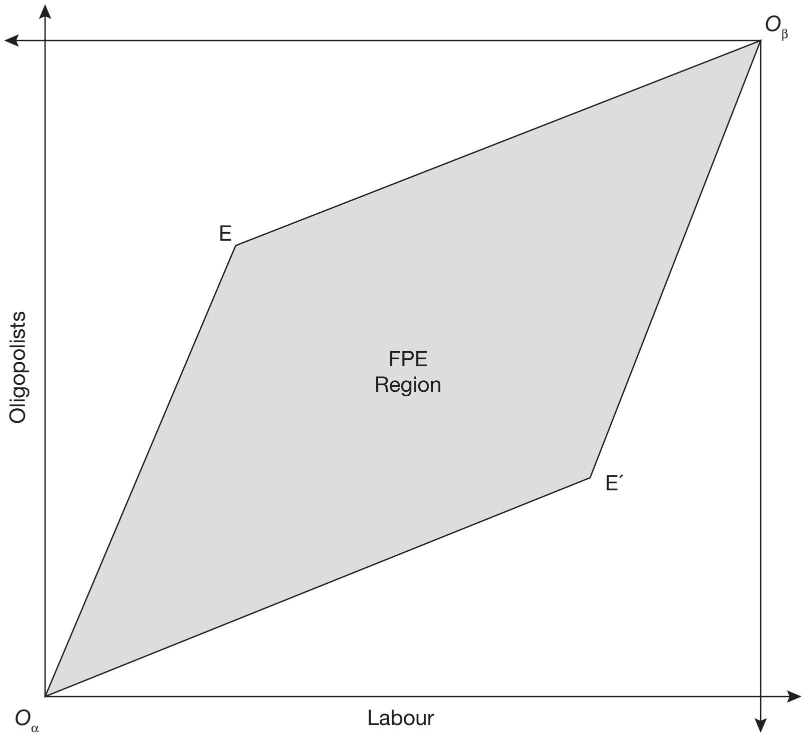


*Figure 14.2*

Taxing our geometrical skills to the limit, we continue into three dimensions. Thus, Figure 14.4 depicts the case of two products (the first oligopolistic, the second competitive) and three primary factors (labour, land and first-industry oligopolists). The integrated equilibrium is at $E$ or $E'$, where $E$ lies in the ceiling of the box and $E'$ lies in the floor. $O_\alpha E$ is the input vector of the first or oligopolistic industry and $O_\beta E$ is the input vector of the second or competitive industry. The flat $O_\alpha E O_\beta E'$ is then the convex FPE region. As required by standard competitive theory, it is of dimension 2, less than full dimension 3.

In Figure 14.5, on the other hand, there is an extra competitive industry (the first is oligopolistic, the second and third are competitive). In the integrated equilibrium, $O_\alpha E_1$ is the input vector of the first industry, $E_1 E_2$ is the input vector of the second industry and $E_2 O_\beta$ is the input vector of the third industry. Points $E_1$ and $E_2$ lie in the ceiling of the box, their counterparts $E_1'$, and $E_2'$ on the floor. The FPE region is obtained by joining all pairs of points on $O_\alpha E_1 E_2 O_\beta E_1' E_2' O$. As required, it is convex and of full dimension 3.

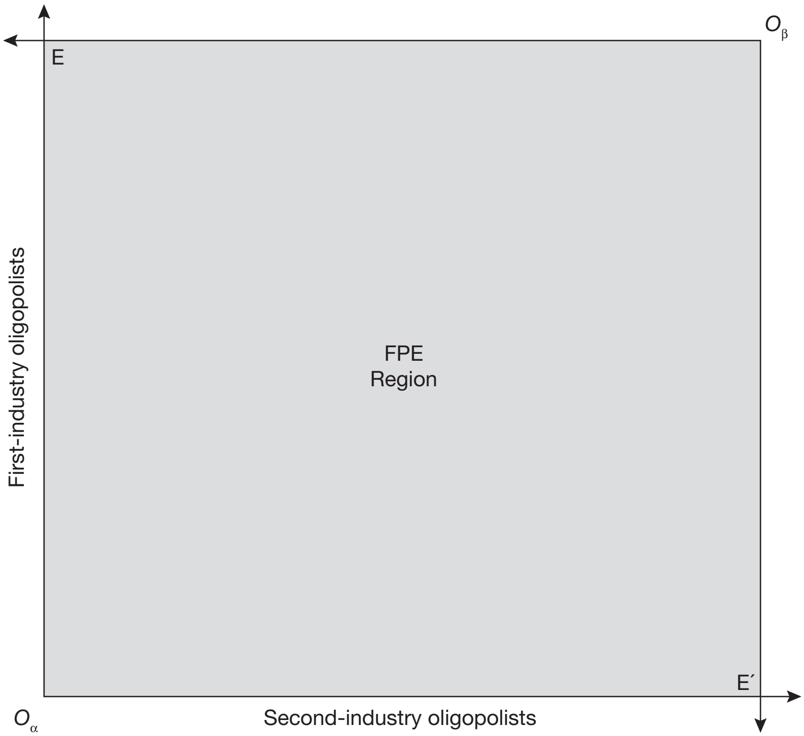


*Figure 14.3*

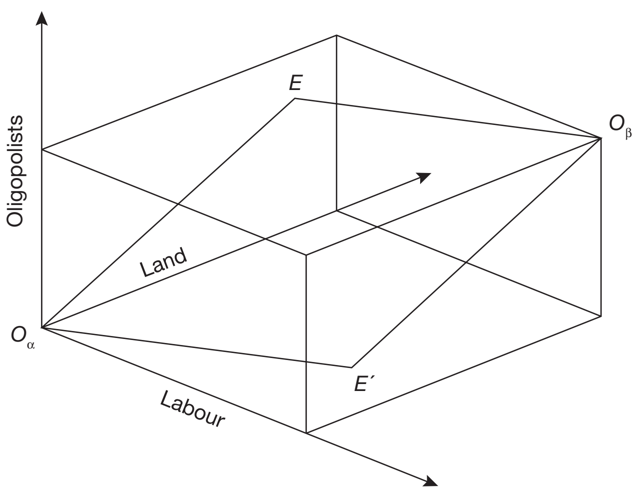


*Figure 14.4*

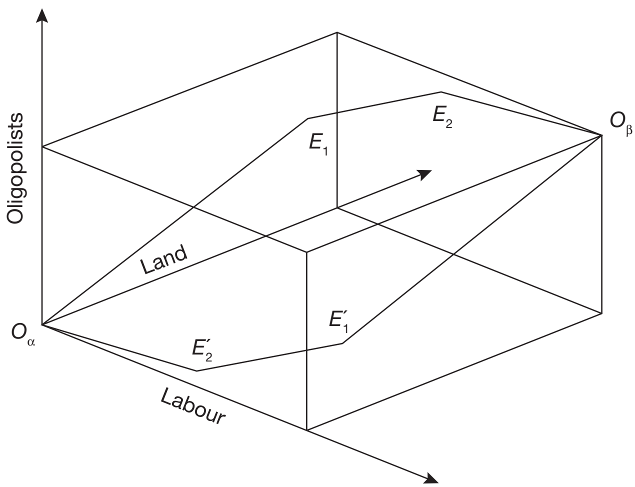


*Figure 14.5*

## 14.3 The general case

To anyone familiar with the standard competitive theory of FPE, it will be apparent that what has been established for the simple cases of Section 14.2 is true also in the more general setting of $m$ products, $n$ primary factors and $q$ countries, provided that the list of primary factors includes all kinds of oligopolists, industry-specific or otherwise. In that more general setting, the convex FPE region is of full dimensionality $n$ if in the integrated equilibrium there are $n$ linearly independent input vectors, each associated with a particular traded good. This condition can be satisfied only if the number of produced and traded goods is not less than the number of primary factors. It cannot be satisfied unless all goods produced with the aid of industry-specific oligopolists are traded on world markets. However, this is the case not because such goods are oligopolistically produced but because the oligopolists who produce the goods are industry-specific. Even when all industries are perfectly competitive, specific factors of production create this small complication.

To this point we have followed textbook tradition and dealt only with no-joint product technologies and with primary factors that are internationally immobile. However, the conclusion of the preceding paragraph is valid without those restrictions. Suppose for example that there are $n$ internationally immobile primary factors and $m$ homogeneous industries, each of which produces, either competitively or oligopolistically, some subset of $s$ traded goods. In some imperfectly competitive industries, production is in the hands of a fixed number of oligopolists, who are specific to that industry. Each type of oligopolist is listed as a primary factor. The convex FPE region is then of full dimensionality $n$ if in the integrated equilibrium there are $n$

linearly independent input vectors, each associated with a particular industry. This condition can be satisfied only if the number of produced goods is not less than the number of immobile primary factors. Moreover, we have followed textbook tradition in ruling out purely intermediate goods and confining attention to FPE between just two trading countries. However, our message, that the existing theory of FPE already covers imperfect competition, remains valid even when these simplifying assumptions are abandoned.[5]

That is all that need be said. However, it might be helpful if we sketch the production structure that lies behind our diagrams and indicate how oligopolists fit into that structure. Beginning with a single imperfectly competitive firm, we can write the production function

$$F(z, v) = \phi(z)\, G(v) \quad (z \text{ integral and non-negative}) \tag{1a}$$

where

$$\phi(z) = \begin{cases} 0 & \text{if } z = 0, \\ 1 & \text{if } z > 0, \end{cases} \tag{1b}$$

and where $z$ denotes the number of oligopolists working in the firm and $v$ is a vector of other factors employed by the firm. In this formulation, $z$ is a limitational factor of production whereas the elements of $v$ may be continuously varied by the firm. $G(v)$ may display constant or decreasing returns to scale. The production function for the industry is then

$$H(Z, V) = ZG(V/Z) \quad (Z \text{ integral and non-negative}) \tag{2}$$

where $Z$ denotes the total number of oligopolists working in the industry and $V$ is a vector of other factors employed by the industry. Evidently $H$ is homogeneous of degree one even when $G$ displays decreasing returns to scale.

Given (1) and (2), in any equilibrium with positive profits there is exactly one oligopolist in each imperfectly competitive firm.

## 14.4 Is FPE 'less likely' under imperfect competition?

In the introduction to this paper, we noted our sense that FPE is commonly viewed as 'less likely' if some industries are imperfectly competitive. However, as far as we know, only Helpman and Krugman (1985: 92–3) have offered a precise statement of this view. Defining the primary factor endowments net of oligopolists, they argue that, if some industries are imperfectly competitive, the set of endowments (one for each country) compatible with FPE 'has full dimensionality only if the number of perfectly competitive industries is at least as large as the number of factors. It differs from the conventional [perfectly competitive] case in that *it places additional constraints* *on the factor allocations*' and thus reduces the relative size of the FPE region (emphasis added). However, their reasoning depends on their exclusion of oligopolists from the list of primary factors; if oligopolists were included in the list, imperfect competition would be seen to place no additional restrictions on the factor allocations compatible with FPE. Thus, by taking as given the allocation of oligopolists, Helpman and Krugman have missed the opportunity to show that one theory covers two quite different market structures.

## 14.5 A final remark

In the present note we have avoided the conventional assumption that, in each country, all agents are identical in preferences, endowments and shareholdings. Economies satisfying that specification are covered by our analysis as special cases. However, from the argument of Kemp and Long (1992) and Kemp and Shimomura (1995), in such cases oligopolists will cooperate to achieve efficient production.

# 15 Factor price equalization when the world equilibrium is not unique

## 15.1 Introduction

This is the last of a trio of short papers that extend the scope of the Factor Price Equalization (FPE) theorem. The first two papers (Kemp and Okawa 1998; Kemp *et al.* 1998) established that factor price equalization, the possibility of which was first demonstrated under conditions of constant returns to scale and perfect competition, is, in a specified sense, just as likely under increasing returns based on production externalities of worldwide incidence and under oligopolistic market structures.

In discussions of the Factor Price Equalization theorem it is customary to assume that the world equilibrium is unique. This tradition leaves it unclear just how the theorem should be formulated to accommodate a possible multiplicity of equilibria. It is my purpose in the present note to extend the relevant theory by allowing for multiple equilibria. For the most part, I confine attention to the case of two factors of production (available as fixed endowments) and two final products, for which a diagrammatic treatment suffices. Again for the most part, I confine attention to the case of constant returns to scale and perfectly competitive markets. However, several extensions of the analysis are indicated in the final section.[1]

## 15.2 Analysis

Consider Figure 15.1, an Edgeworth production box with a little more and a little less than the usual scaffolding. The dimensions of the box are determined by the total or worldwide endowments of the two primary factors, labour and capital. Families of isoquants for the two industries, 1 and 2, are not drawn but may be imagined emanating from the origins $O_\alpha$ and $O_\beta$, respectively. If the world comprises a single country, with each factor perfectly and costlessly mobile between industries and across the face of the globe, then we might indicate an equilibrium by point $E$ on the (undrawn) contract locus, with the output of the first (second) industry proportional to $O_\alpha E$ ($O_\beta E$) and with the capital–labour ratio of the first (second) industry indicated by the slope of $O_\alpha E$ ($O_\beta E$). It is assumed that the two industries

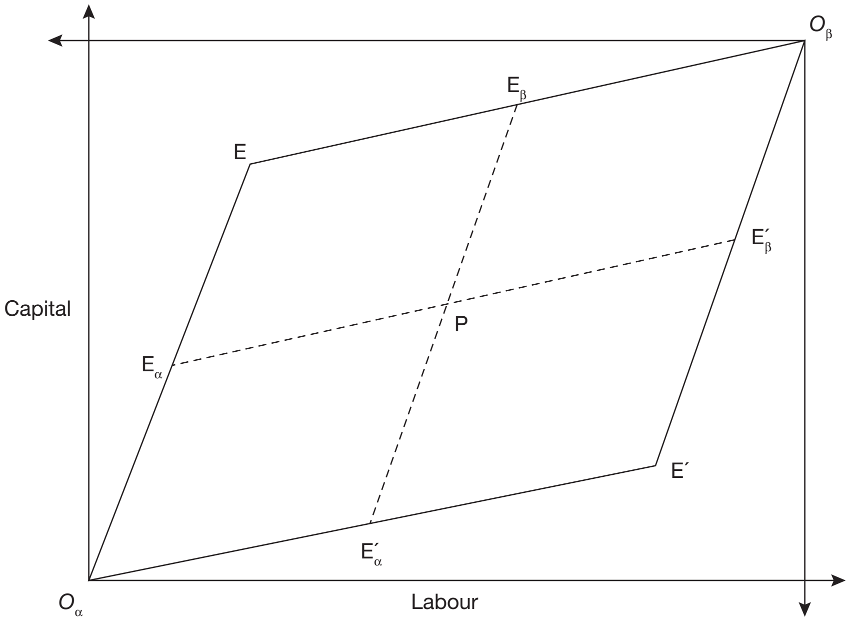


*Figure 15.1*

differ in their factor intensities, so that $E$ lies strictly above or strictly below the diagonal.[2]

Let us now partition the world into two countries, $\alpha$ and $\beta$, with equal access to the pre-existing technology, and with each factor endowment divided into two parts, one part belonging to and located within the boundaries of $\alpha$, the other part belonging to and confined within the boundaries of $\beta$, so that both factors are internationally completely immobile. Then, as is well known,[3] if and only if the division of the world factor endowment can be represented by a point in or on the boundary of the parallelogram $O_\alpha E O_\beta E'$, the aggregate world outputs and commodity and factor prices that prevailed in the pre-partition or integrated world economy must prevail also in the divided world; in particular, the same factor rewards must prevail in each country. For example, if the division of the world factor endowments between $\alpha$ and $\beta$ is represented by point $P$ in Figure 15.1, then the pre-partition prices and aggregate outputs will continue to prevail, with $\alpha$ contributing $O_\alpha E_\alpha$ ($O_\alpha E'_\alpha$) of commodity 1 (commodity 2) and $\beta$ contributing the remainder $EE_\alpha$ ($E'E'_\alpha$).

To this point it has been tacitly assumed that the integrated world equilibrium is unique. Suppose alternatively that there are two equilibria. Choosing the first commodity as numeraire, the prices that prevail in the two equilibria may be denoted by $(p', w', r')$ and $(p'', w'', r'')$, where $p$ denotes the price of the second commodity, $w$ the wage rate, and $r$ the rental of capital. Suppose that $p'' > p'$. If, as in Figure 15.2, commodity 1 is relatively capital-intensive, then, from the Stolper-Samuelson theorem, $w' < w''$ and $r' > r''$; with the double-primed prices, therefore, each industry adopts a more capital-intensive technique. Associated with the single-primed (double-primed) prices are the integrated equilibrium at $E$ $(F)$ and the *FPE* region $O_\alpha E O_\beta E'$ $(O_\alpha F O_\beta F')$ of Figure 15.2. If and only if the division of the primary factors between $\alpha$ and $\beta$ can be represented by a point in or on the boundary of $O_\alpha E O_\beta E'$ $(O_\alpha F O_\beta F')$, then FPE can prevail with prices $(p', w', r')$, $(p'', w'', r'')$. Moreover we notice that if and only if the distribution of primary factors between $\alpha$ and $\beta$ can be represented by a point in or on the boundary of $O_\alpha Z O_\beta Z'$, the region common to the two diamonds $O_\alpha E O_\beta E'$ and $O_\alpha F O_\beta F'$, then, whichever of the two sets of prices prevails, FPE occurs. In the assumed absence of factor-intensity reversal, such a common region always exists.

Proceeding, we may imagine any finite number of integrated equilibria, with prices $(p', w', r')$, $(p'', w'', r'')$, $(p''', w''', r''')$, . . . Let the commodity price

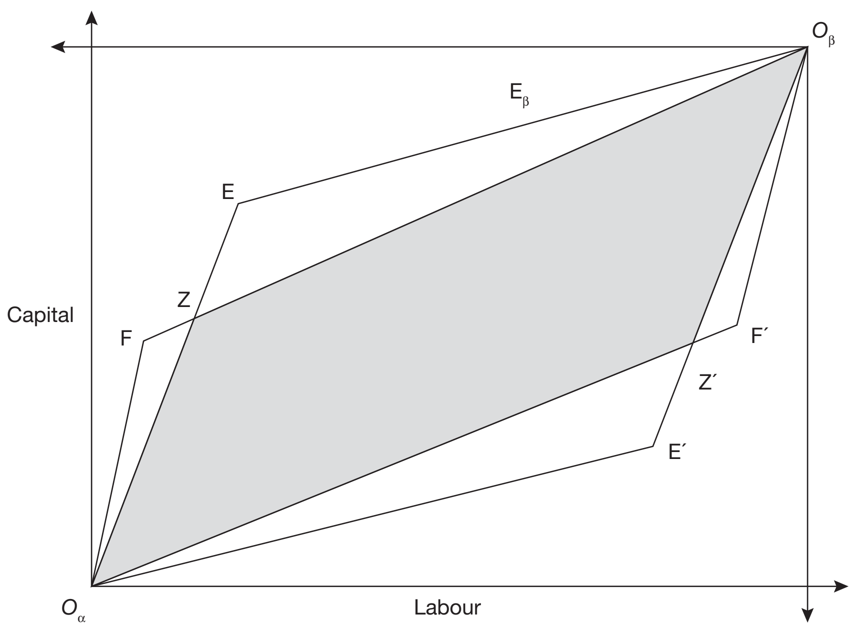


*Figure 15.2*

vectors be so arranged that $p' < p'' < p''' < \ldots$ Then, again applying the Stolper–Samuelson theorem:

$$w' < w'' < w''' < \ldots,$$

$$r' > r'' > r''' > \ldots,$$

and we can construct the associated diamonds:

$$O_\alpha EO_\beta E',\ O_\alpha FO_\beta F',\ O_a GO_\beta G',\ \ldots,$$

with a common region, say $O_a YO_\beta Y'$, which is in fact identical to the intersection of just the first and last of the array of diamonds.

*Proposition 15.1* If and only if the distribution of primary factors between $\alpha$ and $\beta$ can be represented by a point in or on the boundary of the non-convex union of the diamonds $O_\alpha EO_\beta E'$, $O_\alpha FO_\beta F'$, $\ldots$ , then FPE must be associated with at least one of the equilibrium prices $(p', w', r')$, $(p'', w'', r'')$, $\ldots$ If and only if the distribution of primary factors between $\alpha$ and $\beta$ can be represented by a point in or on the boundary of the convex intersection of the diamonds $O_\alpha EO_\beta E'$, $O_\alpha FO_\beta F'$, $\ldots$ , then FPE must prevail and may be associated with any of the equilibrium prices $(p', w', r')$, $(p'', w'', r'')$, $\ldots$

*Corollary* Whatever the multiplicity of the equilibria, the set of international factor assignments compatible with FPE is of full rank 2. In that sense, the 'likelihood' of FPE is independent of the multiplicity of the equilibria.

## 15.3 Extensions

The general message delivered by the proposition remains relevant in the general case of $m$ factors and $n$ products. However, the details depend on cases. Thus suppose that there are three factors and two products; and suppose for the time being that the integrated equilibrium is unique, represented by point $E$ in Figure 15.3. Then the relevant diamond is the two-dimensional flat $O_\alpha EO_\beta E'$. If the distribution of factors between $\alpha$ and $\beta$ can be represented by a point in or on the boundary of $O_\alpha EO_\beta E'$, then FPE is inevitable. If instead there are several integrated equilibria, then there is the same number of diamonds, all intersecting on the diagonal $O_\alpha O_\beta$; hence the non-convex union of the diamonds consists of points that lie in one or more of the two-dimensional diamonds, and the convex intersection of the diamonds consists of all points on the diagonal. If and only if the international distribution of primary factors can be represented by a point on the boundary of the non-convex union of the diamonds, then FPE must be associated with at least one of the equilibrium prices. If and only if the distribution of primary factors can be represented by a point on the diagonal, then FPE must prevail and may be associated with any of the equilibrium prices. As in the simpler

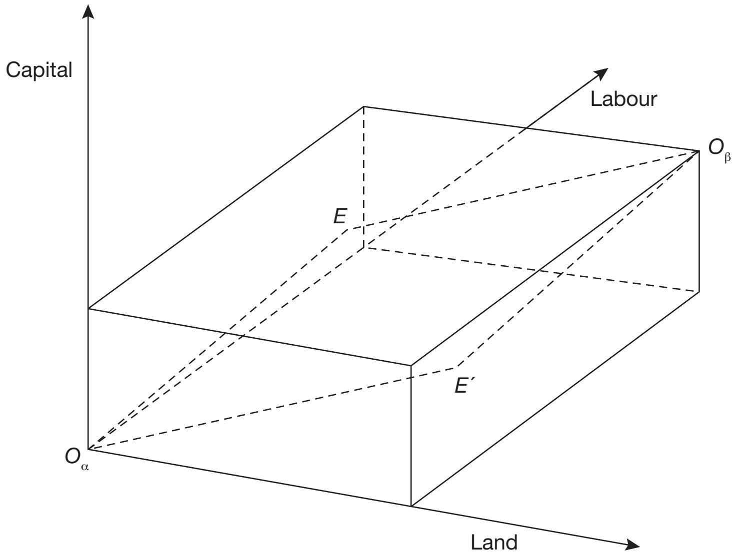


*Figure 15.3*

two-by-two world, the 'likelihood' of FPE is independent of the multiplicity of the equilibria.

Suppose on the other hand that there are three products and two factors; and suppose also, for the time being, that the integrated equilibrium is unique, represented by points $E_1$ and $E_2$ in Figure 15.4.[4] In this case, the relevant 'diamond' is the hexagon $O_\alpha E_1 E_2 O_\beta E_1' E_2'$, with opposite sides parallel and of equal length. If the distribution of factors between $\alpha$ and $\beta$ can be represented by a point in or on the boundary of $O_\alpha E_1 E_2 O_\beta E_1' E_2'$, then FPE is inevitable. If instead there are several integrated equilibria, then there is the same number of 'diamonds', each including all the diagonal points as well as points above and below the diagonal. It is easy to see that the intersection of the 'diamonds' is convex and of dimension 2. In Figure 15.4, which illustrates the case of two integrated equilibria, with $w'' > w'$ and $r'' < r'$, the intersection of the 'diamonds' is the region $O_\alpha Z_1 Z_2 Z_3 O_\beta Z_1' Z_2' Z_3'$. Again the likelihood of FPE is independent of the multiplicity of the equilibria.

Finally, it has been assumed, conventionally, that in each industry there are constant returns to scale and that each market is perfectly competitive. However, it can be inferred from the work of Kemp and Okawa (Kemp and Okawa 1998; Kemp *et al.* 1998), which deals with unique equilibria, that the above proposition can be extended to cases in which some markets are oligopolistically competitive or in which externality-based increasing returns prevail in some industries, with the externalities generated by worldwide outputs.

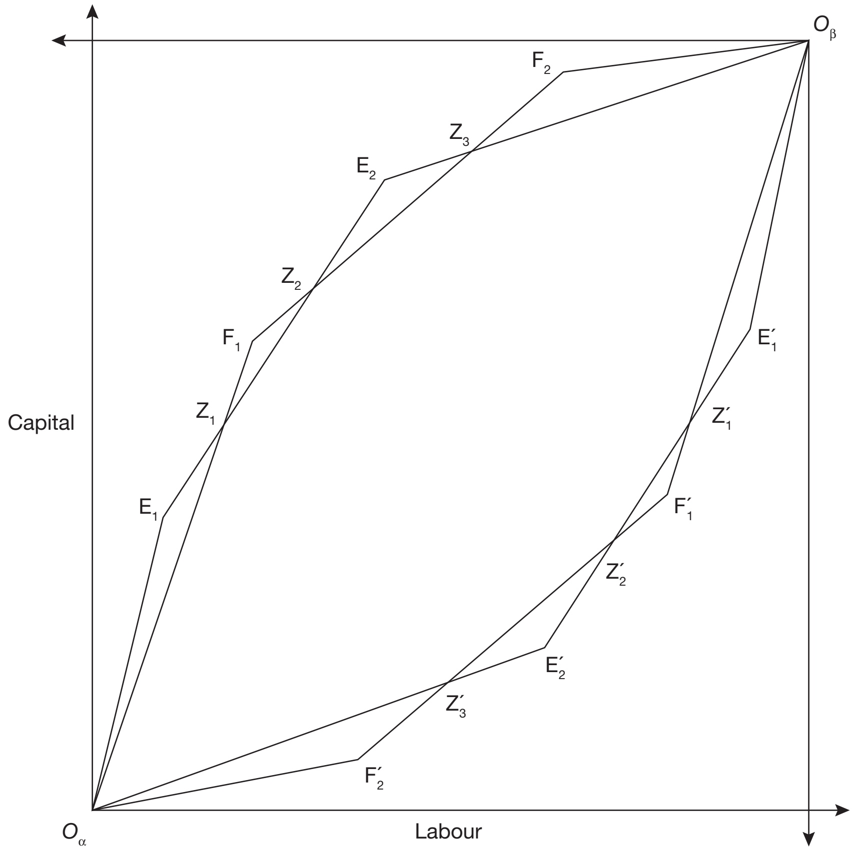


*Figure 15.4*

# 16 Factor price equalization in a world of many trading countries

## 16.1 Introduction

It is now known that if there are more than two trading countries but only the conventional pairs of primary factors and final products, then, given uniform constant-returns technologies and freedom of entry, it suffices for worldwide factor price equalization (FPE) that the factor endowment ratios of the trading countries are all bounded by the factors-in-use ratios[1] of the two industries in an integrated world equilibrium, that is, in an equilibrium of a hypothetical world economy in which both products and factors are perfectly mobile between countries. The sufficiency of the condition was conjectured by Deardorff (1994) and confirmed by Demiroglu and Yun (1999).[2]

If for any given family of trading countries the condition is not met, then FPE at integrated-economy levels is generally not possible, even for a subset of (two or more) countries all with endowment ratios bounded by the factors-in-use ratios of an integrated world equilibrium; even limited FPE, in that sense, is unlikely.

However, this bleak conclusion is forced upon us by the traditional focus on worldwide FPE and worldwide integration. If we are prepared to contemplate less-than-worldwide integration, we can obtain a more general version of the Deardorff-Demiroglu-Yun proposition, a version that allows FPE in a proper subset of the trading countries.

## 16.2 Analysis

Suppose that there are $N$ trading countries, and let us define $\boldsymbol{N} = \{1, \ldots, N\}$. We may then consider any subset $\boldsymbol{M}$ of $\boldsymbol{N}$, where $\boldsymbol{M}$ contains at least two elements, and examine conditions for FPE restricted to members of $\boldsymbol{M}$. To this end, we further define a hypothetical $\boldsymbol{M}$-integrated world economy in which all factors are mobile between countries in $\boldsymbol{M}$ but not between countries at least one of which is in $\boldsymbol{N}\backslash\boldsymbol{M}$, that is, in $\boldsymbol{N}$ but not in $\boldsymbol{M}$. (It is emphasized that all $\boldsymbol{N}$ countries, those in $\boldsymbol{M}$ and those in $\boldsymbol{N}\backslash\boldsymbol{M}$, are part of the $\boldsymbol{M}$-integrated world economy.) Finally, by reasoning very like that of Demiroglu and Yun, we obtain the following generalization of their proposition.

*Generalized FPE proposition* For FPE within $\boldsymbol{M}$ it is sufficient that the factor endowment ratios of member countries are all bounded by the factors-in-use ratios of member countries in an equilibrium of the $\boldsymbol{M}$-integrated world economy.

*Proof* Consider any equilibrium of an $\boldsymbol{M}$-integrated world economy. In that equilibrium, factor prices are equalized in $\boldsymbol{M}$ (but not necessarily at $\boldsymbol{N}$-integrated levels). Suppose that, in the $\boldsymbol{M}$-integrated equilibrium, the factor endowment ratio of each member of $\boldsymbol{M}$ is bounded by the common factors-in-use ratios of the member countries. Then, by an easy extension of the Deardorff-Demiroglu-Yun proposition, the chosen $\boldsymbol{M}$-integrated equilibrium is also an equilibrium when factors are immobile between member countries.

It is now apparent that not all is lost simply because Deardorff's potentially severe condition is not satisfied. FPE is still possible within a set $\boldsymbol{M}$ smaller than $\boldsymbol{N}$. Indeed FPE can be considered for any subset of $\boldsymbol{N}$ with at least two members. However the levels at which factor prices are equalized in general depend on the choice of $\boldsymbol{M}$. In an extreme case, $\boldsymbol{M} = \boldsymbol{N}$. Moreover, there may be several disjoint subsets of $\boldsymbol{N}$ in each of which factor prices are equalized but at levels that differ from subset to subset.

## 16.3 Final remark

Our attention has been confined to the conventional case of two primary factors and two products. However, it has recently been shown that the Deardorff-Demiroglu-Yun proposition remains valid in higher dimensions provided that either the rank of the in-use matrix of an integrated world equilibrium is 2 (see Qi 2003) or the number of products is not greater than the number of factors and the factors-in-use matrix is of full rank (see Demiroglu and Yun 1999). It follows that our generalized FPE proposition can be extended to higher dimensions under the same assumptions. Notice, however, that the extension is obtained by placing restrictions on endogenous variables rather than, conventionally, on exogenous variables. For an unsympathetic appraisal of extensions obtained in this way, see Kemp and Wan (2005).

# 17 Heckscher-Ohlin theory

Has it a future?

## 17.1 Introduction

During the last half century, that is, during my own professional lifetime, economic theorists of the general-equilibrium persuasion have worked in an intellectual environment dominated by two bodies of thought. The first body of thought is based on the pioneering work of Léon Walras, Kenneth Arrow, Gérard Debreu and Lionel McKenzie. The second body of thought is based on the work of Eli Heckscher, Bertil Ohlin, Abba Lerner and Paul Samuelson. For brevity, but un-historically, unfairly and ungrammatically, I will refer to these two bodies of thought as 'Walrasian' and 'Heckscher-Ohlin'.

Within this environment, the work of theorists has for the most part been of the comparative static kind, with their normative calculations rooted in the Walrasian paradigm and their descriptive calculations rooted in the Heckscher-Ohlin paradigm.

This neat division of labour is, at first sight, puzzling for, after all, the Heckscher-Ohlin model is merely a special case of the Walrasian. Why, then, do we persist with the Heckscher-Ohlin? The answer is painfully obvious: the Walrasian model yields no descriptive comparative statics; more precisely, it yields no descriptive comparative statics that are easy to interpret and also profound, that is, answer questions that are interesting and non-trivial. To obtain descriptive results of that kind we have taken on board the family of special Heckscher-Ohlin assumptions, and we have been rewarded with the Stolper-Samuelson, Rybczynski, Factor Price Equalization, Heckscher-Ohlin and Hicks-lkema propositions, along with modern versions of older propositions such as the Mill-Edgeworth result concerning the possibility of impoverishing growth.

Most of the Heckscher-Ohlin assumptions are well known. One easily remembers constant returns to scale, non-joint production and small numbers of just about everything; and we have long known that each of the Heckscher-Ohlin propositions must be qualified if those assumptions are relaxed. However, there are other assumptions, assumptions that are rarely mentioned, even in formal expositions of the Heckscher-Ohlin theory. Here I think especially of zero costs of factor reallocation, representative agents and the existence of autarkic equilibria for all trading countries. What we are now beginning to understand is that if the hitherto hidden assumptions are relaxed, the descriptive Heckscher-Ohlin propositions crumble and entirely disappear, leaving us, however, with the normative Walrasian propositions for comfort.[2]

With the Ohlin centennial celebrations fresh in our memories, this must be a suitable time to pause and reflect on the future role of the Heckscher-Ohlin theory.

## 17.2 The possible non-existence of autarkic equilibria

In open-economy theory it is customary to assume that all trading countries possess autarkic equilibria. However, casual observation suggests that even wealthy trading nations may lack equilibria. In particular they may lack the climate and fertile land needed for subsistence food production; one thinks of Holland, Belgium, Ireland, Singapore, and even the United Kingdom and Japan. Such countries were once able to survive in autarky; indeed, for them there once may have been no alternative to autarky. Over the years, however, natural resources may have been degraded, and trade-based wealth may have induced substantial increases in population, to the point where the economies could not survive in autarky.

It is easy to verify that, if a country has no autarkic equilibrium, and if there are just two traded goods, the offer curve of that country consists of two disjoint segments; and this in turn suggests that, if there are just two countries, the offer curves of those countries may fail to intersect. Thus the absence of autarkic equilibrium, even for one country, may ensure the absence of a world trading equilibrium.

Clearly the Heckscher-Ohlin depiction of world equilibrium in terms of an intersection of continuous offer curves must be revised. By implication, the common belief that save in exceptional cases, any distortion-free Arrow-Debreu economy must benefit from the opening of its frontiers, whatever the characteristics of other economies, must be reconsidered. Finally, the reliability of open-economy econometric estimates, which rely on the assumption of sustained market-clearing, must be questioned.[3]

## 17.3 The costly reallocation of factors

Let me now change tack and briefly consider another of the hidden Heckscher-Ohlin assumptions, that any reallocation of factors is costless. If that assumption is abandoned, so that a factor of production may earn different rewards in different industries or in different firms in the same industry, even in a stationary equilibrium, then, quite simply, all five Heckscher-Ohlin propositions must be abandoned.

Pale shadows of those propositions survive but only in truly dynamic versions of the Heckscher-Ohlin model and only for particular parametric specifications. Thus let us extend the Heckscher-Ohlin model to accommodate costly and time-consuming processes of factor reallocation and let us adopt the conventional assumption that each factor owner seeks to maximize the present value of the stream of instantaneous utilities. Then, if and only if the factor owners share a zero rate of time preference, the Heckscher-Ohlin propositions are *asymptotically* valid.[4] The commonsense of this result is not hard to find. Thus suppose, for the sake of argument, that the migration of a particular factor has stopped short of the complete equalization of earnings in all occupations. Then a further once-over and finite burst of migration towards better-paid occupations would yield an unbounded increase in the present value of that factor's earnings but would entail a once-over and therefore finite increase in the cost of moving. Hence, contrary to supposition, further migration will take place.

## 17.4 The representative agent

Changing direction again, let us now consider the assumption that, in each country, all agents are identical in all respects – preferences and endowments (including information). Invariably, but implicitly, the assumption is supported by the companion assumption that the agents are unaware that they are identical.

However in customary static or steady-growth contexts, an essentially unchanging market game is played repeatedly; hence alert and intelligent agents will soon be aware that they are identical. Thus, in customary contexts, the supporting assumption is implausible and must be replaced by the alternative supposition that agents are aware that they are identical.

But if intelligent agents know that they are identical, then they will know also that they will choose the same strategy. Hence each agent will choose the strategy that, if all other agents choose the same strategy, is socially optimal. In effect, the agents will enter into an enforceable agreement and, in the sense of Harsanyi and Selten (1988: Section 1.2), will play a cooperative game.

Evidently the foregoing argument strikes at that part of economic theory that relies on a representative agent ignorant of his own identity. Several illustrations, especially from the theory of international trade, have been provided by Kemp and Long (1992), and by Kemp and Shimomura (1995, 2000c and 2002b). More recently, it has been shown that the Ramsey-Pigou-Samuelson general-equilibrium theory of tax incidence and the modern theory of endogenous growth are also vulnerable; see Kemp and Shimomura (2007) and Kemp *et al.* (2006).

## 17.5 Final remarks

I have given reasons why the Heckscher-Ohlin theory should be displaced from its dominant position as the workhorse of general equilibrium. Should we do away with it altogether? I do not think so. For the theory effectively highlights the role of factor endowments in determining the pattern of world trade. But we need competing theories, each emphasizing a particular subset of processes and collectively providing a set of hypotheses from which econometricians can choose. Perhaps some of them will do without the three hidden assumptions of the Heckscher-Ohlin.

# Part III

# Normative trade theory

# 18 On a misconception concerning the classical gains-from-trade proposition

## 18.1 Introduction

Existing proofs of the classical doctrine that free trade is potentially beneficial to each trading nation are typically based on the Arrow-Debreu or McKenzie model of general equilibrium, extended to accommodate any finite number of trading countries, each with a country-specific scheme of lump-sum compensation installed; see, for example, Grandmont and McFadden (1972) and Kemp and Wan (1972, 1993). In those models, all markets clear at an initial point of time, with all exchanges, present and future, agreed upon at that moment. There is no room for decisions based on imperfect information.

If we depart from the strict Arrow-Debreu or McKenzie formats, allowing for sequential decision-making and hence for the possibility that agents base their decisions on imperfect and changing information, the potential gainfulness of trade is no longer assured, even in the sense of expected values. It is now understood that international trade, either in short and long term securities or in conventional commodities such as cloth and wine, can give rise to socially harmful but privately profitable behaviour. See for example, Kemp and Sinn (2000), to which I will later return.

It is therefore surprising to find well-known economists responding to the recent East Asian collapses by writing in a quite a contrary vein. Jagdish Bhagwati (2000), for example, has advanced the following three-step argument:

(a) Whereas freedom of capital movements has sometimes been associated with highly destructive 'panics' and 'manias', '[n]o one of sound mind can seriously sustain the notion that . . . trade in goods and services leads to such problems . . .' (p. 14).
(b) Direct foreign investment, which brings with it 'skills and technology', can achieve most of any benefits associated with general capital mobility (p. 15).
(c) It follows from (a) and (b) that the international mobility of short-term capital should be restricted.

It is desirable that a vigorously argued, wide-ranging and unconventional proposal from the pen of an influential economist should be carefully assessed. I hope that this brief chapter will contribute to that end.

However, it is not my purpose to finally settle the question whether a general case can be made for the worldwide restriction of short-term capital movements. I am willing to concede that there are circumstances in which a well-informed government might be justified in intervening in the market for short-term capital and that there are circumstances in which a poorly informed government might do harm by intervening.

Instead I shall focus on the details of Jagdish Bhagwati's own case for restriction. It will be argued that he has failed to justify his proposal, that, in particular, he has erred in taking for granted that the classical gains-from-trade proposition, valid for Arrow-Debreu and McKenzie economies, is also valid for sequential economies.

## 18.2 The argument dissected

Part (a) of Bhagwati's syllogism is simply not correct, for Kemp and Sinn, at least one of whom is of 'sound mind', have shown that barter trade in goods and services can easily give rise to privately profitable but socially harmful speculation, even to 'panics' and 'manias'. They constructed a very simple general-equilibrium and barter model of pure sequential exchange, with stochastic endowments and missing markets, and then proceeded to show that, for that model, speculation in the forward market may be (privately) profitable yet generate (social) welfare losses. Speculators gain from the mechanics of Jensen's inequality. However, they also generate negative pecuniary externalities that, in a context of missing markets, may, from a social perspective, outweigh the private gains. (Here the essential reasoning is that of Scitovsky (1954).) In short, Kemp and Sinn showed that there are circumstances in which profitable speculation is worse than useless. In those circumstances, the closing down of the forward market would increase social welfare.

Part (b) of Bhagwati's syllogism may be valid if attention is restricted to special cases. But can (b) be accepted as a general proposition? I do not think so. Certainly Bhagwati has offered no evidence in support of the notion.

Thus Bhagwati's conclusion (c) rests on shaky grounds, both logical and empirical. A valid case for the restriction of short-term capital must be sought elsewhere.

# 19 Recent challenges to the classical gains-from-trade proposition[1]

## 19.1 Introduction

The classical gains-from-trade conjecture, first formulated in the eighteenth century, was finally given a thorough Arrow-Debreu proof in 1972. It was then and remains virtually the only easy-to-interpret comparative-static proposition valid for any Arrow-Debreu economy.[2] Since 1972 the proposition has been repeatedly challenged, sometimes on grounds that in no respect violate Arrow-Debreu assumptions; and therein lies a puzzle on which we focus in the first part of our paper. The remaining and most recent challenges rest on the assertion that some of the Arrow-Debreu assumptions are unnatural and unnecessarily restrictive or on the imposition of artificial restrictions on the set of permissible compensatory transfers; these challenges will be examined in the remainder of the paper. It is our general conclusion that none of the challenges can be sustained.

## 19.2 The Thurow-Tompkinson challenge

The first challenge, historically speaking, was launched by Lester Thurow (1980). Thurow noted that workers typically find some occupations more pleasant than others and claimed that occupational disparities in ‘psychic income’ constitute externalities that distort the allocation of resources and, in particular, might render free trade potentially harmful in the sense of Pareto. However, Thurow’s claim was denied by Katz and Syrquin (1982), who pointed out, correctly in our opinion, that any ‘externalities’ associated with psychic income are market-mediated and therefore completely compatible with efficient allocation.

Recently, Paul Tompkinson (1999) has reopened the earlier debate. After noting that Thurow and Katz and Syrquin had confined their attention to the case in which workers share the same preferences both over produced commodities and over occupations, he argued that if workers prefer some occupations to others and differ from one another in those preferences then Thurow’s conclusions are valid. As Kemp and Shimomura (2000a, 2000b) argued, however, the point made by Katz and Syrquin remains valid whether or not workers differ in their preferences over occupations. Occupational preferences are no more disruptive of the allocation of resources than are preferences over possible places in which to eat an apple. In fact, Tompkinson's model economy excludes all known sources of market failure (non-pecuniary externalities, public goods, increasing returns to scale, closed entry, asymmetries of information, and commodity taxes). It is therefore a special case of the Arrow-Debreu model. In fact, Tompkinson's challenge to the classical proposition is based on a serious error, uncovered by Kemp and Shimomura (2000a, 2000b). Recognition of the error suffices to reconcile his and traditional conclusions. Implicitly, Tompkinson has assumed that, after the payment of compensation, all workers remain in their free-trade but pre-compensation occupations. In particular, those workers who – after the opening of trade but before the payment of compensation – move to less preferred occupations remain in their new occupations even after the payment of compensation. He then showed that under this restrictive assumption compensation may be infeasible. However, such an assumption is unwarranted. It played no part in the 1972 proofs of the gains-from-trade proposition.

## 19.3 The Newbery-Stiglitz challenge

A second challenge was launched by David Newbery and Joseph Stiglitz (1984), who showed that, if some markets are missing, the opening of trade might leave every household in a country worse off than in autarky. They concluded that '[t]he belief that free trade is Pareto optimal . . . may not be well founded'. Now only the ill-informed have ever claimed that, in general, free trade is Pareto optimal. We must assume, therefore, that Newbery and Stiglitz meant to assert that the belief that *suitably compensated* free trade is Pareto optimal may not be well founded. However, even that reformulated conclusion would have been too hasty. For, as was later shown by Kemp and Wong (1995), models that are Arrow-Debreu except for missing markets always allow of compensation of the losers – even when, before compensation, all households are losers. The central point is quite simple: if some markets are missing, the locus of competitive equilibrium utilities need not be everywhere negatively sloped.

## 19.4 More recent challenges

The two most recent challenges differ qualitatively from their predecessors. They rest not on the assumed characteristics of the trading economies but either on artificial restrictions imposed on the set of permissible compensatory transfers or on dissatisfaction with assumptions of Arrow-Debreu type. It is to these latest challenges that we devote the remainder of our paper.

Some years ago, Tito Cordella and Luigi Ventura (1992) claimed that standard arguments for the gainfulness of (compensated) free trade fail if the compensatory transfers are implemented after the opening of trade. Their argument rests on two numerical examples.

However, neither of the examples has a time dimension. It is therefore impossible to identify, in the examples, compensation that is implemented before or after the opening of trade. Indeed close inspection of their examples reveals that the argument of Cordella and Ventura rests not on the relative timing of compensation and the opening of trade but on the assumption that the set of feasible and efficacious compensatory transfers for any particular country is determined by the state achieved by that country as part of a world equilibrium without compensation.

This assumption differs radically from its counterparts in standard proofs of the gains-from-trade proposition; see Grandmont and McFadden (1972) and Kemp and Wan (1972). In those proofs, the set of feasible and efficacious compensatory transfers for any particular country depends on the primitive characteristics of every trading country and on the compensatory schemes adopted by its trading partners. The assumptions underlying the Cordella-Ventura paper are therefore an inappropriate basis for a challenge to standard theory.

Nevertheless, the Cordella-Ventura conclusion (that a delay in the payment of compensation may render all feasible schemes of compensation ineffective) is correct; only their demonstration is at fault. To appreciate that this is so, recall again that the 1972 proofs of the gains-from-trade proposition were constructed in a context that is essentially Arrow-Debreu but extended to accommodate several trading countries and compensatory transfers within at least one of them. In an Arrow-Debreu economy commodities are distinguished by date of delivery; that is, time enters essentially. In particular, each feasible and efficacious vector of compensatory transfers contains dated commodities. Hence any delay imposed on the vector changes it in essentials and may render it infeasible or ineffective. In an extreme case, a delay might render the set of feasible and efficacious transfers null because of the severe harm done to some individuals in the temporary absence of compensation.

The latest challenge, by Tito Cordella, Enrico Minelli and Heracles Polemarchakis (1999), is directed to the Arrow-Debreu assumptions of strictly monotone preferences and strictly positive endowments, which they consider to be not ‘natural’. To accommodate weaker assumptions about preferences and endowments, Cordella *et al.* were driven to adopt Lionel McKenzie’s (1959, 1961) assumption of resource relatedness.

That the gains-from-trade proposition can be established under alternative assumptions is a useful discovery. However, the assumption of resource relatedness, in the hands of Cordella *et al.*, is quite severe. Applied once, it is weaker than the combined Arrow-Debreu assumptions of strictly monotone preferences and strictly positive endowments. But Cordella *et al.* apply the assumption twice, under autarky and under free trade without compensation; and, in addition, they assume that resource unrelatedness holds for the world economy ‘after the autarkic endowments are modified to coincide with an autarkic equilibrium allocation’. It is not clear that their assumptions are less strict than those of Arrow and Debreu.

## 19.5 Conclusion

All of the challenges considered in earlier sections are defective, each in its own way. The challenges mounted by Thurow and Tompkinson rest on the mistaken belief that disparities in the extent to which alternative occupations generate psychic income are a source of distorting externalities. The challenge mounted by Newbery and Stiglitz rests on the mistaken belief that, in a context of missing markets, the competitive locus of household utilities is necessarily of conventionally negative slope. The challenge by Cordella and Ventura rests on the artificial and inappropriate assumption that the set of feasible and efficacious compensatory transfers available to a country is determined by the state achieved by that country as part of a world equilibrium without compensation. And, finally, the challenge of Cordella *et al.* rests on the unjustified claim that they have established the traditional gains-from-trade proposition under assumptions that are less strict than those employed in 1972.

# 20 Trade gains

## The end of the road?

### 20.1 Introduction

I recently offered a brief survey of the progress made over two and a half centuries in answering the fundamental questions of Montesquieu (1749) concerning the gainfulness of international trade for individual trading countries; see Kemp (2003b: ix–xii). The survey began by recalling the inadequacy of the normative contributions of Adam Smith (1776) and David Ricardo (1817), the result of their dependence in crucial passages on the assumption of a representative agent in each trading country. Given that assumption, the question of trade gains becomes quite trivial: either all households benefit from trade or all households suffer from trade or, the singular case, all households are indifferent to trade. And yet the analysis provided by Smith and Ricardo formed the foundation of nearly all academic discussion of trade gains throughout the nineteenth century and indeed well into the twentieth century.

The need to develop models that admit a link between international trade and the distribution of income within countries and the implied need to specify compensation as a necessary condition of trade gains were explained by Vilfredo Pareto (1894) but were unknown to or misunderstood by English speaking economists until Paul Samuelson enlightened them in 1950. After Pareto and Samuelson, it remained only to establish the existence of a world trading equilibrium under free trade and lump-sum compensation of the losers. That step was taken by Jean-Michel Grandmont and Daniel McFadden (1972) and by Kemp and Henry Wan (1972), each working with assumptions of Arrow-Debreu type.

The survey continued by noting available extensions of the 1972 propositions: to accommodate economies with missing markets, overlapping finite generations, infinite time horizons, some kinds of monetary arrangements, chaotic competitive equilibria and states of information dependent on the opportunities to engage in foreign trade; and, subject to existence, to accommodate economies characterized by increasing returns to scale and oligopolistic market structures.[2] Thus, during the post-1972 period, the theory of trade gains bifurcated. There is now a ‘finite’ branch of Arrow-Debreu type, with populations that do not vary from one period to the next, and there is a branch with infinite horizons and overlapping mortal generations based on the early closed-economy work of Maurice Allais (1947) and Samuelson (1958).

The list of extensions is impressive. As I noted in the survey, however, the list contains three serious gaps. Thus it is doubtful if it will ever be possible to accommodate both overlapping generations and inter-generational caring (of parents for their children and/or children for their parents). For example, the strategic relationships of two pairs of parents-in-law, each pair a potential source of bequests and aware of the relationship, will give rise to resource misallocation, notably and directly between consumption and investment; and the extent of the misallocation may be exacerbated by the introduction of free trade. Moreover, it is unlikely that it will be possible to completely accommodate internally increasing returns to scale and the market power based on them. Here the stumbling block is the inability to establish the existence of equilibrium when the quantities produced are strategic variables drawn from non-convex sets. Finally, it is quite unlikely that normative trade theory will ever be able to accommodate sequential economies and the false expectations associated with them.

The survey summarized the state of our art in the year 2002. Since its appearance in print, however, several readers have suggested that I should provide a more detailed account of the first of the three gaps identified. Such an account is now provided in Section 20.3. Moreover, very recently it has been shown that the finite Arrow-Debreu model of general equilibrium, which inspired the 1972 proofs, is internally inconsistent if households are taken to be perfectly rational and perfectly informed about the economy of which they are part. This discovery raises questions concerning the acceptability of the 1972 proofs. The questions are discussed in Section 20.2.

The outcome of our audit is mixed. It is shown, in Section 20.2, that the Arrow-Debreu model, and therefore the Grandmont-McFadden and Kemp-Wan models, make complete sense only if households are endowed with suitably *imperfect* knowledge or suitably *imperfect* rationality or both. If one insists on complete information and complete rationality then the Arrow-Debreu assumption of price taking behaviour must be abandoned and reliance placed on existence propositions that rest not only on the specification of the pre-market world economy in terms of preferences, technologies and endowments but on the specification of the world market equilibrium itself. It is then shown, in Section 20.3, that the first of the three concessions can be moderated if each household is sufficiently integrated across generations and engages in a multi-dimensional search whenever a member of the household reaches a marriageable age. However, active search by a family generates positive informational externalities with incidence among other searching families. Unless offsetting subsidies are provided by the government, the externalities will ensure that expenditure on search is less than optimal; and that the distortion might be exacerbated by free trade with other countries.

It is then noted that, even if it is optimally conducted, search is costly and may remain incomplete. It follows that, usually, the two pairs of parental in-laws associated with any household will find themselves in a strategic relationship to each other, suggesting that their individual and collective bequests, and their aggregate savings, will also be suboptimal.

Section 20.4 offers a revised summary of the state of the art.

## 20.2 The assumption of price taking by households and firms[3]

Much of normative general-equilibrium trade theory rests on the foundations provided by Kenneth Arrow and Gérard Debreu (1954). In the Arrow-Debreu model:

(i) Households and commodities are finite in number and constant over time, both in number and identity.
(ii) Each household conceives of itself as a price taker in all markets.
(iii) Each household seeks to maximize its own utility.
(iv) The production set of each firm is convex.
(v) The endowment point of each household is in the interior of its consumption set.

However, if households are finite in number and if the endowment vector of each household is in the interior of its consumption set, then, in any equilibrium, each household exercises market power, directly or through firms in which it owns shares or both. That is, given the equilibrium net offers of all other households, any change in the net offer of a particular household (say, household *j*) would disturb the set of market clearing relative price vectors. Arrow and Debreu place virtually no restrictions on the distribution of endowments over households. Hence the extent of household *j*'s market power might be considerable. Or it might be very small – indeed it might approach zero as the number of households goes to infinity; but it cannot be zero for any finite population, even if the distribution of endowments is uniform. It follows that, if it is perfectly informed and rational in the double sense that it seeks to maximize its own utility and can appreciate that (i) and (v) imply market power, then household *j* cannot in equilibrium conceive of itself as a price taker in every market.

It further follows that if the Arrow-Debreu model is internally consistent then each household must be incompletely informed and/or incompletely rational. And this in turn suggests that the Arrow-Debreu analysis rests on an implicit understanding that households are:

(a) unaware that they are finite in number; and/or
(b) incompletely rational in the sense that they cannot appreciate that (i) and (v) imply market power.

Without that understanding, the Arrow-Debreu assumptions (i)–(v) would be mutually inconsistent, with implications laid bare by Debreu's own pithy remark, 'A deductive structure that tolerates a contradiction does so under the penalty of being useless, since any statement can be derived flawlessly and immediately from that contradiction' (Debreu 1991: 2).

With that understanding and paradoxically, the familiar existence theorem and the two fundamental welfare propositions of 1954 remain intact – and so do the gains-from-trade propositions of 1972. Thus a little carefully delineated ignorance and/or irrationality can be viewed as a good thing. If the assumption of imperfect knowledge and/or rationality is unacceptable then one must specialize the model proposed by Kemp and Koji Shimomura (2001a) by excluding non-convex production sets while continuing to admit market power on the part of individual households. Appeal may then be made to the single-economy existence result of Kazuo Nishimura and James Friedman (1981: Theorem 1). Presumably, the Nishimura-Friedman result can be extended to accommodate several free-trading countries, as well as schemes of country-specific lump-sum compensation. However, it must be borne in mind that the Nishimura-Friedman result is based on assumptions unlike those of Arrow and Debreu in that they are imposed on households' best replies to the strategies of other households, which are normally viewed as endogenous variables, not directly on the customary defining elements of an economy (preferences, technologies and endowments [including information]). To introduce such assumptions in the course of debate is to change the question debated.

Throughout this section we have focused on the uncertainty-free Arrow-Debreu model of 1954. However, that model and those of the two papers of 1972 can be extended to accommodate uncertainty while retaining the assumption of price-taking behaviour and while remaining finite in scope; see Arrow (1953, 1964), Debreu (1959) and Grinols (1986). Not surprisingly, the more general models obtained in this way are open to comments very like those directed to the parent models.

In summary, the Arrow-Debreu model, and therefore the Grandmont-McFadden and Kemp-Wan models, make no sense if households are endowed with perfect knowledge and perfect rationality; but the models make complete sense if households are endowed with imperfect knowledge of the economy of which they are part and/or imperfect rationality in the sense of failure to perceive that (i) and (v) imply market power.[4] Evidently imperfect knowledge and imperfect rationality are more likely to be encountered in short-run than in long-run applications of the theory. If imperfect knowledge and imperfect rationality, in the above senses, are unacceptable, then market power and strategic behaviour must be accepted and with them, substantial non-Arrow-Debreu assumptions of Nishimura-Friedman type.

## 20. 3 Overlapping generations with bequests and/or gifts *inter vivos*

We know that, in a context of overlapping generations, infinite horizons and price taking behaviour by households and firms, free trade is potentially gainful for a country if the economy of that country is competitive and irreducible and if in equilibrium there are no inter-generational bequests or gifts *inter vivos* (including dowries); see Kemp and Nikolaus Wolik (1995). However, in a context of inter-generational caring, we cannot be sure of the existence of autarkic and free-trade equilibria; and, even if the existence of equilibria were guaranteed, we could not be sure that free trade is beneficial, even potentially (that is, after compensation).

To better understand the nature of the problem, let us focus on bequests from parents to their children and to their children's spouses. Then, associated with each pair of newly weds, there are two pairs of parental in-laws. In the introduction to this paper, it has been maintained that the in-laws stand in an allocation-distorting strategic relationship to each other, with the extent of the distortion possibly greater under free trade than under autarky. Let us examine that claim in detail.[5]

Going back a century or so, we can find many countries in which whole families participated in the search for marriage partners and in which the objective of the search was to find a compatible family with which to form an alliance. Not only was the young couple to be well matched but the two pairs of parents (and any siblings of either partner) were to be compatible. Even today, there are countries in which families resort to thorough multi-dimensional searches of this kind; one thinks especially of India and Japan.

Suppose that all families in a country engage in costless multi-dimensional searches and that the searches are uniformly successful in achieving close matches. Under such favourable conditions, compatible in-laws might be expected to play a cooperative rather than a non-cooperative game, with bequests and gifts *inter vivos* agreed upon at the time of marriage, perhaps regulated by common law. Any misallocation associated with bequests and gifts would be negligible.

However, multi-dimensional search is difficult, as readers of Vikram Seth's *A Suitable Boy* will understand. For that reason, a searching family will not only seek guidance from friends; it will also avail itself of the search facilities of specialized agencies ('match makers' or 'go betweens'). Thus search is costly; in particular, it is time-consuming. In practice, therefore, families will halt the search before a complete match is achieved. On the other hand, active search by any family makes it easier for other searching families with similar characteristics to achieve an acceptable match; that is, the active search of each family generates informational externalities with incidence among the searchers from other households, and this is so whether or not professional go-betweens have been employed. The externalities may be offset by the subsidization of search. In general, however, even optimally subsidized search is incomplete, that is, fails to achieve a complete match. Thus, any search, whether privately financed or subsidized by government, will fall short of a perfect match, leaving residual scope for strategic behaviour by in-laws and for the associated sub-optimality of bequests and savings.[6] Indeed the scope for strategic behaviour and the sub-optimality of bequests and savings might be greater under free trade than under autarky. Clearly one cannot expect to establish a *general* gains-from-trade proposition even if search is optimally subsidized.

Is there any way around this conclusion? As noted in Section 20.2, Kemp and Shimomura (2001a) have shown that, even when families possess market power and behave strategically, any free-trade equilibrium is potentially gainful to each participating country, that is, gainful after compensating transfers; and Nishimura and Friedman (1981) have provided a set of sufficient conditions for the existence of equilibrium in a single economy in which households possess market power and behave strategically. However, the Kemp-Shimomura and Nishimura-Friedman results apply only to static, finite economies. There remains the task of extending their findings to economies with overlapping generations and infinite horizons. Moreover, as already noted in Section 20.2, the Nishimura-Friedman result is based on assumptions unlike those of Arrow and Debreu in that they are imposed on households' best replies to the strategies of other households, which are normally viewed as endogenous variables, not directly on the customary defining elements of an economy (preferences, technologies and endowments).

It is often said that, in the modern West, children choose their marriage partners independently of the wishes of their parents and siblings, thus creating the misallocation of resources alluded to in the introduction. However that common assessment seems to me to be exaggerated. Children are inevitably influenced by the opinions of their parents and siblings. Moreover, in the case of wealthy families, there is a strong financial incentive for family members to join forces in multi-dimensional searches not unlike those found in some Eastern societies. I do not suggest that the West is as successful as modern India and Japan in coordinating inter-generational decision-making. However, the difference seems to be one of degree. In both East and West, some misallocation results from the failure of families to completely coordinate decision-making across generations and from the externalities generated by search.

Implicitly I have assumed that each family contains two parents. Without that assumption, as in Plato's Utopia or in extreme matriarchal societies in which children and fathers are invariably unknown to each other, the misallocation identified above does not arise.[7] Its place is taken by a potential misallocation of resources to the activity of child raising.[8]

Finally, the issues discussed in an Arrow-Debreu context arise also in a context of overlapping generations and infinite horizons and can be discussed in similar terms if assumptions (i)–(v) are imposed. In both contexts, the assumption of price taking makes sense if, in each period, agents are fully informed of market-clearing prices but unaware of their own power to influence those prices.

## 20.4 The present state of the art

In the introduction, I distinguished two potential difficulties in further developing the normative theory of international trade. The first potential difficulty flows from the incompatibility of the conventional assumptions that all households are price takers and yet completely rational and completely informed about the economies of which they are part. It was shown in Section 20.2 that, at least in short-run applications, this difficulty can be brushed aside by abandoning the assumption of complete knowledge and/or complete rationality. However, the paradoxical quality of the rescue was noted. If the assumptions of complete knowledge and complete rationality are retained, one must abandon the assumption of price taking and rest content with a gains-from-trade result qualified by the non-Arrow-Debreu assumptions provided by Nishimura and Friedman.

The second potential difficulty is peculiar to the dynamic branch of the theory and flows from the allocation-distorting strategic relationships associated with inter-generational bequests and gifts *inter vivos.* In Section 20.3 it was acknowledged that distortions of this kind persist throughout the world and might substantially reduce the gains from trade. However it was suggested that this risk would be less in countries where closely knit families engage in multi-dimensional searches of the kind described in that section.

As barriers to the formulation of a general gains-from-trade proposition there remain the sequential nature of modern economies and the absence of general existence results for economies characterized by internally increasing returns and strategic behaviour. About those barriers I have little of a constructive nature to offer.

## 20.5 Final remark

In the present paper, the focus has been on the normative implications of the replacement of autarky by free trade. It will be clear, however, that the difficulties in assessing the opening of trade have counterparts in the normative assessment of any decision of government.

# 21 Tariff reform

## Some pre-strategic considerations

### 21.1 Introduction

GATT/WTO tariff negotiations are multilateral and piecemeal, subject to no formal rules other than the most-favoured-nation (MFN) clause. In the absence of additional rules, it is not possible to say anything definite about the necessary characteristics of feasible agreements. However, one does discern an additional informal but widely acknowledged objective – that each participating country should on balance benefit from any agreement. Indeed, this objective may be detected in the preamble to the GATT itself, for there the hope is expressed that the member countries will enter into 'reciprocal and mutually advantageous arrangements'. Now by post-Paretian convention the well-being of a single country is said to increase as the result of an agreement if and only if no resident of that country is left worse off and at least one resident is left better off. In the present paper, therefore, our focus is on the characteristics of tariff reforms that accommodate this informal constraint, interpreted in the sense of Pareto and, for that reason, referred to as Pareto-improving. Indeed, we go a step further and require that tariff reforms leave the world on its contract locus, in a Pareto-optimal position. Thus our focus is on reforms that satisfy a two-edged Paretian rule.

It will be shown that the two-edged Paretian rule restricts the set of feasible reforms, and in unexpected directions. For this purpose, it suffices to focus on the familiar case of two commodities and two countries, in which case the MFN clause plays no role. Specifically, it will be shown that the set of tariff reforms that satisfy the Paretian rule:

(a) is always non-empty;
(b) might include no reforms that end in worldwide free trade;
(c) always includes reforms that are incompatible with free trade and, in particular, always includes reforms that impose negative import duties and/or positive export duties;
(d) might include reforms that support a Pareto-optimal and Pareto-improving allocation but that also support other allocations without either of those characteristics.

Here (a) is our basic *existence* proposition for two-by-two economies. Conclusion (b) affirms that there are circumstances in which free trade is ruled out by the two-edged Paretian rule. Thus the frequently heard remark that the GATT rules are conducive to free trade is inaccurate.[2] Conclusion (c) states that, whether or not free trade is attainable, there are always available reforms that impose negative import duties and/or positive export duties. Finally, conclusion (d) draws attention to a fundamental obstacle to the *attainability* of particular allocations associated with tariff reform.

These findings establish a sharp contrast between redistribution attainable by means of distorting tariffs and redistribution attainable by non-distorting lump-sum Grandmont-McFadden-Grinols (GMG) compensation. Moreover, they carry the possibly disturbing implication that an import subsidy and/or an export tax may be necessary elements of a pure tariff reform, that is, a tariff reform unaccompanied by international transfers. Finally, they generalize the classical gains-from-trade proposition, in which the initial tariffs are jointly prohibitive and in which all new tariffs are zero; see Kemp and Wan (1972: Theorem 1). They also generalize a more recent gains-from-trade proposition, in which the initial tariffs are jointly prohibitive for each country and in which all new tariffs, whether on imports or exports, are non-negative and jointly prohibitive for no country; see Kemp and Wan (1972: Theorem 1′). However, these generalizations are available only in a two-by-two setting; Kemp and Wan (2005) have provided a three-by-three example in which propositions (a) and (c), and therefore the above generalizations of the two gains-from-trade propositions, do not hold.

Of course, each trading country must accept a particular level of well-being and a tariff vector that helps support that level. To that extent, our finding relies on the *cooperative* behaviour of the trading countries. However, even the classical proposition relies on each country to cooperate in trading freely or, at least, in imposing non-prohibitive tariffs.

In a well-known earlier contribution, Wolfgang Mayer (1981) studied some of the questions posed in the present paper. In particular, he anticipated our conclusions (b) and (c). However, Mayer confined himself to the special case in which, in the initial pre-reform equilibrium, each country imposes its optimal tariff and in which both the pre-reform and the post-reform equilibria are unique.

## 21.2 The basic model

Consider two pure-exchange economies, the home and the foreign, each with a single representative agent. Possibly, the two commodities differ only in the point in time at which they become available; thus international borrowing and lending are accommodated. The home country has an endowment of one unit of commodity 1; the foreign country has an endowment of one unit of commodity 2. The two agents share a symmetrical, increasing and strictly quasi-concave utility function; for example, they might share the function

$$u = x + y + \alpha xy \quad \alpha > 0$$

where $x$ and $y$ denote the amounts consumed.

In the unit Edgeworth box of Figure 21.1, $E$ is the initial endowment point and the contract locus coincides with the diagonal joining the home and foreign origins, $0_H$ and $0_F$. At all points on the contract locus, the two marginal rates of substitution are equal to one.

The unique free-trade equilibrium is represented by point $C$, where the two offer curves, $EH$ and $EF$, intersect and where two dashed indifference curves, one for each country, are tangential. The equilibrium world price ratio is equal to one, and each country exports half of its endowment, consuming the vector (0.5, 0.5).

Suppose alternatively that each country imposes a tariff on its imports. The tariffs are non-negative but otherwise arbitrary. Possibly but not

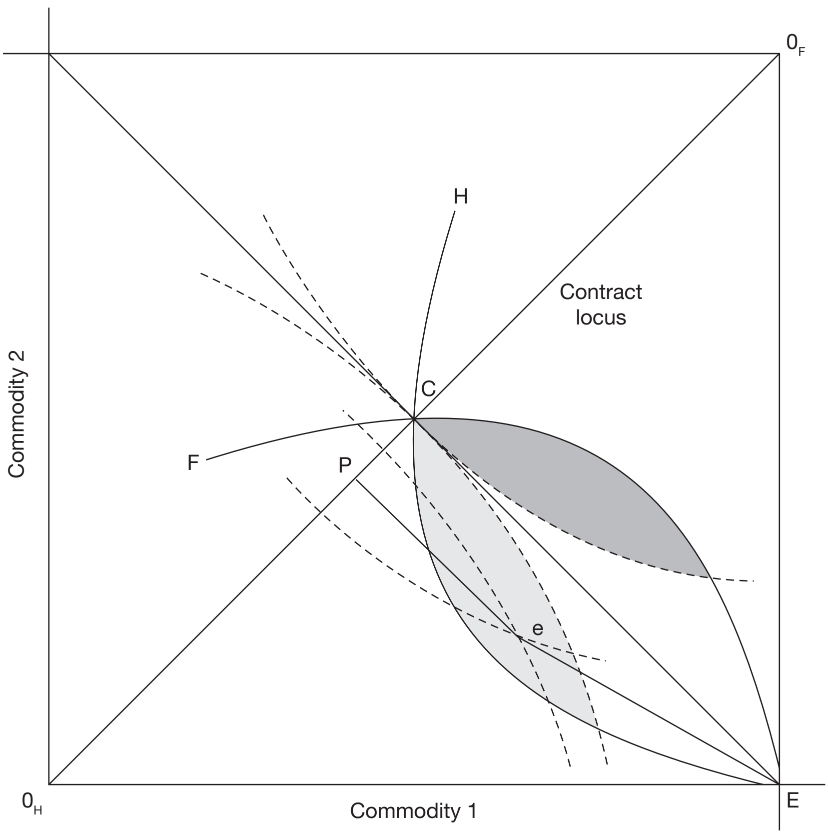


*Figure 21.1*

necessarily they form a Nash solution to a tariff war; possibly one is optimal, the other zero; possibly they are jointly prohibitive. The tariff-distorted curves intersect at a point *e* in the 'lens' *CE* formed by the free-trade offer curves.

If, exceptionally, each tariff is initially imposed at the same rate, *e* must lie in the open segment *EC*. In that case both countries benefit from any equi-proportional reduction of the two tariffs; in particular, this is so if the tariffs are eliminated in favour of free trade. Moreover, any Pareto-optimal and Pareto-improving point other than *C* can be reached by negotiating a tariff pair one element of which is positive, the other negative.

If, on the other hand, the two tariffs are initially imposed at different rates, then it is possible that one country will be harmed by a retreat to free trade. Indeed this outcome will emerge if and only if the tariff-distorted point *e* lies in the interior of either of the shaded regions of Figure 21.1. If that condition is met, therefore, the two countries will not be able to agree on the free-trade outcome without a side payment by one country to the other. However, whether or not *e* lies in the interior of a shaded region, the countries will be able to reach a Pareto-optimal and Pareto-improving point by negotiating a tariff pair one element of which is positive, the other negative. Thus worldwide Pareto-optimality is attainable in a context of positive and negative import duties. In effect, the tariffs are *equivalent* to the side payment mentioned above. Thus we may add to the familiar roles of tariffs (in raising revenue, in redistributing income [Stolper-Samuelson] and in raising national well-being [Edgeworth-Bickerdike optimal tariffs]) the new *fiscal* role of extending (and camouflaging) foreign aid.[3]

These are interesting findings since they suggest that negotiating countries should not constrain their negotiations by imposing equi-proportionality, thus ruling out import subsidies. Without recourse to import subsidies it is generally impossible to achieve a world allocation that is both Pareto-optimal and Pareto-improving; in particular, it is generally impossible to reach the free-trade point *C* without harming one country.

The proof of the proposition is straightforward. Consider any point *e* in a shaded region of Figure 21.1 and any Pareto-optimal and Pareto-improving point *P*. At *P* there is a shared marginal rate of substitution ($MRS = 1$), which differs from the terms of trade ($p_2/p_1 \neq 1$). Suppose that *P* can be attained by means of an ordered semi-positive (non-negative and non-zero) pair of specific tariffs ($t^H$, $t^F$). Then

$$(p_1 + t^F)/p_2 = p_1/(p_2 + t^H).$$

Since the pair of tariffs is semi-positive, however,

$$(p_1 + t^F)/p_2 > p_1/(p_2 + t^H)$$

a contradiction.

## 21.3 Extensions

The analysis has been based on several simplifying assumptions. These can now be relaxed. Thus we have assumed that the two agents share the same symmetrical utility function, ensuring that the contract locus coincides with the positively sloped diagonal of Figure 21.1. The assumption is not necessary. Thus in Figure 21.2 the assumption is abandoned but our conclusion remains intact. In particular, from any point *e*, whether it is in the shaded or unshaded region of the lens *EC*, it is possible by adopting new tariffs to move to any point *P* that is Pareto-optimal and Pareto-preferred to *e*. If *e* lies in the straight segment *CD* and *P* coincides with *C*, then the new tariffs will be zero; that is, free trade will obtain. Otherwise, one of the new tariffs must be positive, the other negative.

Nor is it necessary to assume that there is a single agent in each country. For we can interpret the indifference contours of Figures 21.1 and 21.2 as Scitovsky community indifference contours based on the individual contours of any number of heterogeneous agents, with a GMG scheme of lump-sum

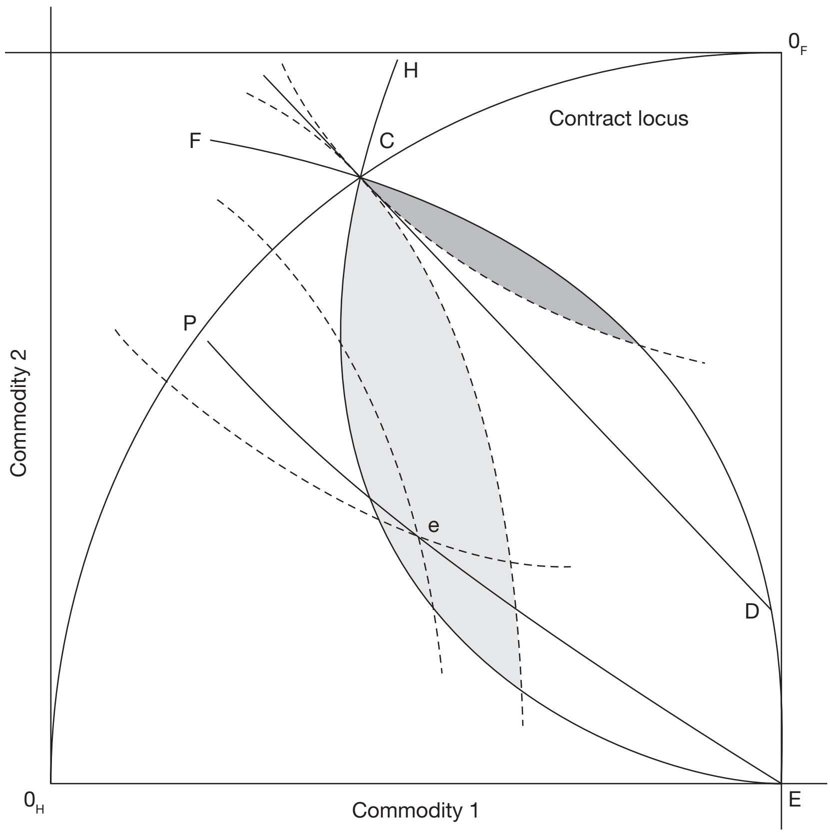


*Figure 21.2*

compensation ensuring that, in the general context of tariff-cum-subsidy reform as in the traditional special context in which free trade replaces autarky, the economies move to ever-higher Scitovsky contours. Thus point *P* in Figure 21.1 or Figure 21.2 is not only Pareto-optimal and Pareto-improving in relation to point *e* but readily implementable by means of GMG compensation.

We have focused on a particular endowment point. However, it is possible to accommodate any initial endowment point compatible with autarkic subsistence and, by reinterpreting the indifference curves as *trade* indifference curves, to accommodate production.

It is also possible to accommodate initial tariffs that are jointly prohibitive. We need only recall that the free-trade allocation is Pareto-optimal and Pareto-preferred to *E* which, if the tariffs are prohibitive, coincides with *e*.

We have assumed that the free-trade and tariff-distorted world equilibria are unique. Suppose that this is not so. In particular, suppose that there are three free-trade equilibria, as in Figure 21.3. If the home country imposes a positive import duty, its offer at each terms of trade contracts (perhaps to

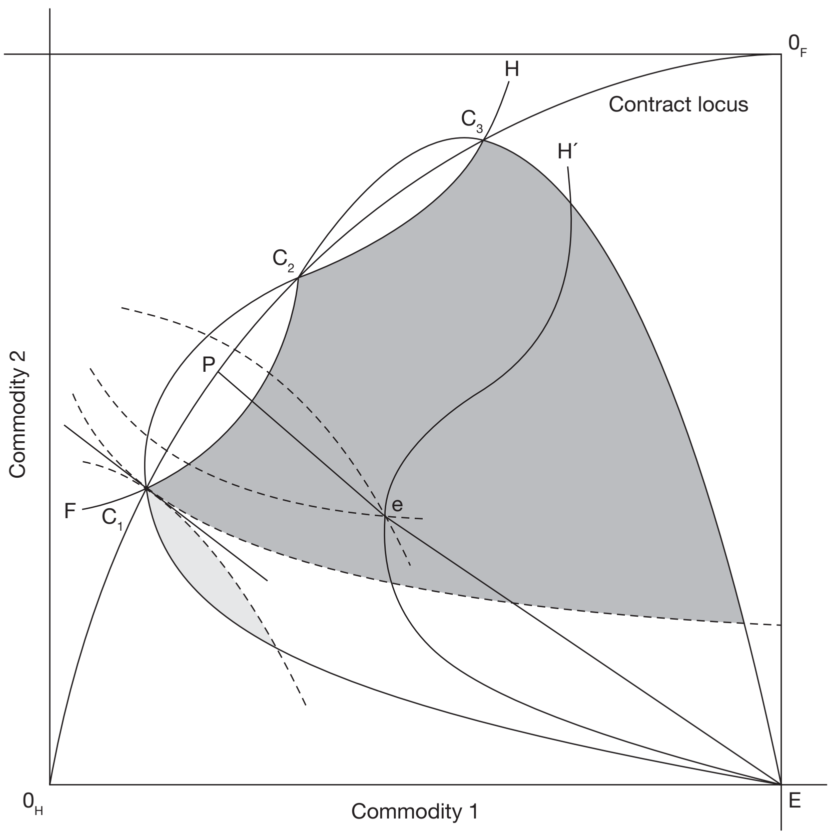


*Figure 21.3*

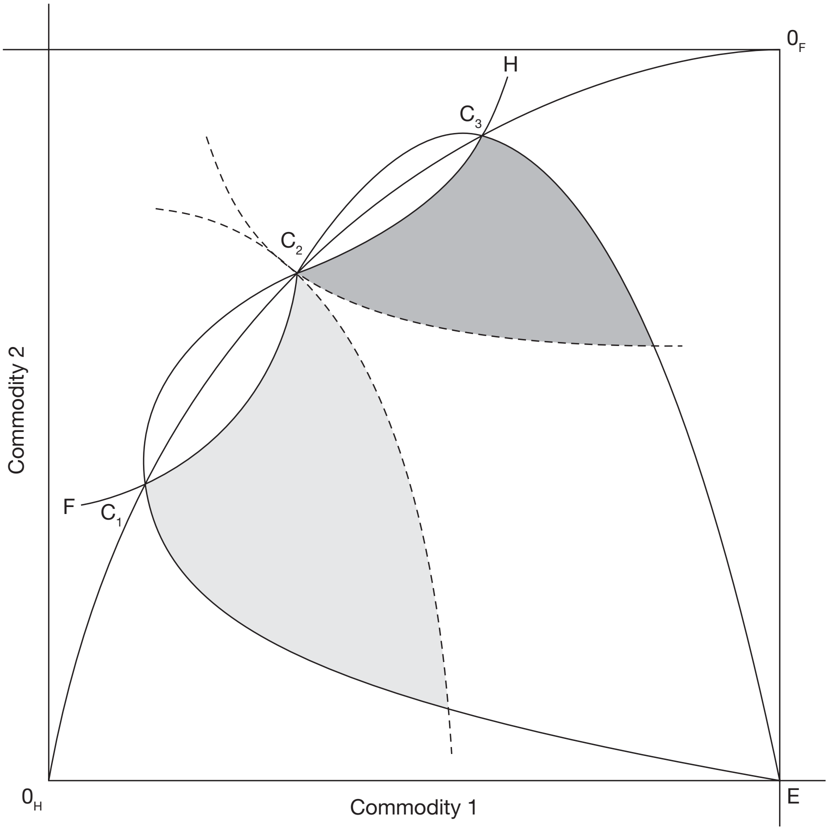


*Figure 21.4*

zero), so that its new tariff-distorted offer curve $EH'$ lies uniformly 'inside' its free-trade offer curve $EH$; similarly for the foreign country. Thus, corresponding to each pair of positive tariffs there is a pair of tariff-distorted offer curves. The curves may or may not intersect in the interior of the region inside both $EH$ and $EF$; and, if the curves do intersect, they may intersect less than or more than three times. Now consider any point $e$ which is an interior equilibrium for some pair of positive tariffs. The shaded region of Figure 21.3 is associated with the free-trade equilibrium $C_1$, the shaded region of Figure 21.4 is associated with the free-trade equilibrium $C_2$, and the shaded region of Figure 21.5 is associated with the free-trade equilibrium $C_3$. Evidently, the three shaded regions are not disjoint; they overlap, so that point $e$ might lie in as many as three shaded regions. But, however that may be, our proposition survives: Given $e$ in a shaded region and any Pareto-optimal and Pareto-improving point $P$, there exists a new tariff pair, with one member positive, the other negative, that is compatible with the world

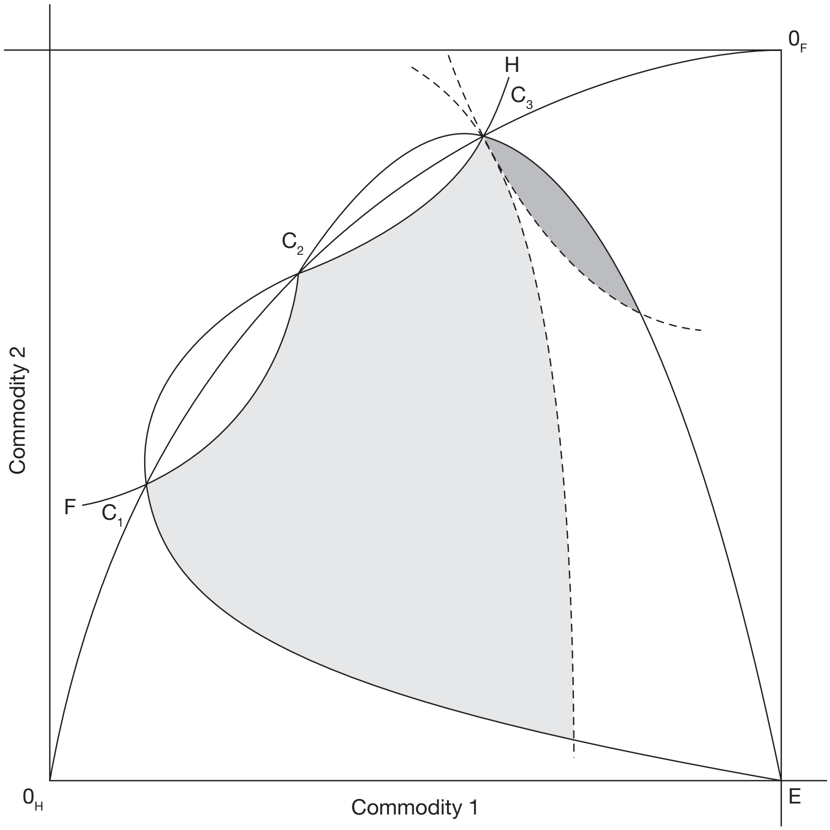


*Figure 21.5*

equilibrium at $P$; and, given $e$ in a non-shaded region, there exists a Pareto-optimal and Pareto-improving free-trade point $C_j$.

Of course, the mere adoption of a tariff pair compatible with equilibrium at $P$ does not ensure that the world economy will settle at that point. And the mere adoption of free trade does not ensure that the economy will settle at a Pareto-optimal and Pareto-improving point $C_i$; it might settle at point $C_j (j \neq i)$, which is Pareto-optimal but not Pareto-improving. In other words, $C_i$ need not be replicable.

Finally, it has been assumed that all tariffs are imposed on the imported commodity and are initially non-negative. Neither assumption is logically required; they have been adopted for simplicity only. As the reader may easily verify, the initial equilibrium point $e$ may lie in any of the four quarters of Figure 21.1; and any change in the commodity to be taxed by a country will change the sign of the tax.

Thus we arrive at our proposition.

*Proposition 21.1* Suppose that each of two economies produces and trades in two final goods subject to tariffs on its imported or exported goods. The tariffs may be positive or negative; some, but not all, may be zero; collectively, they may be prohibitive. Given any initial tariff-ridden equilibrium e, there exists a non-empty set $\Lambda$ (e) of feasible allocations which are Pareto-optimal and Pareto-improving. Any member $\lambda$ of $\Lambda$ is supportable by (i) an (e, $\lambda$)-dependent pair of tariffs and (ii) an information-parsimonious GMG scheme of lump-sum compensation in each country. If $\Lambda$ (e) contains the free-trade allocation, then that allocation can be supported by free trade; all other allocations in $\Lambda$ (e) can be supported by pairs of tariffs, each pair with one member positive, the other negative.

## 21.4 A final remark

We have focused on several fundamental questions associated with tariff reform. All of the questions have been handled in terms of the conventional two-by-two theory of international trade. The same questions could have been posed in the broader context of *m* countries and *n* commodities without changing our main conclusions – that a free-trade agreement is not generally Pareto-improving and that a Pareto-improving and Pareto-optimal outcome generally requires that, in some countries, some imports be subsidized or some exports be taxed. However, in the broader context there is a new possibility – that a Pareto-improving and Pareto-optimal reform is not available. This possibility is discussed in Kemp and Wan (2005). It is there shown that Mayer's proposition can be extended to accommodate any number of commodities but breaks down if there are more than two countries.

# 22 On the existence of equivalent tariff vectors

## When the status quo matters[1]

### 22.1 Introduction

Consider a competitive world economy, free trading and with no non-tariff market distortions, but supporting an arbitrary feasible system of international lump-sum transfers. It has long been known (at least to some) that, in the simplest case in which just two countries trade in just two commodities, with each country exporting one commodity, the non-distorting transfers can be replaced by a pair of individually distorting but collectively *equivalent import duties*, one positive and the other negative; see Mayer (1981: 142).[2]

This is a remarkable finding. For it seems to provide governments with alternative lump-sum and non-lump-sum means of efficiently redistributing the world's income. In tandem with the Second Welfare Theorem, it seems to imply that a Pareto-improving reform of the world's tariffs is always available and that any improvement in technologies or endowments, in whatever country, can be converted into a worldwide Pareto-improvement by first abandoning any initial tariffs and then cooperatively adopting a programme of international aid or a matrix of transfer-equivalent tariffs. It even suggests that there may be a sound theoretical case for agricultural subsidies, so typical of the period after the Second World War.

But is the proposition valid for more ample world economies? Many have cited Mayer's result in the course of broad discussions of coordinated tariff reform under the GATT, leaving the impression that generalization is possible and implementability not a problem; see, for example, Bagwell and Staiger (2002: Chapter 2). Moreover, Nakanishi (1991) and, later, Turunen-Red and Woodland (2001) have already provided sufficient conditions for generalization; and in each paper it is suggested that those conditions are acceptable as a basis for policy formation, implying implementability. Thus Nakanishi holds that 'tariffs should be (re)considered as a policy instrument not only for [the] efficiency of international resource allocation but also for international equity' (1991: 95); and Turunen-Red and Woodland repeatedly describe their assumptions as 'mild'.

However Nakanishi requires that countries and commodities be equal in number and that, in any competitive world equilibrium, *either* each country imports a single commodity, specific to that country, and exports all other commodities *or* each country exports a single commodity, specific to that country, and imports all other commodities, so that each country specializes as an importer or as an exporter. The assumptions are analytically convenient, allowing appeal to the properties of matrices with dominant diagonals; but they are, as Nakanishi understands, very special. However, the second assumption merits our special attention, for it is an *ex post* restriction on the world market equilibrium rather than an *ex ante* or pre-market restriction on the specification of the world economy in terms of endowments, technologies and preferences. Turunen-Red and Woodland also impose a mixture of *ex ante* and *ex post* restrictions; in particular, they restrict the matrix of competitive equilibrium net exports. Thus both Nakanishi and Turunen-Red and Woodland impose *ex post* restrictions on the world market equilibrium instead of relying in the customary way on *ex ante* restrictions alone.

A decisive objection to *ex post* restrictions is that they do not assist policy-makers, either in understanding their own unilateral tariff reforms or in negotiating international reforms. For, in the absence of detailed knowledge of preferences and technologies, to verify that any particular *ex post* restrictions are satisfied one must first introduce the reforms, a time-consuming and potentially costly experimental procedure not accommodated in the models of Nakanishi and Turunen-Red and Woodland.[3] Existence therefore remains an open question, inviting further study in terms of strictly *ex ante* conditions.

Our further purposes are two-fold. First, we provide a fully specified three-by-three example. The example has no extraordinary features; in particular, it is subject to no *ex post* restrictions. It is shown that, for this example, there are no equivalent import duties, that in this respect the example is robust and that therefore the existence of equivalent import duties cannot be taken for granted. Second, and more briefly, we emphasize some of the difficulties in implementing equivalent tariffs even when they exist, whether in the two-by-two case or in more general cases and whether they are supported by *ex ante* restrictions or by *ex post* restrictions. Specifically, we focus on the difficulties of implementation in a context of multiple world equilibria. These findings together with our earlier discussion of *ex ante* and *ex post* restrictions, suggest that Mayer's result cannot be developed into a useful tool of policy.

Analytically, whether price concessions and grants-in-aid are equivalent means of realizing Paretian allocations is a non-trivial issue. In Appendix 22.1 it is shown that the robustness of our non-equivalence example is rooted in the non-negativity of equilibrium world relative prices.

## 22.2 A counter-example based exclusively on *ex ante* restrictions

Consider a pure-exchange world economy containing three countries, *A, B* and *C*, and three commodities, *x* (grain), *y* (oil), and *z* (gas), with the worldwide endowment of each commodity equated to unity by an appropriate choice of units and with grain chosen as the numeraire. Let **E** denote the world endowment matrix, component $E_{ij}$ indicating the amount of the *j*th commodity with which the *i*th country is endowed.

Now suppose that an initial 'no trade' or autarkic equilibrium gives way to a free-trade equilibrium. Let $\mathbf{r} = (1, p, q)$ denote the vector of equilibrium world prices under free trade, let

$$\mathbf{D} = \begin{pmatrix} x_A & y_A & z_A \\ x_B & y_B & z_B \\ x_C & y_C & z_C \end{pmatrix}$$

denote the matrix of equilibrium world consumption under free trade, and let $\mathbf{e}$=(1,1,1) denote the unit row vector in three dimensions. Evidently

$$(\mathbf{D} - \mathbf{E})\mathbf{r} = \mathbf{0} \text{ (balance of payments)} \tag{1}$$

and

$$\mathbf{e}(\mathbf{D} - \mathbf{E}) = \mathbf{0} \text{ (market clearance)} \tag{2}$$

Let us at this point introduce a particular assumption concerning preferences.

*Assumption 22.1* Whatever their countries of residence, all individuals share the same CES (constant elasticity of substitution) utility index:

$$u_i = (x_i^{1/s} + y_i^{1/s} + z_i^{1/s})^s \quad s \geq 1;\ i = A, B, C \tag{3}$$

We note for later use that $u_i$ is *y-z symmetric* in the sense that, for any $x_i$, $u_i(x_i, y_i, z_i) = u_i(x_i, z_i, y_i)$.

*Lemma 22.1* Any worldwide Pareto optimum is associated with a consumption matrix

$$\mathbf{D} = \begin{pmatrix} a\boldsymbol{e} \\ b\boldsymbol{e} \\ (1-a-b)\boldsymbol{e} \end{pmatrix} \quad (a, b, 1-a-b > 0) \tag{4}$$

where

$$\mathbf{e}(\mathbf{D} - \mathbf{E}) = \mathbf{0} \tag{2}$$

and

$$u_A/a = u_B/b = u_C/(1 - a - b) \tag{5}$$

and with a shadow price vector

$$\mathbf{r} = \mathbf{e}' \tag{6}$$

where the prime indicates the transpose.

*Proof* Straightforward, given Assumption 22.1.

Let us now replace the 'no trade' initial equilibrium with a 'distorted trade' initial equilibrium, with the distortions provided by import and/or export duties imposed by each of the three countries. The equilibrium consumption matrix $\mathbf{D}^0$ is compatible with market clearing:

$$\mathbf{e}(\mathbf{D}^0 - \mathbf{E}) = \mathbf{0} \tag{7}$$

However $\mathbf{D}^0$ is not necessarily Pareto-optimal.

*Lemma 22.2*[4] There exists a Pareto-optimal *transfer-based free-trade equilibrium* with a consumption matrix

$$\mathbf{D}^* = (a^*\mathbf{e}, b^*\mathbf{e}, (1 - a^* - b^*)\mathbf{e}) \tag{8a}$$

where

$$\mathbf{e}(\mathbf{D}^* - \mathbf{E}) = \mathbf{0}, \tag{8b}$$

with supporting prices $\mathbf{r} = \mathbf{e}'$ and with a balance of payments vector

$$(\mathbf{D}^* - \mathbf{E})\mathbf{e}' = (\mathbf{D}^0 - \mathbf{E})\mathbf{e}' \tag{9}$$

Moreover, the system of transfers $(\mathbf{D}^0 - \mathbf{E})\mathbf{e}'$ is self-financing:

$$\mathbf{e}(\mathbf{D}^0 - \mathbf{E})\mathbf{e}' = \mathbf{0} \tag{10}$$

*Proof* Consider the *equivalent exchange economy* with the endowment matrix $\mathbf{D}^0$. Under free trade, the equilibrium consumption matrix of such an economy is $\mathbf{D}^*$ with equilibrium prices $\mathbf{r} = \mathbf{e}'$. To establish that economy and that equilibrium, it is required only that the transfer vector be set equal to $(\mathbf{D}^0 - \mathbf{E})\mathbf{e}'$.

By routine computation,

*Lemma 22.3* $a^* = (x_A^0 + y_A^0 + z_A^0)/3$ and $b^* = (x_B^0 + y_B^0 + z_B^0)/3$.

We can now approach the central question: Given the 'distorted trade' initial equilibrium, with the ordered pair $(\mathbf{D}^0, \mathbf{E})$, can one find a *tariff-based equilibrium* equivalent to the *transfer-based equilibrium* $(\mathbf{D}^*, \mathbf{D}^0)$ already discussed. More specifically, (i) given a matrix $\mathbf{D}^*$ satisfying (9), does there exist a post-tariff world price vector $\mathbf{r}^* \geq 0$ such that $(\mathbf{D}^* - \mathbf{E})\mathbf{r}^* = \mathbf{0}$?

(ii) If not, is non-existence a problem only in 'highly special' circumstances? Question (i) can be resolved by means of a single example of non-existence. Question (ii) is more difficult to resolve. However one can at least dispel the belief that non-existence requires a disparity in the numbers of goods and countries or extreme endowment proportions. Moreover, we can show that any counter-example is robust: If there is no equivalent tariff at a particular configuration of endowments, then, in general, the same is true after an infinitesimal variation of the configuration.

For our counter-example, we further specify the world endowment matrix.

*Assumption 22.2* The world endowment matrix takes the form

$$\mathbf{E} = \begin{pmatrix} 0 & v & v+w \\ 0 & 1-v & 1-v-w \\ 1 & 0 & 0 \end{pmatrix}$$

where $1 > v > 0$ and $-v < w < 1 - v$.

When $w = 0$, $\mathbf{E}$ is $y$-$z$ symmetric, in the sense that each country has equal endowments of $y$ and $z$; in that case the matrix is singular. If $w \neq 0$, $\mathbf{E}$ is non-singular. In Figure 22.1, points $\mathbf{E}$ and $\mathbf{E}'$ represent non-singular and singular matrices respectively.

*Assumption 22.3* Individuals in country A seek maximum utility as a cartel, but individuals in countries B and C are price-takers.

Readers may be left uneasy by this assumption: If individuals in *A* collaborate in the pursuit of maximum utility, why do not individuals in *B* and *C* do likewise? The assumption might be defended in terms of international disparities of information or in terms of historical accident. However the assumption can be justified, without going outside the model, simply by specifying that in *A* all individuals are identical, know themselves to be so, and therefore, by the familiar reasoning of Kemp and Shimomura (1995), behave cooperatively. Individuals in *B* and *C* remain price takers because, while they are *representative consumers* with the same CES preferences, either they have endowments that differ both within and across countries and therefore are not completely *representative agents* or they are indeed representative agents but are unaware of their status. Finally, we note without proof that the asymmetrical Assumption 22.3 is not needed. Even if, symmetrically, *each* country is inhabited by representative agents who are aware of their status, so that the initial tariff-distorted trading equilibrium is the outcome of a Johnsonian tariff war, it still can be shown that equivalent tariffs are not generally available.

We next apply the method of Lagrange to a special case of the endowment matrix, to derive a unique numerical solution; and then demonstrate by a continuity argument the robustness of the solution to small changes in the matrix.

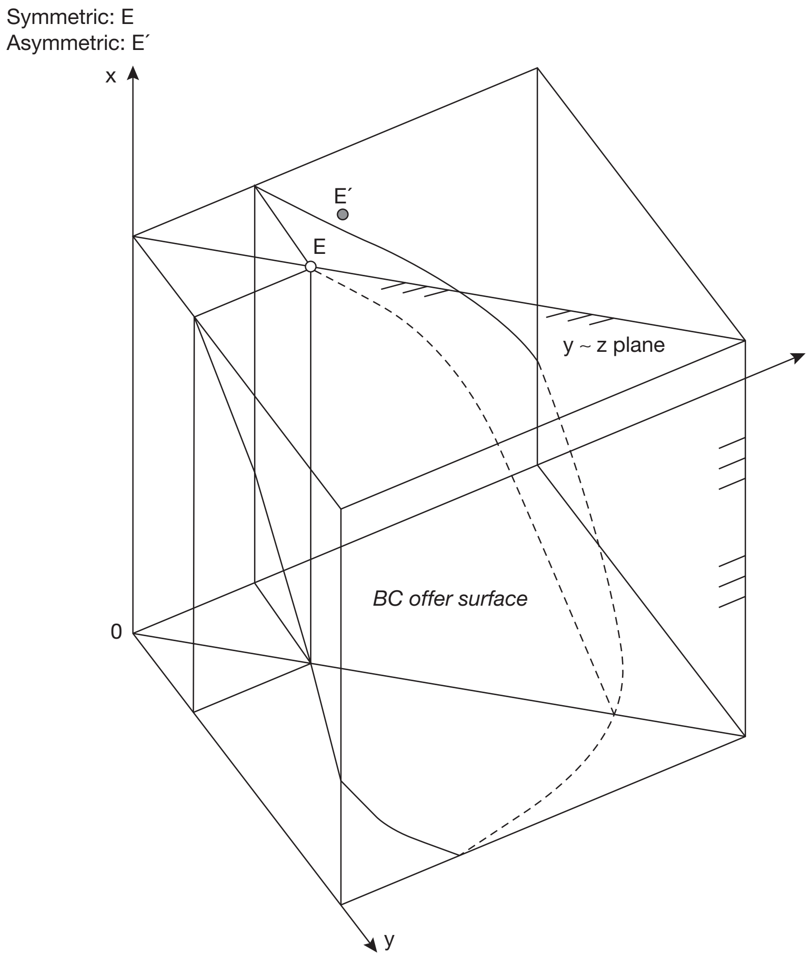


*Figure 22.1*

*Lemma 22.4* The collective behaviour of countries B and C is determined as the solution to the problem

$$(P_{BC}) \max u = (x^{1/s} + y^{1/s} + z^{1/s})s$$

subject to

$$x + py + qz = 1 + (1 - v)(p + q) - qw$$

where the right-hand side of the constraint is the value of their collective endowment vector $(1, 1–v, 1–v–w)$.

*Proof* Obvious.

For ($P_{BC}$), the Lagrangean is

$$L_{BC} = (x^{1/s} + y^{1/s} + z^{1/s})^s + \mu\{(x + py + qz) - [1 + (1 - v)(p + q)] + qw\} \quad (11)$$

and the first-order conditions are

$$\partial L_{BC} / \partial x = (x^{1/s} + y^{1/s} + z^{1/s})^{s-1} x^{(1-s)/s} + \mu = 0 \quad (12a)$$

$$\partial L_{BC} / \partial y = (x^{1/s} + y^{1/s} + z^{1/s})^{s-1} y^{(1-s)/s} + \mu p = 0 \quad (12b)$$

$$\partial L_{BC} / \partial z = (x^{1/s} + y^{1/s} + z^{1/s})^{s-1} z^{(1-s)/s} + \mu q = 0 \quad (12c)$$

$$\partial L_{BC} / \partial \mu = (x + py + qz) - [1 + (1 - v)(p + q)] + qw = 0 \quad (12d)$$

From (12a)–(12c),

$$p = (x / y)^{(s-1)/s}, \; q = (x / z)^{(s-1)/s}, \; q / p = (y / z)^{(s-1)/s} \quad (13)$$

which, when substituted into (12d), yields the locus of consumption possibilities for $B$ and $C$:

$$x + (x / y)^{(s-1)/s}(y - 1 + v) + (x / z)^{(s-1)/s}(z - 1 + v + w) - 1 = 0 \quad (14a)$$

Bearing in mind that the net import vector for $B$ and $C$ is ($x$–1, $y$–1+$v$, $z$–1+$v$+$w$), (14a) also yields the offer locus of $B$ and $C$.

Given our objective (a counter-example to the general existence of equivalent tariffs), the choice of $s$ is critical. If $s = 1$, $A$ would opt for free trade; and, if $s \geq 2$, $A$ would threaten to cut off all trade and in fact would obtain nearly all trade gain. Let us therefore set $s = 3/2$, so that (14a) reduces to

$$x + (x / y)^{1/3}(y - 1 + v) + (x / z)^{1/3}(z - 1 + v + w) - 1 = 0 \quad (14b)$$

Since

$$x_A = 1 - x, \quad y_A = 1 - y \quad \text{and} \quad z_A = 1 - z, \quad (15)$$

the values of $x$, $y$ and $z$ are determined by the solution to $A$'s problem:

($P_A$) max $u_A = [(1 - x)^{2/3} + (1 - y)^{2/3} + (1 - z)^{2/3}]^{3/2}$

subject to (14b).

Let $F(x, y, z; v, w) \equiv x + (x / y)^{1/3}(y - 1 + v) + (x / z)^{1/3}(z - 1 + v + w) - 1$

Then, at any unique interior solution to $(P_A)$, the use of the Lagrangean method and the subsequent elimination of the multipliers yield the three first-order necessary conditions:

$$F(x, y, z; v, w) = 0 \tag{14c}$$

$$(\partial F / \partial x) / (\partial F / \partial y) = [(1-y)/(1-x)]^{1/3} \tag{14d}$$

$$(\partial F / \partial x) / (\partial F / \partial z) = [(1-z)/(1-x)]^{1/3} \tag{14e}$$

From the smoothness of these conditions and the Implicit Function Theorem, any sufficiently small changes in the parameters $v$ and $w$ induce continuous changes in the values of the three variables $x$, $y$ and $z$ and, by Lemma 22.3, in $a^*$ and $b^*$.

To complete the counter-example, we now offer a convenient specific case and show numerically that $(P_A)$ has a unique, interior solution.

*Assumption 22.4* $(v, w) = (1/2, 0)$, so that

$$\mathbf{E} = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \end{pmatrix}$$

Given Assumption 22.4, the offer locus is $y - z$ *symmetric* in the sense that if at prices $(1, p, q)$ the amounts demanded are $(x, y, z)$ then at prices $(1, q, p)$ the amounts demanded are $(x, z, y)$, $B$ and $C$ enjoying the same utility before and after the change in prices. Moreover, equation (14b) can be depicted in Figure 22.1 as a surface symmetric to the plane $y = z$.

*Lemma 22.5* Given Assumptions 22.1–22.4, $(P_A)$ has a solution. Moreover, in the solution, $y_A = z_A$; hence $y = z$.

*Proof* Existence follows from the symmetry of goods $y$ and $z$ and from the strict quasi-concavity of $u_A$ when $s > 1$. Suppose that the solution is asymmetric, that is, that $(y, z) = (h', h'')$, $h' \neq h''$. Then, by $y$-$z$ symmetry, $(h', h'')$ is also a solution, implying in turn that both solutions are inferior to $((h' + h'')/2, (h' + h'')/2)$, a contradiction.

Bearing in mind Lemma 22.5, we now form the Lagrangean

$$L_A = [(1-x)^{2/3} + 2(1-y)^{2/3}]^{3/2} + m[x + 2(x/y)^{1/3}(y - 1/2) - 1]$$

The first-order conditions necessary for a maximum are

$$[(1-x)^{2/3} + 2(1-y)^{2/3}]^{1/2}(1-x)^{-1/3} = m[1 + (2/3)(x/y)^{-2/3}(y - 1/2)/y] \tag{16a}$$

$$[(1-x)^{2/3}+2(1-y)^{2/3}]^{1/2}(1-y)^{-1/3}=m(x/y)^{1/3}[1-(1/3)(y-1/2)/y] \tag{16b}$$

Thus, deleting the multiplier from (16) and recalling (14b) with $y = z$, we obtain:

*Lemma 22.6* The optimal values of $x$ and $y$ satisfy

$$[(1-y)/(1-x)]^{1/3}=[1+(2/3)(x/y)^{-2/3}(y-1/2)/y]/\{(x/y)^{1/3}[1-(1/3)(y-1/2)/y]\}$$

$$1-x=2(x/y)^{1/3}(y-1/2)$$

Moreover, the equilibrium price and incomes are

$$p=(x/y)^{1/3};$$

$$I_A=(1-x)+2(x/y)^{1/3}(1-y),\ I_B=(x/y)^{1/3},\ I_C=1;$$

$$a=(1-x)+2(1-y),\ b=(x+2y)I_B/(I_B+I_C),\ c=(x+2y)I_C/(I_B+I_C)$$

*Proof* Straightforward, bearing in mind (13) and Lemma 22.5.

As our final step, we establish the relationship between the aggregate demands of $B$ and $C$ and the given price $p$. To this end, we return to (12), the first-order conditions for $(P_{BC})$. With $s = 3/2$, and $y = z$, those conditions reduce to

$$(x^{2/3}+2y^{2/3})^{1/2}x^{-1/3}+\mu=0 \tag{17a}$$

$$(x^{2/3}+2y^{2/3})^{1/2}y^{-1/3}+\mu p=0 \tag{17b}$$

$$x+2p(y-1/2)-1=0 \tag{17c}$$

or, after the elimination of $\mu$, to

$$x-p^3y=0 \tag{18a}$$

$$x+2py=p+1 \tag{18b}$$

Solving for $x$ and $y$, we obtain:

*Lemma 22.7* $x = [p^3 + p^2]/(p^2 + 2) > 0$ and $y = [p + 1]/[p(p^2 + 2)] > 0$.

Thus, by substitution,

$$\begin{aligned} u_A &= [(1-x)^{2/3}+2(1-y)^{2/3}]^{3/2} \\ &= \{[1-(1+p)p^2/(p^2+2)]^{2/3}+2[1-(1+p)/(p(p^2+2))]^{2/3}\}^{3/2} \end{aligned}$$

$$= \{[p(2-p^3)]^{2/3} + 2[p^3 + p - 1]^{2/3}\}^{3/2} / [p(p^2+2)]$$

$$\equiv U(p) \tag{19}$$

As shown graphically by Figure 22.2, $U$ is concave in $p$ over the range $p = [0.68, 1.26]$, with country $A$ losing all control at each limit of the range. Thus, at 0.68, the positive root of the equation $p^3 + p - 1 = 0$, countries $B$ and $C$ would satisfy their own reciprocal wants and $A$ would be priced out of the market; and, at 1.26, the positive root of $p^3 - 2 = 0$, country $A$ would exhaust its entire endowment in exports, leaving nothing for consumption. The unique and interior optimal monopoly price for $A$ is $p = 1.06$, which exceeds the free trade price, $p = 1$.

It is now straightforward to compute the *domestic* prices facing individuals in country $A$. These are $(1, p^A, p^A)$, where

$$\begin{aligned} p_A &= [(1-x)/(1-y)]^{1/3} \\ &= 0.877 \\ &\neq 1.06 \end{aligned} \tag{20}$$

Thus we arrive at:

*Lemma 22.8* For our example, in which $v = 1/2$ and $w = 0$, there is a unique tariff-distorted trading equilibrium. In that equilibrium, $u_A < u_B$.

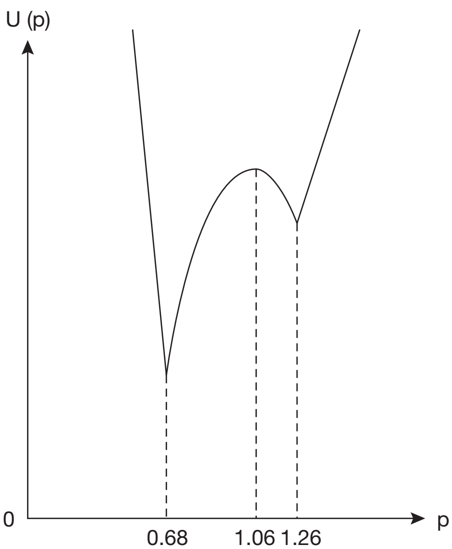


*Figure 22.2*

The commonsense of Lemma 2.8 emerges when it is recalled that while $A$ and $B$ have the same endowments and trade at the same world prices, $A$'s consumers trade at tariff-distorted prices. The monopolist $A$ suffers a distortion loss but the free-rider $B$ does not.

*Lemma 22.9* Corresponding to the tariff-distorted trading equilibrium there is a Pareto-optimal, transfer-based free-trade equilibrium with $a^* < b^*$, that is, $1 > a^*/b^*$.

*Proof* The lemma follows from Lemmas 22.2 and 22.8.

*Proposition 22.1* For our example, in the special case $(v, w) = (1/2, 0)$, there is no tariff-based equilibrium equivalent to the transfer-based free-trade equilibrium.

*Proof* From (19), (20) and the free-trade equilibrium price vector $\mathbf{e}$, we can compute the consumption matrices

$$\mathbf{D}^0 = \begin{pmatrix} 0.259 & 0.378 & 0.378 \\ 0.382 & 0.320 & 0.320 \\ 0.359 & 0.302 & 0.302 \end{pmatrix}$$

$$\mathbf{D}^* = \begin{pmatrix} 0.338 & 0.338 & 0.338 \\ 0.341 & 0.341 & 0.341 \\ 0.321 & 0.321 & 0.321 \end{pmatrix}$$

where all entries are rounded to the third decimal place. Introducing the scaling vector $\lambda = (1, -1, 0)$, we denote the differential endowment ($A$ over $B$) by $\lambda\mathbf{E}$ and the differential consumption ($A$ over $B$) in the tariff-ridden initial equilibrium and in the transfer-based free-trade equilibrium by $\lambda\mathbf{D}^0$ and $\lambda\mathbf{D}^*$, respectively. In the present case, $\lambda\mathbf{E}$ is the null row vector and $\lambda\mathbf{D}^*$ is a strictly negative vector. Hence $\lambda(\mathbf{E} - \mathbf{D}^*)$ is a strictly positive vector; moreover, correction for the rounding errors would not reverse this inequality. On the other hand, the existence of equivalent tariffs implies that, for some semi-positive price vector $\mathbf{r}^*$, $(\mathbf{E} - \mathbf{D}^*)\mathbf{r}^* = \mathbf{0}$, contradicting the strict positivity of $\lambda(\mathbf{E} - \mathbf{D}^*)$.

To this point we have relied on Assumption 22.4, which requires that $w = 0$. We now abandon that assumption, retaining all other assumptions, and replace $\mathbf{E}$ with

$$\mathbf{E} + \Delta\mathbf{E} = \begin{pmatrix} 0 & (1/2) & (1/2)+w \\ 0 & (1/2) & (1/2)-w \\ 1 & 0 & 0 \end{pmatrix}, \; w > 0$$

We already know that, for $v = 1/2$ and $w = 0$, there exists a unique, interior and tariff-ridden initial equilibrium such that $u_A < u_B$. We now note that, within a small neighbourhood of $(v, w) = (1/2, 0)$, there is a unique solution, a smooth function of $v$ and $w$. This allows us to invoke the Implicit Function Theorem and infer the robustness of our conclusion for sufficiently small values of $|v - (1/2)|$ and $w$. Replacing $\mathbf{D}^0$ and $\mathbf{D}^*$ with $\mathbf{D}^0 + \Delta\mathbf{D}^0$ and $\mathbf{D}^* + \Delta\mathbf{D}^*$, we find that $\lambda(\mathbf{E} + \Delta\mathbf{E})$ is now semi-positive while $\lambda(\mathbf{D}^* + \Delta\mathbf{D}^*)$ remains strictly negative. From this point, the assumption that equivalent tariffs exist can be shown to be self-contradictory, as in the proof of Proposition 22.1.

*Proposition 22.2* Proposition 22.1 is robust in the sense that it remains valid for all sufficiently small changes in the distribution of oil or gas between countries A and B.

We draw attention to the fact that $w$ has been restricted to be positive. Without that restriction we could not have shown that our counter-example is robust. A demonstration may be found in Appendix 22.1.

## 22.3 Implementability

We have shown that if reliance is placed on *ex ante* restrictions only, then Mayer's proposition cannot be extended beyond the two-by-two case. That does not mean that equivalent tariffs exist only in the two-by-two case. However, even when equivalent tariffs exist, whether in the two-by-two case or in more ample cases, policy-makers may be unable to replicate the free-trade, transfer-based equilibrium from which the tariffs have been calculated. For the equivalent tariffs might support several world equilibria, some of which do not even lie on the world contract locus. In these circumstances, the mere announcement of equivalent tariffs does not ensure the replication of the free-trade transfer-based equilibrium for which the tariffs have been tailored. Possibly for that reason, Mayer (1981) and later authors (such as Nakanishi 1991, Turunen-Red and Woodland 2001 and Bagwell and Staiger 2002) have simply assumed uniqueness.[5]

Of course, no useful discussion of implementability can be conducted in the context of purely static models such as those of Nakanishi and Turunen-Red and Woodland. One must move to a dynamic model and seek to extract from it a time line along which are arrayed the steps that governments might take in implementing an equivalent tariff vector.[6]

## 22.4 Final remark

We must give up any thought of developing an exact equivalence between lump-sum and non-lump-sum (or tariff-based) means of redistribution under customary (Arrow-Debreu) specifications of the world economy. However, the development of alternative non-lump-sum and non-equivalent means of redistribution, based on restrictions that are non-Arrow-Debreu but nevertheless *ex ante* and testable, remains an appropriate objective of academic research.

## Appendix 22.1 The positivity of $w$

Consider the endowment matrix

$$\mathbf{E}=\begin{pmatrix} 0 & v & v+w \\ 0 & v & v+w \\ 1 & 0 & 0 \end{pmatrix}$$

where, for the time being, we place no restriction on the sign of $w$; and consider an initial tariff-distorted equilibrium with consumption matrix

$$\mathbf{D}^0=\begin{pmatrix} x_A & y_A & z_A \\ x_B & y_B & z_B \\ 1-x_A-x_B & 1-y_A-y_B & 1-z_A-z_B \end{pmatrix}$$

If there exists an equivalent tariff vector, then there exists also a semi-positive price vector $\mathbf{r}^* = (1\ p^*\ q^*)$ such that

$$(\mathbf{E}-\mathbf{D}^0)\mathbf{r}^*=\mathbf{0}$$

At most, only two of the conditions are independent. One of the conditions (the balance of payments of $C$) can be written as

$$\underset{\text{(Imports of } C)}{p^*(1-y_A-y_B)+q^*(1-Z_A-Z_B)}=\underset{\text{(Exports of } C)}{a^*+b^*}$$

so that, in view of Lemma 22.1,

$$p^*(1-a^*-b^*)+q^*(1-a^*-b^*)=a^*+b^*$$

or

$$p^*=(a^*+b^*)/(1-a^*-b^*)-q^* \tag{A1}$$

If there is a second independent condition, it can be obtained by subtracting the balance of payments of $B$ from that of $A$ and again recalling Lemma 22.1:

$$(a^*-b^*)+p^*(a^*-b^*)+q^*(a^*-b^*)=2wq^* \tag{A2}$$

whence, substituting for $p^*$ from (A1), solving for $q^*$ and recalling that $1 – a^* – b^* > 0$,

$$q^* = (a^* - b^*)/[2w(1 - a^* - b^*)] \tag{A3}$$

For sufficiently small and positive $w$, $a^* – b^* < 0$, implying that $q^* < 0$, in contradiction of the hypothesis that equivalent tariffs exist. On the other hand, if $w < 0$, $q^* > 0$ and no contradiction emerges.

# Part IV

# Methodology

# 23 The representative agent in economic theory[1]

In many branches of economic theory, notably in public economics, international economics, labour economics and industrial organization, it is conventional to assume that all members of some class of agents are identical in their preferences and endowments (including information).[2] Each member of such a class may be said to 'represent' the class. The convention is especially prominent in analyses that purport to be general-equilibrium in scope, for it allows appeal to the properties of the substitution matrix of a representative household and to the properties of the production set of a representative firm.

However, invariably, and always implicitly, the assumption that members of a class of agents are representative is supported by the companion assumption that the members are unaware that they are identical. What appears to have been overlooked is that if each agent in a class is like every other agent of the class, if all agents of the class know this to be so, and if in any choice situation each agent's preferred alternative is unique, then each agent will make its choices on the understanding that all other agents in the class will make the same choices. More important, no agent in the class will choose its strategy on the assumption that the strategies of all other agents in the class are given; that is, the Nash equilibrium concept must be abandoned. Aware of their representativeness, members of the class will resolve their coordination problem by entering into an enforceable agreement to each choose the group's optimal strategy.[3] In effect, any equilibrium will reflect the cooperative behaviour of members of the class. However, even if non-members of the class are also utility maximizers, the decisions of the representative agents will not necessarily be optimal for society as a whole. Indeed only in the limiting case in which the set of representative agents contains all agents will the chosen alternative be globally or socially optimal. In that limiting case there is no need of intervention by a benevolent government, even when the economy is characterized by externalities, increasing returns, and/or public goods.

In a multi-country world, it is implausible to assume that all agents, worldwide, are representative. In such a world, the limiting case is more usefully defined as the case in which, *within each country*, all agents are identical. If, in *that* limiting case, each agent is aware of its status, then, in *world* markets, each country will be aware of and exercise its market power; hence the world equilibrium will be inefficient.

Thus the simplifying assumption that all agents in a class are identical, combined with the assumption that the representativeness of each agent is known to every agent in the class, undermines much of existing economic analysis, whether descriptive or normative. Most of existing analysis is based on the assumption of perfectly competitive behaviour. For that analysis to be plausible, it is necessary that each agent feels, in some measure, independent of other agents. If, by assumption, agents are identical, and know it, then conventional analysis is irrelevant and must be replaced by new analysis based on cooperative behaviour.

Of course, I do not believe that models embodying a representative agent are realistic. However, I do hold that if a representative agent is assumed, then all implications of the assumption must be recognized.

It has been suggested to me that the models with price taking representative agents are justified if it is desired to abstract from the consequence of differences between agents in order to focus on other issues. However, to assume that all members of a particular group are identical is to introduce a *new* complication, namely, the incentive for members of the group to cooperate. To ignore that complication is to defend the use of internally inconsistent models.

It has also been suggested that the companion assumption, that all representative agents are aware of their representative status, is unrealistic. This objection is more plausible, at least in some contexts. However, in the contexts of infinite-horizon development and indefinitely repeated games, and given the uniqueness of each agent's optimal choice in each choice situation, agents must soon become aware of their representativeness. For many purposes, therefore, it is appropriate to begin analysis at a point in time at which awareness is already complete.

# 24 Price taking in general equilibrium[1]

## 24.1 Introduction

For eighty years the Walrasian theory of general equilibrium suffered from a serious deficiency. It lacked an existence proposition. That deficiency was removed fifty years ago by the appearance of two remarkable papers, Arrow and Debreu (1954) and McKenzie (1954), and on those two papers most of us have since relied in our ventures into general equilibrium.[2] Nowadays, however, the Arrow-Debreu and McKenzie papers are thought by some to be passé, mainly because of their assumption (inherited from Walras [1874]) of non-strategic price taking behaviour on the part of households and firms. On the other hand, it is not entirely clear why that assumption might now be unacceptable. The pioneering authors (Walras, Arrow, Debreu and McKenzie) simply assumed price taking, without justification or apology. Modern texts do address the issue, but without complete clarity. For example, Mas-Colell *et al.* (1995: 315) content themselves with the vague observations that '. . . if market participants' desired trades are small relative to the size of the market, then they will have little incentive to depart from market prices. Thus, *in a suitably defined equilibrium*, they will act *approximately* like price takers' (italics added). Each sentence lacks precision and proof.

Our first purpose in the present note is to explain why the assumption of price taking behaviour might be found to be unacceptable. The explanation proceeds by establishing that price taking by households implies that each household is incompletely rational and/or incompletely informed about the economy of which it is part. It then follows that the assumption of price taking is unacceptable if incomplete information and incomplete rationality are unacceptable. Our point is not that the assumption of price taking is unrealistic in some sense. Nor do we suggest that the 1954 papers are logically defective. We suggest only that, when combined with other assumptions common to the two papers, the assumption of price taking implies incomplete information or incomplete rationality.

Our second purpose is to demonstrate, paradoxically perhaps, that if the Arrow-Debreu-McKenzie assumption of price taking is validated by the recognition of ignorance and/or irrationality, then their existence propositions, remain intact as do the two fundamental welfare propositions for competitive economies.

Throughout, our analysis will focus on the Arrow-Debreu model. It might have been restricted to McKenzie's model, with the same outcome.

## 24.2 Analysis

In any Arrow-Debreu (1954) economy:

(i) Households and dated commodities (endowments and outputs) are finite in number.
(ii) Each household conceives of itself as a price-taker in all markets.
(iii) Each household seeks to maximize its own utility.
(iv) The production set of each firm is convex.
(v) The endowment vector of each household is finite and lies in the interior of its consumption set.

However, if households are finite in number and if the endowment vector of each household lies in the interior of its consumption set, then, in any equilibrium and in every market, each household possesses market power, directly and/or through firms in which it owns shares. That is, given the equilibrium net offers of all other households, any change in the net offer of household *j* would disturb the set of market-clearing relative price vectors. Arrow and Debreu place virtually no restrictions on the distribution of endowments over households. Hence the extent of household *j*'s market power might be considerable. Or it might be very small; but it cannot be zero for any finite population.

Thus far, we are on familiar ground; see, for example, Romer (1986: 1016). However, matters cannot be left there, for it immediately follows from the foregoing argument that, if it is perfectly informed and rational in the double sense that it seeks to maximize its own utility *and can appreciate that* (*i*) *and* (*v*) *imply market power*, then household *j* cannot in equilibrium conceive of itself as a price taker in every market.[3] Thus we can state our first proposition.

*Proposition 24.1* If the Arrow-Debreu model is internally consistent then each household must be incompletely informed and/or incompletely rational.[4]

This suggests that the Arrow-Debreu analysis rests on an implicit understanding – that households are unaware that the economy is finite and/or that they are incompletely rational in the sense that they cannot appreciate that (i) and (v) imply market power. Without that understanding, assumptions (i)–(v) would be mutually inconsistent, with implications clearly spelled out by Debreu (1991: 2):

Being denied a sufficiently secure experimental base, economic theory has to adhere to the rules of logical discourse and must renounce the facility of internal inconsistency. A deductive structure that tolerates a contradiction does so under the penalty of being useless, since any statement can be derived flawlessly and immediately from that contradiction.

With that understanding and paradoxically, the familiar existence theorems and the fundamental welfare propositions remain intact.

*Proposition 24.2* If households are unaware that the economy is finite in number and/or they are incompletely rational in the sense that they cannot appreciate that assumptions (i) and (v) imply market power, then existence is assured and the two fundamental welfare propositions of competitive economies remain intact.

Some post-1954 writers have sought to counter the problem of market power by assuming that all households are domestically price takers but some firms are price makers in some markets; see Gabszewicz and Vial (1972), Roberts and Sonnenschein (1977), Hart (1985) and Stahn (1999). However all firms are owned, ultimately, by households; and, given the convexity of production sets – assumption (iv) – there is no reason why a firm cannot be owned by a single household. Why then should a household that is aware of its market power as a shareholder forget its power when it buys the household's groceries or sells the household's labour or other primary factors? To that question there appears to be no answer.

Others have sought to eliminate market power by assuming that the set of households forms a continuum of price taking agents; see, for example, Aumann (1964, 1966) and the later developments of Aumann's ideas by Gabszewicz and Mertens (1971) and Shitovitz (1973). To take that path, however, is effectively to assume away the problem posed in the present paper.

Some readers might feel that a place can be found in the Arrow-Debreu model for learning about market power. That is not so; for, in the Arrow-Debreu world, markets need open only once.

## 24.3 Final remarks

We have focused on the uncertainty-free 1954 article of Arrow and Debreu. As is well known, their model can be extended to accommodate uncertainty while retaining the assumption of price taking and while remaining finite in scope; see, for example, Arrow (1953) and Debreu (1959). We note that the more general models obtained in this way remain subject to Propositions 24.1 and 24.2.

Bertrand Russell (1946: 637) has remarked that '[n]o one has succeeded in inventing a philosophy at once credible and self-consistent'.[5] We have suggested that, for the consistency of the Arrow-Debreu and McKenzie models of competitive general equilibrium, it is necessary that, incredibly, *each* household is incompletely informed and/or incompletely rational. By way of contrast, we note that, for the consistency of oligopolistic general equilibrium, it is necessary that *some* but not all households be incompletely informed and/or incompletely rational; thus, without at least one price taking consumer in its market, it is impossible to define a Cournot oligopolist's market power.

# 25 Generality versus tractability

Normative trade theory has developed in a general competitive setting of Walras-Arrow-Debreu-McKenzie (WADM) type, where generality is reckoned in terms of the potentially large numbers of products and primary factors recognized, in terms of the weakness of the restrictions placed on the relationships in WADM models, in terms of the ease with which (time, place) subscripts accommodate imperfect product and factor mobility, and in other ways. The generality of the WADM models has not prevented the derivation of many propositions of broad scope dealing with the potential benefits of free trade to individual trading nations or with the potential Pareto improvements associated with particular types of free trade associations (including customs unions); see, for example, Kemp and Wan (1972, 1976), Kemp and Shimomura (2001b) and Kemp (2005). Moreover, in recent years, the focus of normative trade theory has drifted away from perfectly competitive to oligopolistic structures but without serious diminution in the generality of the models employed or of the conclusions derived; see, for example, the two Kemp-Shimomura (K-S) papers of 2001 (2001a, 2001c).

However, the generality of the WADM models has undoubtedly hindered attempts to deal with other policy issues – notably, issues involving the partial revision of protectionist tax structures – and this has persuaded many trade theorists that, on occasion, they must be prepared to sacrifice generality and, for a class of issues, to work with a single special but 'sufficiently realistic' model. This point of view was clearly expressed many years ago by Ivor Pearce (1970: 17): 'Our purpose in short is to bring the reader not to the point where he understands a great many models but to the point where he understands a great deal about one model.'

More recently, Peter Neary (2003a: 246), with a finite oligopolistic world in mind, has suggested that '[i]f we want to answer real-world questions, we must trade off generality for tractability'. Evidently this suggestion, like that of Pearce, rests on a confident belief that awkward issues can be resolved in terms of a single model that lacks generality but remains 'sufficiently realistic'.[1] Others confidently employ monopolistically-competitive general-equilibrium models based on quadratic or CES utility functions; see, for example, Avinash Dixit and Joseph Stiglitz (1977) and Neary (2003b). Still others rely on economic geography, the modern forms of which rest on very special utility and production functions; see, for example, Masahisa Fujita *et al.* (1999).

When we impose special restrictions on a model, however, we incur a responsibility to indicate how the solutions derived from the restricted model respond to alternative relaxations of the restrictions. At the very least, we should re-solve the model under a substantial variety of alternative restrictions. Until that responsibility has been discharged, we will not have convincingly answered any 'real-world questions'.

It might be suggested that, as long as the special restrictions are empirically supported, so that the restricted model 'fits the data', the model may yet serve as a basis for policy formation. However, such a defence is fragile, for each of several restricted models might fit its model-specific data equally well, precisely because the sets of exogenous or explanatory variables are highly correlated. A government guided by a particular restricted model might then find that its policies are grossly counter-effective.

Let us now pull these thoughts together in three tentative precepts:

(1) There are many national and international policy issues that cannot be resolved in terms of the general WADM or K-S models.
(2) To resolve these awkward issues trade theorists must resort to restricted versions of the WADM and K-S models. The restrictions may apply to the dimensions of those models or to the mathematical form of the model relationships or to both.
(3) Reliance should never be placed on a single restricted model.

Pearce and Neary (and many other economists) seem to have rejected the third precept.

# Notes

## 1 The Torrens-Ricardo Principle

1 We have linked Torrens with Ricardo, just as we might have linked Walras with Arrow, Debreu and McKenzie. As Jacob Hollander (1911) pointed out long ago, Torrens did not achieve a complete statement of the Principle. However, Torrens did perceive one of the most striking implications of the Principle – that a free-trading country might import a commodity in the production of which it has an absolute advantage and that if it did so, then it would benefit from doing so. Moreover Torrens' *Corn Trade* (1815) appeared two years before Ricardo's *Principles* (1817); and it appears from Ricardo's correspondence with Malthus that Ricardo arrived at the Principle only in October 1816. (On the latter point, see Ruffin 2002.) Ricardo therefore had ample time to absorb Torrens' contribution before completing his own. Nevertheless, whether Ricardo used the time to read *Corn Trade* remains unknown. Hollander also noted that Torrens failed to state a vital assumption – that factors of production are internationally immobile. However, Torrens was writing during the last years of the Napoleonic Wars. It is therefore not surprising and perhaps forgivable that he was not explicit on the subject of factor mobility.

2 This assumption is only implicit in Torrens and Ricardo and, indeed, in most textbook presentations.

3 The Torrens-Ricardo Principle does not accommodate non-tradable commodities. In fact, however, the Principle (as well as the extensions derived in sections 1.3 and 1.4) remain valid whatever the number of non-tradable commodities. Thus suppose that there are three commodities (wine, cloth and housing). In autarky, none of the commodities is traded internationally; under free trade, wine and cloth are traded, but housing is not traded. Then in only one small detail does our analysis need modification: when the world price of wine in terms of cloth moves away from its autarkic level in a country, the relative price of housing and the labour devoted to housing might change; hence the labour available to the other industries might also change. In contrast, under Torrens-Ricardo assumptions, the labour available for cloth and wine production is always constant. Even if some non-tradable commodities are indispensable inputs to the production of some non-tradable commodities, the Principle and its extensions remain valid whatever the number of non-tradable commodities.

4 It remains true that the equilibrium world price ratio must lie between the two equilibrium autarkic price ratios. However, the latter are no longer uniquely determined; each takes any value in the set determined by the relevant ratio of marginal labour costs and the relevant marginal rate of substitution in consumption. Thus the bounds of the equilibrium world price ratio implied by equilibrium autarkic price ratios are fuzzy and less informative than those imposed by the autarkic marginal rates of substitution.

5 Similar remarks apply to the relatively uninformative inner products (correlations) derived by Deardorff (1980) and by Dixit and Norman (1980).

## 2 Gottfried Haberler's Principle of Comparative Advantage

1 This representation can be justified by the assumption that all households are identical in all respects but are unaware of the fact or by introducing a family of post-compensation Scitovsky indifference curves.

2 Figure 2.3 is drawn on the assumption that $MRT_j^A \neq MRS_j^A$, $j = E, P$. If that assumption is put aside, we return to the familiar textbook case in which the offer curves are free of kinks so that, if only $MRT_E^A \neq MRT_P^A$, trade and gains from trade are assured.

## 4 Production and trade patterns under uncertainty

1 In a valuable pioneering paper, Brainard and Cooper (1968) have studied the implications of price uncertainty for the patterns of trade and production of a small country of the Heckscher-Ohlin type. Their paper differs from ours not only in its assumptions about production but also in taking price uncertainty as given, without relating it to the underlying randomness of preferences, technology or factor endowments, and in its reliance on quadratic utility functions and the mean-variance analysis of choice under uncertainty.

2 Conclusions of this type could have been obtained simply by confronting the small country with randomly fluctuating prices, without inquiring into the source of the randomness. However, while this partial-equilibrium approach would have spared us some tedious calculation, it would have left the job half done. Moreover, to display a world trading equilibrium under conditions of uncertainty seemed in itself to be a useful exercise. Finally, we wished to preserve symmetry in our treatments of spot and futures markets.

3 See Arrow (1953) and Debreu (1959).

## 5 International trade without autarkic equilibria

1 The introductory material of Sections 5.1 and 5.2 is based on Kemp (2003a).

2 As noted in Section 5.1, the positioning of the subsistence social indifference curve depends both on the total population and on the age distribution of the population. One might argue that the subsistence curves of families are generated by a single homothetic function, differing from each other only in scale, so that any inter-family differences in preferences are revealed only by supra-subsistence indifference curves. However, that refinement is not needed here.

3 Of course, the *identification* of the membership of any particular feasible club would require a vast amount of information about each of the $n$ economies, information not usually available.

4 Other components of the Principle presuppose an autarkic equilibrium in each country; for details, see Kemp and Okawa (2006).

## 6 Impoverishing technical and preferential improvements

1 Both Mill and Edgeworth explicitly allowed for two or more primary factors; the assumptions of factor-neutral technical improvements and non-inferior consumption goods were implicit in their analyses. Formally, Edgeworth (1894a) confined his attention to the limiting case in which the progressive country is, under free trade, completely specialized in producing the exported commodity and in consuming the imported commodity. Informally, in each of his articles,

he considered other cases. Johnson (1955) and Bhagwati (1958) later extended Edgeworth's formal analysis to accommodate incomplete specialization of both consumption and production.

2 Another segment of the English offer curve is confined to the third quadrant of Figure 6.2; see Kemp (2003a). However, that segment plays no role in our present analysis.

## 7 A dynamic Heckscher–Ohlin model

1 We acknowledge with gratitude the helpful comments of an anonymous referee, Ngo Van Long, Alan Woodland and Masatoshi Yamada.

2 The sole descriptive comparative static proposition states that the set of equilibria is independent of the means of price normalization.

3 The costs of reallocation may be of many and diverse types ranging from the price of a one-way train ticket to the psychological costs of adjusting to new workplace and social cultures and of the weakening of family ties. On the other hand, reallocation does not always imply physical movement. This is obvious in the case of land but true also of labour.

4 The function $G$ has as its arguments produced commodities only. However, the production of those commodities requires inputs of primary factors.

5 Samuelson's proposition may be found in a brief comment on a paper of Leontief's; see Leontief (1936) and Samuelson (1947; p. 29n). Leontief's model contained just two commodities. However, Samuelson's proposition was later shown to be valid for any number of traded commodities; see Safra (1983). And, building on Safra, it can be shown that the proposition is valid for any number $k$ ($k > 2$) of produced commodities even if only $k'$ ($2 \leq k' < k$) of them are tradable on world markets.

6 The notation $\lim\limits_{x \to +a}$ and $\lim\limits_{x \to -a}$ denote the right and left limits, respectively.

## 8 A second correspondence principle

1 It may be useful to recall three pioneering studies of the costly reallocation of a single primary factor of production. Mussa (1978) analyses the dynamic process of costly movement of a factor service. It is assumed that capital moves between two industries with the aid of labour input and that the mobile factor moves towards the industry in which it is relatively well paid. Thus, workers are employed in three industries. Therefore, if we consider that not only capital but also labour is mobile, keeping the above two assumptions intact, then there may arise a situation where labour is best paid in two industries so that the industry into which it flows becomes equivocal. On the other hand, our approach, expressed in (8.4), of which a direct predecessor is Kemp and Wan (1974), is free of such a technical difficulty. Furthermore, Mussa (1978) is concerned with the dynamic analysis of a costly adjustment process and has little to say about the implications of his dynamic analysis for the comparative static propositions of the Heckscher-Ohlin models. Long (1978) also studies the costly regional reallocation of labour based on the maximization of the sum of the discounted future utilities. However, Long (1978) was not interested in the robustness of the Heckscher-Ohlin comparative statics.

## 9 A theory of involuntary unrequited international transfers

1 We thank an anonymous referee for valuable comments on an earlier version of this paper.

2 It is already well known that, in a context of factor market distortions, transfer paradoxes are possible without bystanders (see Wang 1985). The novelty of our

result is that it relies on international utility externalities that, unlike factor market distortions, are inseparable from the problem posed.

3 The same conclusion would emerge if Assumption 9.2 were weakened so that, for some $j$ and $k$, $j \neq k$, $\partial E^j/\partial u^k = 0$.

**10 A reply to Carlos da Costa**

1 We are grateful to Carlos da Costa for his helpful comments on earlier drafts.

2 Towards the end of his paper, da Costa goes beyond our intentions in exploring the normative implications for each country of the strategic internalization of consumption externalities. The game is played by two representative agents, one from each country and each fully aware that the members of his constituency are identical. However to endow the two players with such information is incompatible with da Costa's earlier assumption that all agents are completely unaware that they are identical. In the present response we focus on that part of da Costa's comment that is directed to the questions posed in our own paper.

3 The assumption of representative households is widespread, as is the assumption of price taking. However, the assumptions are mutually compatible only if the households are unaware that they are identical. For a detailed discussion, see Kemp and Shimomura (1995).

**11 A theory of voluntary unrequited international transfers**

1 A positively sloped locus is commonplace in a context of externalities or other distortions. In the present context, the externalities vanish only in the singular event that $k^\alpha k^\beta = 1$ and $u^\alpha = k^\alpha k^\beta$ (hence $u^\beta = k^\beta k^\alpha$). It follows that, at most, one point on the locus is Pareto-optimal. Such a singular point must lie on a negatively sloped section of the locus.

**12 Aid tied to the donor's exports**

1 In the singular case in which $\Delta = 0$, the system may be locally stable or unstable. The outcome then depends on the non-linear terms in the expansion of the functions in (1)–(3) about the equilibrium point.

2 The principle of second best goes back to Samuelson (1947: 252–3) and to Boiteux (1956); see also Lipsey and Lancaster (1956).

**13 Variable returns to scale and factor price equalization**

1 The present chapter is companion to Kemp *et al.* (1998). In that paper it was shown that the dimensionality of the set of national factor endowments compatible with factor price equalization is independent of market structure. In the present chapter it is shown that, under specified conditions, dimensionality is independent of scale returns.

2 For readers unfamiliar with this construction, we offer a brief explanation. Any point $P$ in the Edgeworth box can be interpreted as an assignment of primary factors to $\alpha$ and $\beta$, with the endowment of $\alpha$ indicated by the vector $O_\alpha P$ and the endowment of $\beta$ by the vector $O_\beta P$. If, as in Figure 13.1, $P$ lies in the parallelogram $O_\alpha E O_\beta E'$ then the integrated world equilibrium can be replicated without international factor mobility. In that unintegrated world equilibrium, $\alpha$ produces the proportion $(O_\alpha E_\alpha)/(O_\alpha E)$ [respectively, $(O_\alpha E'_\beta)/(O_\alpha E')$] of the world output of the first [respectively, the second] commodity, and $\beta$ produces the balance.

3 If the public goods are not traded on international markets, they may be privately *or* publicly provided.

## 14 Market structure and factor price equalization

1 We acknowledge with gratitude the useful comments of Kar-yiu Wong and William Schworm.
2 Dixit and Norman (1980) and Helpman and Krugman (1985) are admirable exceptions. However, neither text takes up our theme that the existing theory of FPE can be reinterpreted to accommodate any mixture of perfectly and imperfectly competitive product markets.
3 The question of likelihood will be briefly addressed in Section 14.4.
4 In Figure 14.1 and in later figures, we ignore the fact that oligopolists come only in integral numbers, just as we conventionally ignore the imperfect divisibility of other primary factors.
5 The extension of FPE theory to accommodate any number of trading partners has been undertaken by Deardorff (1994).

## 15 Factor price equalization when the world equilibrium is not unique

1 The existence of equilibrium is assured by the well-known sufficient conditions of Arrow and Debreu (1954) and McKenzie (1959).
2 Throughout the paper it is assumed, as is customary, that production is non-joint. However, the logic of the FPE theorem is independent of the jointness or non-jointness of production; see McKenzie (1955) and Chang *et al.* (1980). Similarly, only the details of the present analysis would change if joint production were allowed.
3 My manner of formulating the FPE theorem can be traced to the pioneering work of Lancaster (1957) and Travis (1964). Their approach has been adopted by Dixit and Norman (1980) and by Kemp and Okawa (1998) and Kemp *et al.* (1998).
4 In Figure 15.4, $O_\alpha E_1$ is the input vector of industry 1, $E_1E_2$ is the input vector of industry 2, and $E_2O_\beta$ is the input vector of industry 3.

## 16 Factor price equalization in a world of many trading countries

1 This term is introduced to differentiate the industry-specific equilibrium factor ratios from the economy-wide factor endowment ratios.
2 Deardorff also suggested that his condition is necessary for worldwide FPE. In his supporting argument, however, the condition is buttressed by additional specifications that are in fact unnecessary. Thus among the additional specifications one finds (i) constant-returns technologies and (ii) freedom of entry and exit which, in turn, suffice for (iii) perfect competition. In contrast, it is now known that, if there are just two countries, FPE is possible in a context of externality-based increasing returns and strategic market behaviour; indeed FPE is just as likely (in a specific sense) as in the conventional Lerner-Samuelson setting. See Kemp and Okawa (1998) and Kemp *et al.* (1998).

## 17 Heckscher-Ohlin theory

1 A brief preliminary version of this paper appeared in the *Kobe Economic and Business Review* 46: 1–3. I am grateful to Professor Junichi Goto, the present editor of the *Review*, for his ready permission that I rework some of the earlier material. I am also grateful to audiences at Kobe University, Nanyang Technological University and the National University of Singapore for their lively comments on the two papers.

2 One thinks here of the well-known propositions concerning the gainfulness of free trade for individual trading countries and concerning the existence of Pareto-improving free trade associations.

3 These remarks are developed in greater detail and generality in Kemp (2003a).

4 For a detailed demonstration of the Stolper-Samuelson version of the result, see Kemp and Shimomura (2003a).

## 19 Recent challenges to the classical gains-from-trade proposition

1 We acknowledge with gratitude the useful comments of Kar-yiu Wong and William Schworm.

2 There exist formally similar propositions in which the disturbance is an improvement in technology or an increase in the endowment of factors.

## 20 Trade gains

1 I acknowledge with gratitude the helpful comments of Geoffrey Fishburn, Ngo Van Long and John Zerby.

2 It can now be added, in 2006, that the 1972 propositions remain valid if each individual displays a 'love of variety', as emphasized in the modern theory of monopolistic competition; see, for example, Jean-Pascal Benassy (1996). They remain valid also if primary and other factors of production are imperfectly mobile between occupations.

Less obviously, the 1972 propositions remain valid if to conventional budget constraints are added the time constraints introduced by Hermann Heinrich Gossen (1854). To see that this is so, recall that in the conventional theory of consumer demand each individual is assumed to consume each commodity at a steady rate during each period, with no allowance for joint consumption with other individuals (waltzing and tennis), for externalities generated by consumption or for the individuals' need of alternating variety in consumption. The task of individual $i$ is to maximize utility subject to an income constraint:

$$\max\ c^i\ u^i\ (c^i)$$
$$\text{s.t. } pc^i \leq y^i$$

where $c^i$ is the consumption vector of $i$, $p$ a given price vector and $y^i$ the given income of $i$. Let us now modify the conventional theory by recognizing that consumption takes time and that the time available for consumption is limited. The task of individual $i$ is then to find

$$\max_{c^i}\ v^i\ (c^i, t^i)$$
$$\text{s.t. } pc^i \leq y^i$$

where $t^i$ is the time available to $i$. If, for each individual $i$, $v^i$ viewed as a function of $c^i$ has properties similar to those of $u^i(c^i)$, then the admission of time constraints has no bearing on the existence of a free-trade equilibrium or on the gainfulness of free trade. This remains true even when waltzing and tennis are admitted, and even when consumption is lumpy and variable through time, but not when consumption externalities are recognized.

3 Section 20.2 draws on the earlier analysis of Kemp and Shimomura (2005).

4 Going beyond the perspectives of Arrow and Debreu, we can consider a sequence of Arrow-Debreu games, each played over an unchanging finite period and in each of which households are not less well informed and not less rational than in the preceding game. That is, unanticipated learning is possible. If, after a finite number of plays one or more households, in whatever country, are completely informed, completely rational and therefore well aware of their market power,

the dynamic game comes to an end. The gainfulness of trade is assured, but only for a finite period.

5 The 'optimality-*destroying* public-goods nature of families (each family has two pairs of grandparents)' was noted long ago by Kemp and Long (1982: 206, n. 3), in a context of closed economies.

Here I make no attempt to quantify the potential misallocation of resources. However, readers will recall the estimation by Laurence Kotlikoff and Lawrence Summers (1981) that intergenerational transfers account for as much as two-thirds of aggregate US capital accumulation. On the *scope* for misallocation, they might also consult William Gale and John Scholz (1994) and Michael Hurd and James Smith (2002). On the other hand, the *degree* of misallocation depends also on the type of game played by the in-laws, and that in turn depends on their willingness to divulge the terms of their wills *inter vivos*.

6 It is easy enough to construct static Cournot-Nash examples in which both pairs of in-laws strategically reduce their bequests. This suggests (but does not imply) that their strategic behaviour will be associated with a reduction in their rates of saving.

7 Commenting on the US, Gordon Tullock (2002: 107) has noted that '[a]mong black families more than half of the children are fatherless'.

8 This may be an appropriate point at which to remark on the scope of the 'Ricardian equivalence theorem', discussed in recent times by Martin Bailey (1962), Robert Barro (1974) and many others. That proposition presupposes not that there are single-parent families but that the two pairs of parents-in-law are identical in all respects, are aware of that fact and therefore cooperate to choose a joint bequest that maximizes their joint welfare; for a more detailed argument, see Kemp and Shimomura (1995).

## 21 Tariff reform

1 We acknowledge with gratitude the helpful comments of Chen Kang and Michihiro Ohyama.

2 The GATT itself is silent on this question. In his illuminating paper (Ohyama 2002: 72), Michihiro Ohyama makes the more moderate claim that 'the basic rules of the GATT/WTO are economically meaningful and useful for creating freer trade'. However, he interprets the 'reciprocal and mutually advantageous arrangements' of the preamble to the GATT quite strictly, in terms of mutual tariff reductions that hold relative world prices at their initial values. He points out that, if all tariffs are initially positive and remain positive at all stages of the negotiations, such reductions leave all negotiating countries better off. Evidently reductions constrained in this way can never yield a Pareto optimum. We are grateful to Professor Ohyama for his clarifying remarks on this point.

3 In general models, which accommodate any number of countries and any number of commodities, tariffs can play a fiscal role only under special assumptions; see Kemp and Wan (2005).

## 22 On the existence of equivalent tariff vectors

1 We acknowledge with gratitude the helpful remarks of Noritsuga Nakanishi, pioneer in the study of equivalent tariffs, and of Koji Shimomura and Kenji Fujiwara.

2 Mayer fails to note the necessity of ruling out transfer-ridden world equilibria in which the recipient country imports both commodities.

3 This objection applies equally to an earlier paper co-authored by one of the present authors; see Kemp and Negishi (1970).

4 More general forms of Lemma 22.2 may be found in Kemp and Wan (1993, 1999).

5 In a more recent paper, on the joint reform of commercial and environmental policy, Turunen-Red and Woodland (2004) have again relied on the availability and implementability of equivalent import duties; accordingly, they have again imposed *ex post* restrictions on the matrix of competitive equilibrium net exports and have again assumed that the world equilibrium is unique. Their new paper is therefore subject to reservations similar to those noted in the present paper.

6 In an overlapping-generations context, the construction of such a line has been examined by Wan (1997) and Kemp and Wan (1999).

## 23 The representative agent in economic theory

1 For an alternative, more detailed exposition, the reader may consult Kemp and Shimomura (1995).

2 The assumption is almost as old as economic theory. For example, it is implicit in the discussions of trade gains by Adam Smith and David Ricardo. However it was only after the First World War that its use became widespread and systematic.

3 In a repetitive context, an agreement is enforceable if it forever excludes any participant who once deviates from the agreement.

## 24 Price taking in general equilibrium

1 We acknowledge with gratitude the helpful comments of M. Ali Khan, Ngo Van Long and Peter Slezak.

2 We focus on the best known of the four pioneering papers which, together, resolved the existence problem. The less well known papers are Gale (1955) and Nikaido (1956).

3 Romer (1986: 1016) is content to work in terms of ‘the usual approximation for a large but finite number of agents’.

4 Irrationality as here defined should not be confused with bounded rationality in the sense of Herbert Simon. The latter refers to the *optimality* of restricting the time- and resource-using search for a solution to a problem. It can be applied only to *dynamic* models of choice, not to *static* models of Arrow-Debreu and McKenzie type.

5 Russell added: ‘Locke aimed at credibility, and achieved it at the expense of consistency. Most of the great philosophers have done the opposite. A philosophy which is not self consistent cannot be wholly true, but a philosophy which is self-consistent can very well be wholly false. The most fruitful philosophies have contained glaring inconsistencies, but for that very reason have been partially true. There is no reason to suppose that a self-consistent system contains more truth than one which, like Locke’s, is obviously more or less wrong.’

## 25 Generality versus tractability

1 In an earlier chapter, Neary and Ronald Jones (Jones and Neary 1984: 2–3) were more explicit: ‘Positive trade theory uses a variety of models, each one suited to a limited but still important range of questions.’ However, in that chapter, the authors were focused on positive trade theory. It is not clear whether they believed that their statement is valid also for normative trade theory.

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# Index